Scaling factors in Resampling.

Note Title 5/25/2011

Define: average power

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \underbrace{\int_{-T}^{T} |x(t)|^2 \, dt}_{\text{energy}}$$

Define: RMS value:

$$\bar{x}_{RMS} = \sqrt{P_x}$$

RMS = Root Mean Square

Discrete Time

- $$P_x = \lim_{N \to \infty} \frac{1}{2N\cancel{T_s}} \sum_{m=-N}^{N-1} |x[m]|^2 \cancel{T_s}$$

- $$\bar{X}_{RMS} = \sqrt{P_x}$$

Downsampling

$$x[n] \longrightarrow \boxed{\downarrow D} \longrightarrow y[n] = x[nD]$$

Assume: no aliasing,

i.e.

$$X(\omega) = 0 \quad \text{when} \quad \frac{\pi}{D} < |\omega| \leq \pi$$

Recall:

$$Y(\omega) = \frac{1}{D} X\left(\frac{\omega}{D}\right) \qquad -\pi < \omega \leq \pi$$

Then: by Parseval's theorem

$$\sum_{m=-\infty}^{+\infty} |y[m]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |Y(\omega)|^2 d\omega =$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{1}{D} X\left(\frac{\omega}{D}\right)\right|^2 d\omega =$$

call $\lambda = \frac{\omega}{D}$, $\omega = D\lambda$

$$\sum_{m=-\infty}^{+\infty} |y[m]|^2 = \frac{1}{2\pi} \frac{1}{D^{\cancel{2}}} \cancel{D} \int_{-\frac{\pi}{D}}^{\frac{\pi}{D}} |X(\lambda)|^2 \, d\lambda$$

$$=$$

$$= \frac{1}{D} \sum_{m=-\infty}^{+\infty} |x[m]|^2$$

$$=$$

$$P_x \, 2ND$$

$$\uparrow$$

Consequence

$$P_y = \lim_{N \to +\infty} \frac{1}{2N} \sum_{m=-N}^{N-1} |y[m]|^2 = \frac{1}{2N} \frac{1}{D} \sum_{m=-ND}^{ND-1} |x[m]|^2 =$$

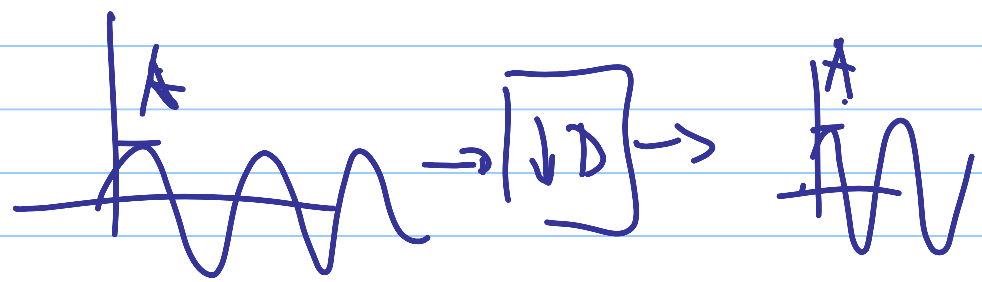
$$= P_x$$

$$x[n] \longrightarrow \boxed{\downarrow D} \longrightarrow y[n]$$

P_x $\qquad\qquad$ $P_y = P_x$

$\bar{x}_{RMS}$ $\qquad\qquad$ $\bar{y}_{RMS} = \bar{x}_{RMS}$

<u>Ex:</u>



Note Title 5/26/2011

Scaling in Upsampling

$$x[n] \longrightarrow \boxed{\uparrow M} \xrightarrow{v[n]} \boxed{H(\omega)} \longrightarrow y[n]$$

Problem: Relate $\bar{y}_{RMS}$ with $\bar{X}_{RMS}$

$$\bar{X}_{RMS} = \sqrt{\lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N-1} |x[n]|^2}$$

$$\sum_{m=-\infty}^{+\infty} |v[m]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

since all nonzero values of $v[m]$ are the same as $x[n]$

$\blacksquare$

$$\sum_{m=-\infty}^{+\infty} |y[m]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y(w)|^2 \, dw \qquad (1)$$

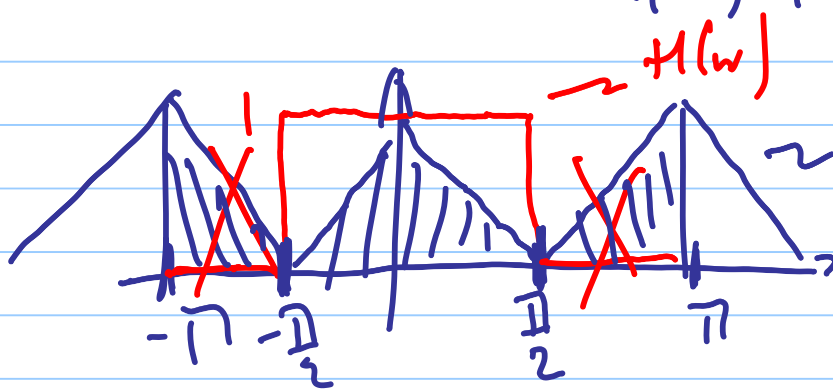
by Parseval's theorem.

But $Y(w) = H(w)\, V(w)$

$= H(w)\, X(Mw)$

Ex 1

$M = 2$



Then

$$\sum_{m=-\infty}^{+\infty} |y[m]|^2 = \frac{1}{2\pi}\int_{-\frac{\pi}{M}}^{\frac{\pi}{M}} |Y(\omega)|^2 \, d\omega =$$

$$= \frac{1}{M}\,\frac{1}{2\pi}\int_{-\pi}^{\pi} |V(\omega)|^2 \, d\omega$$

$$= \frac{1}{M}\sum_{m=-\infty}^{+\infty} |v[m]|^2 = \frac{1}{M}\sum_{n=-\infty}^{+\infty} |x[n]|^2$$

$$|\bar{y}_{RMS}|^2 = P_y$$

$$P_y = \lim_{N \to +\infty} \frac{1}{2N} \sum_{m=-N}^{N-1} |y[m]|^2 =$$

$$= \lim_{N \to +\infty} \frac{1}{2N} \frac{1}{M} \left(\underbrace{\sum_{m=-\frac{N}{M}}^{\frac{N}{M}-1} |x[m]|^2}_{P_x \frac{2N}{M}} \right)$$

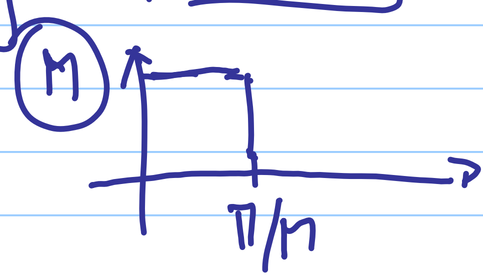
$$= \lim_{N \to \infty} \frac{1}{\cancel{2N}} \frac{1}{M} \frac{\cancel{2N}}{M} P_x = \frac{1}{M^2} P_x$$

$$\Rightarrow \quad P_y = \frac{1}{M^2} P_x$$

$$\boxed{\bar{y}_{RMS} = \frac{1}{M} \bar{x}_{RMS}}$$

If we want same RMS value

$$x[n] \longrightarrow \boxed{\uparrow M} \longrightarrow \boxed{H(w)} \longrightarrow y[n]$$



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