CH 3: Differentiation Rules

3.1 Derivatives of Polynomials and Exponential Functions

1. $\frac{d}{dx}f(x)$ is another notation for $f'(x)$ that points out the variable you must take the derivative with respect to.

2. constants: $\frac{d}{dx}(\text{constant}) = 0$

3. the **power rule**: $\frac{d}{dx}(x^n) = nx^{n-1}$, for all real numbers n, particularly $\frac{d}{dx}(x) = 1$

4. note that the power rule is a shortcut in finding the slope of the tangent line of the curve x^n

5. the **sum rule**: $\frac{d}{dx}f(x) + \frac{d}{dx}g(x) = \frac{d}{dx}(f(x) + g(x))$. Example: $\frac{d}{dx}(x^7 + x) = 7x^6 + 1$

6. the **difference rule**: $\frac{d}{dx}f(x) - \frac{d}{dx}g(x) = \frac{d}{dx}(f(x) - g(x))$. Example: $\frac{d}{dx}(\sqrt{x} - 3) = \frac{1}{2}x^{-\frac{1}{2}} - 0 = \frac{1}{2\sqrt{x}}$

7. the **constant multiple rule**: $\frac{d}{dx}(c \cdot f(x)) = c\frac{d}{dx}f(x)$. Example: $\frac{d}{dx}(3x^{-7}) = 3 \cdot (-7)x^{-6} = -21x^{-6}$

8. recall e! The special property that e has is that the function e^x has a tangent line with a slope of 1 at the point $x = 0$, i.e. if $f(x) = e^x$, then $f'(0) = \lim_{h \to 0} \frac{e^h - e^0}{h} = 1$

9. the derivative of the **natural exponential function**: $\frac{d}{dx}(e^x) = e^x$,

10. recall: the **secant line** to a curve is the line that goes through 2 points on the graph (if the graph represents distance, then its slope is the average speed between the two points)

11. recall: the **tangent line** to a curve is the line that touches the graph exactly once at that point (if the graph represents distance, then its slope is the instantaneous speed at that point)

12. we define the **normal line** to a curve is the line that is perpendicular to the tangent line (if we have a 3-dimensional surface, then we have a tangent plane that touches the surface at exactly one point, and the normal line is perpendicular to each line in this tangent plane and it helps find the equation of the plane – you'll see this in MA1115)

3.2 The product and the quotient rules

1. first note that $\frac{d}{dx}(f(x) \cdot g(x)) \neq \frac{d}{dx}f(x) \cdot \frac{d}{dx}g(x)$ (i.e. $(f \cdot g)' \neq f' \cdot g'$). Check this on $f(x) = x^3$ and $g(x) = x^{10}$

2. the **product rule**: $\frac{d}{dx}(f(x) \cdot g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$ (i.e. $(f \cdot g)' = f \cdot g' + f' \cdot g$).

3. similarly note that $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) \neq \dfrac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$ (i.e. $(\frac{f}{g})' \neq \frac{f'}{g'}$). Check this on $f(x) = x^3$ and $g(x) = x^{10}$

4. the **quotient rule**: $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{g^2(x)}$ (i.e. $(\frac{f}{g})' = \frac{f' \cdot g - f \cdot g'}{g^2}$)

3.3 Derivatives of the Trig functions

1. $(\sin x)' = \cos x$ $\qquad$ $(\sec x)' = \sec x \cdot \tan x$
2. $(\cos x)' = -\sin x$ $\qquad$ $(\csc x)' = -\csc x \cdot \cot x$
3. $(\tan x)' = \sec^2 x$ $\qquad$ $(\cot x)' = -\csc^2 x$
4. $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$

3.4 The Chain Rule

1. The chain rule: Let f and g be two functions. If g differentiable at x, and f is differentiable at $g(x)$, then the composition $F = f \circ g$ is defined and its derivative is

$$F'(x) = f'\Big(g(x)\Big) \cdot g'(x).$$

2. $(a^x)' = a^x \ln a$, for $a > 0$