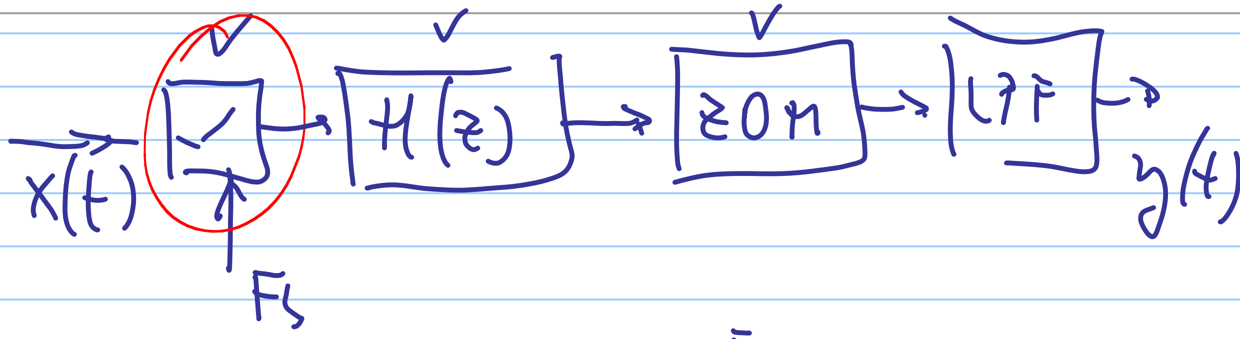
PROB. 2.5/2.6

Note Title 5/3/2011



$$Y(F) = \cancel{F_s}\, H(\omega)\Big|_{\omega = \frac{2\pi F}{F_s}} \cancel{T_s}\, e^{-j\pi \frac{F}{F_s}}\, \mathrm{sinc}\left(\frac{F}{F_s}\right) X(F)$$

(no aliasing)

$$Y(F) = \bar{H}(F)\, X(F)$$

Example:

if $X(F) = Ae^{j\alpha}\delta(F-F_0) + Ae^{-j\alpha}\delta(F+F_0)$

since no aliasing

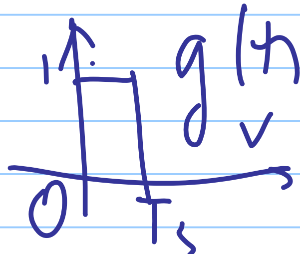
$$-\frac{F_s}{2} < F_0 < \frac{F_s}{2}$$

$$Y(F) = Ae^{j\alpha}\overline{H}(F_0)\,\delta(F-F_0) + $$
$$+ Ae^{-j\alpha}\overline{H}(-F_0)\,\delta(F+F_0)$$

$$y[n] \rightarrow \boxed{ZOH} \rightarrow y(t)$$

$$Y(F) = G(F)\, Y(\omega)\Big|_{\omega = 2\pi \frac{F}{F_s}}$$

$$G(F) = FT\{g(t)\}$$



$$g(t) \xrightarrow{FT} \underline{\underline{T_s \,\mathrm{sinc}\left(\frac{F}{F_s}\right) e^{-j\cancel{2}\pi \frac{F T_s}{\cancel{2}}}}}$$

$$\mathrm{rect}\left(\frac{t - \frac{T_s}{2}}{T_s}\right)$$

If no aliasing

$$x(t) \rightarrow \boxed{\quad} \rightarrow x[n]$$

$$X_{DTFT}(\omega)\Big|_{\omega = \frac{2\pi F}{F_s}} = F_s \, X_{FT}(F)$$

$$\sum_n x[n] e^{-j\omega n} \qquad \int x(t) e^{-j2\pi F t} \, dt$$

Example with aliasing

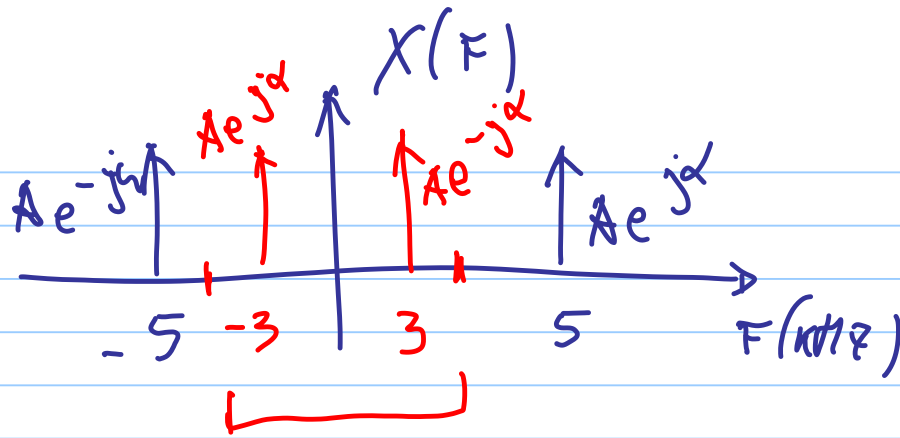
$$X(F) = Ae^{j\alpha}\delta(F-F_0) + Ae^{-j\alpha}\delta(F+F_0)$$

Ex:

$F_0 = 5\ kHz$ ✓

$F_s = 8\ kHz$ ✓

Step 1: find the aliased freq. F_1



$$X_1(F) = Ae^{-j\alpha}\delta(F-3000) + Ae^{j\alpha}\delta(F+3000)$$

the proceed as before

$$Y(F) = H(3000)\,Ae^{-j\alpha}\delta(F-3000) - \ldots$$

PROBLEM 3.2c

$$\text{DTFT}\{n\,x[n]\} = j\frac{d}{dw}X(w)$$

since:

$$j\frac{d}{dw}X(w) = j\frac{d}{dw}\sum_n x[n]e^{-jwn}$$

correct

$$= \underbrace{j(-j)}_{1}\underbrace{\sum_n n\,x[n]e^{-jwn}}_{\text{DTFT}\{n\,x[n]\}}$$

Ex: DTFT$\{ n \underbrace{0.8^n u[n]}_{x[n]} \} = ?$

$$X(\omega) = \text{DTFT}\{0.8^n u[n]\} =$$

$$= \frac{1}{1 - 0.8 e^{-j\omega}}$$

apply the property.

Properties of DFT

$$\mathrm{DFT}\{x[(n-m)_N]\} = e^{-j\frac{2\pi}{N}km} X[k]$$

for all $k = 0, \ldots, N-1$

Ex:

$$X = [1, j, 0, -j] \qquad N = 4$$

$$\text{DFT}\{x[(n-2)_4]\} = ?$$

$$= e^{-j\frac{\cancel{2}\pi}{4}k\cancel{2}} X[k], \quad k = 0, \ldots, 3$$

$$= (-1)^k X[k]$$

$$= [1, -j, 0, j] \quad //$$

PROBLEM 3.3d

$$x[n] = \cos(0.25\pi n), \quad n = 0, \ldots, 7$$

$$= \frac{1}{2} e^{j\frac{\pi}{4}n} + \frac{1}{2} e^{-j\frac{\pi}{4}n}$$

$$\text{DFT}\left\{e^{j\frac{\pi}{4}n}\right\} = \sum_{n=0}^{7} e^{j\frac{\pi}{4}n} e^{-jk\frac{2\pi}{8}n} =$$

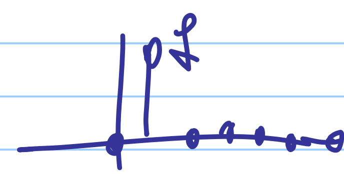
$$= \sum_{n=0}^{7} e^{j\frac{\pi}{4}(1-k)n} = (\text{geometric})$$

$$\sum_{n=0}^{7} \left(e^{j\frac{\pi}{4}(1-k)} \right)^{n} =$$

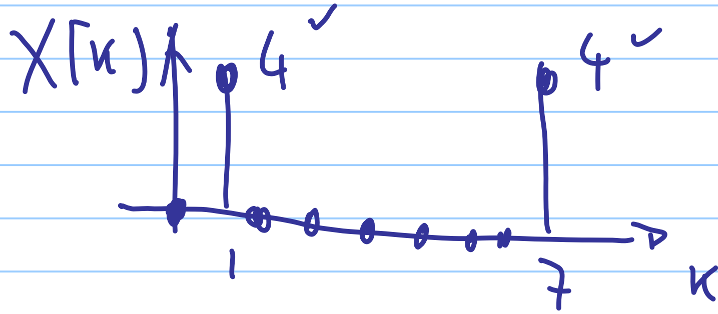
$$= \frac{1 - e^{j\cancel{\frac{\pi}{4}}(1-k)\cancel{8}\;2}}{1 - e^{j\frac{\pi}{4}(1-k)}} = 0, \quad k \neq 1$$

$$= \quad 8 \qquad \text{if} \quad k = 1$$

$$= 8\,\delta[k-1]$$



$$X[k] = \frac{8}{2}\,\delta[(k-1)_8] + \frac{8}{2}\,\delta[(k+1)_8]$$



PROBLEM 3.4 a, b

a) $x[(n-1)_4]$ — same

b) $x[(n+3)_4]$

Since

$$x[(n-1)_4] = x[(n+4-1)_4]$$
$$= x[(n+3)_4]$$

$$x[(n+3)_4] = x[(n-4+3)_4] = x[(n-1)_4]$$

In fact, using DFT:

$$DFT\{x[(n-1)_4]\} = e^{-jk\frac{2\pi}{4}} X[k]$$

$$DFT\{x[(n+3)_4]\} = e^{jk\frac{2\pi}{4}3} X[k]$$

same

Since

$$(-j)^k = j^{3k}$$

$$j^{3k} = j^{2k} j^k = (-1)^k j^k = (-j)^k$$

PROBLEM 2.4

$$y[n] \longrightarrow \boxed{ZOH} \longrightarrow y(t)$$

$$F_s = 4\,\text{kHz}$$

$$Y(F) \qquad -F_s < F < +F_s$$

Recall:

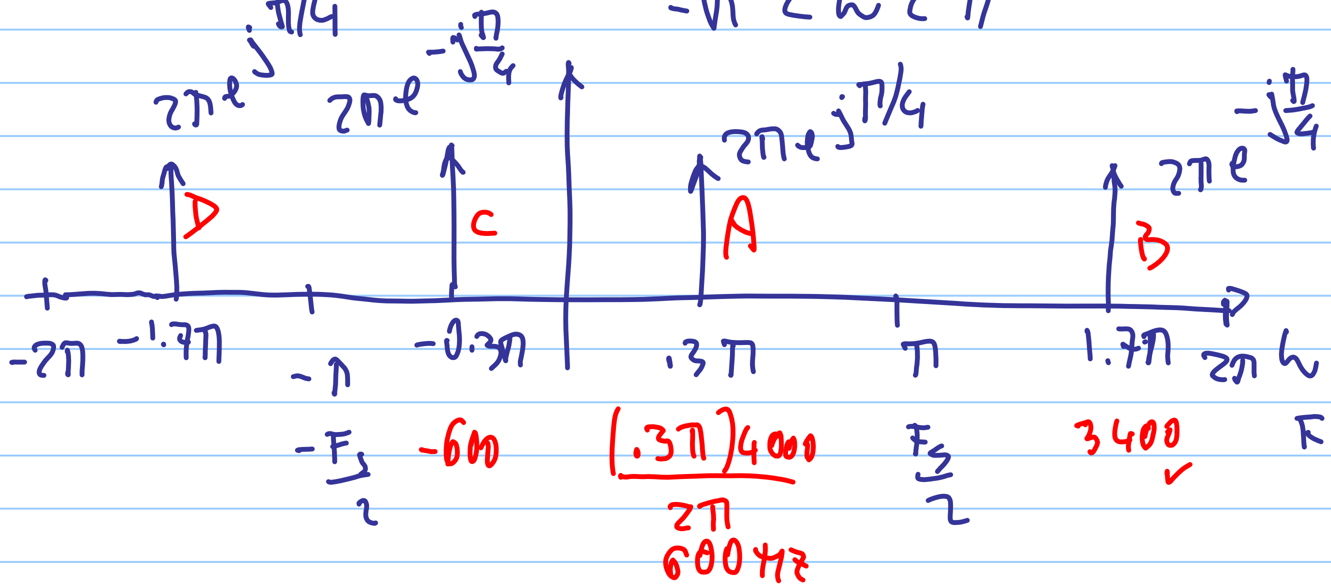
$$Y(F) = T_s\, e^{-j\pi \frac{F}{F_s}}\, \text{sinc}\left(\frac{F}{F_s}\right) Y(w)\Big|_{w=\ldots}$$

$$= G(F)\, Y(w)\Big|_{w=2\pi \frac{F}{F_s}}$$

$$Y(\omega) = \text{DTFT}\left\{ e^{j\frac{\pi}{4}} e^{j0.3\pi n} + e^{-j\frac{\pi}{4}} e^{-j0.3\pi n} \right\}$$

$$= 2\pi e^{j\frac{\pi}{4}} \delta(\omega - 0.3\pi) + 2\pi e^{-j\frac{\pi}{4}} \delta(\omega + 0.3\pi)$$

$$-\pi < \omega < \pi$$



$$Y(F) = \underbrace{\frac{1}{4000} e^{-j\pi\frac{600}{4000}} \operatorname{sinc}\left(\frac{600}{4000}\right)}_{G(600)} \cdot$$

$$2\pi e^{j\frac{\pi}{4}} \delta\left(\frac{2\pi F}{4000} - 0.3\pi\right) + \cdots$$

$$= G(600)\, 2\pi e^{j\pi/4} \frac{4000}{2\pi} \delta(F - 600) + \cdots$$

Chapter 4

4.6 FIR Low pass

Pass band $0 \to 10\,KHz$

STOP $> 11\,KHz$, $A = 50\,dB$

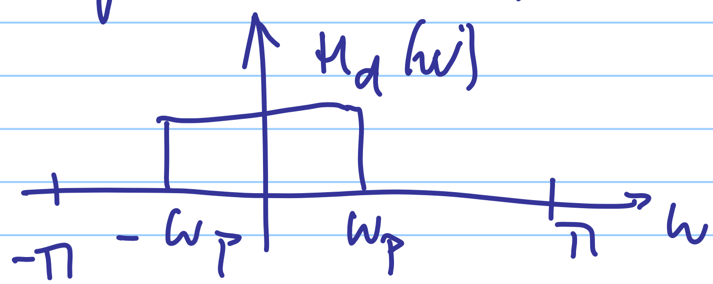
$F_s = 44\,KHz$

① Digital Freq.

$$\omega_p = \frac{2\pi\, 10}{44} = \ldots$$

$$\omega_{stop} = 2\pi \frac{11}{44} = \frac{\pi}{2}$$

② ideal filter $h_d[n]$



$$h_d[n] = \frac{1}{2\pi}\int_{-w_p}^{w_p} e^{jwn}\,dw$$

$$= \frac{w_p}{\pi}\,\text{sinc}\left(\frac{w_p}{\pi} n\right) \quad -\infty < n < +\infty$$

(3) Window: Blackman window

$$\Delta\omega \geq \frac{16\pi}{N}$$

$$\Delta\omega = \omega_{stop} - \omega_{pass} = \dots$$

$$N \geq \frac{16\pi}{\Delta\omega}$$ choose N odd

$$N = 2L+1$$

L = the delay

④ $h[n] = h_d[n-L]\, w_{Blackman}[n]$

$$= \frac{\omega_p}{\pi} \operatorname{sinc}\left(\frac{\omega_p}{\pi}(n-L)\right) w_B[n]$$

$n = 0, \ldots, N-1$

<u>impulse response</u>

$y[n] = h[0]\,x[n] + h[1]\,x[n-1]$ . . . .

Typical Problem:

arbitrary

$$H(\omega) = \begin{cases} j\,|\omega| & |\omega| < \frac{\pi}{4} \\ 0 & |\omega| > \frac{1.2\pi}{4} \end{cases}$$

$A = 40\,dB$

Same kind of solution:

① ideal filter

$$h_d[n] = \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} j\,|\omega|\, e^{j\omega n}\, d\omega$$

Recall

$$|\omega| = \begin{cases} \omega & \text{if } \omega > 0 \\ -\omega & \text{if } \omega < 0 \end{cases}$$

$$h_d[n] = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{0} -j\omega e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\frac{\pi}{4}} j\omega e^{j\omega n}\, d\omega =$$

2 window: hamming from attenuation

$$\Delta\omega \geq \frac{8\pi}{N} \Rightarrow N \geq \frac{8\pi}{\Delta\omega}$$

$$N = 2L+1$$

(3) $h[n] = h_d[n-L]\, w_h[n]$

$$n = 0, \ldots, N-1$$

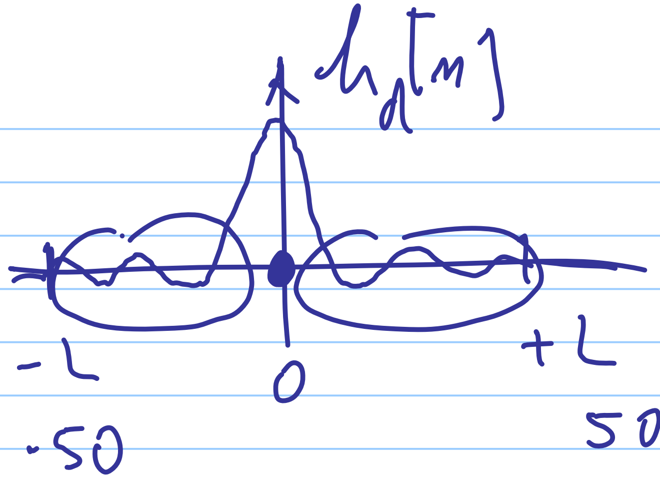
$N \geq 100$

choose $N = 101$ length

$L = \underline{\underline{50}}$ time delay

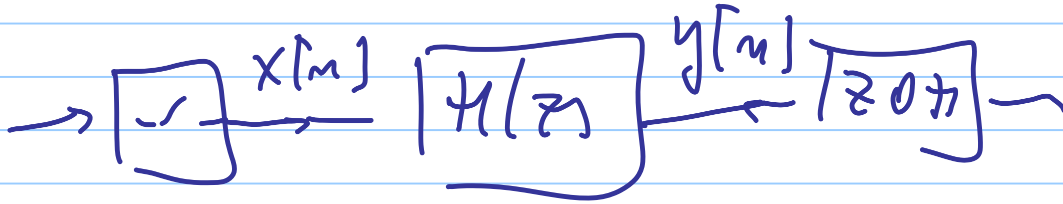
$$\Delta\omega \geq \frac{8\pi}{N}$$

$$\underline{\underline{N}} \geq \frac{8\pi}{\Delta\omega} = 100$$



$N = 50 + 50 + 1 = \underline{101}$

Z-Transform -

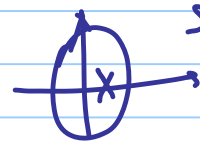


✓

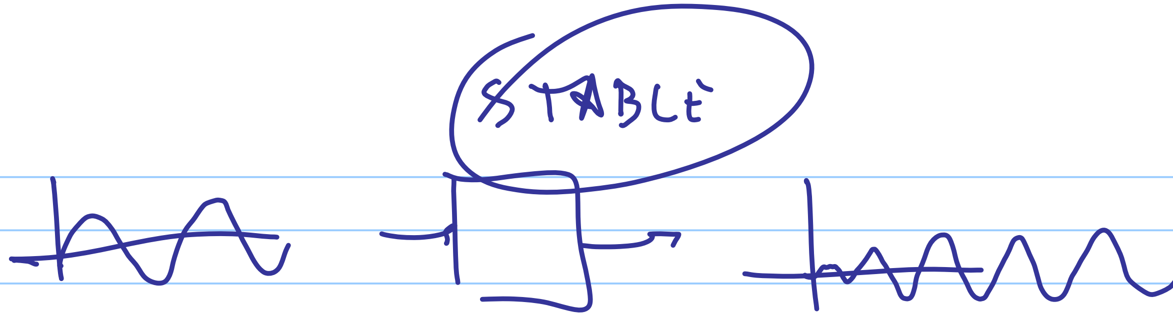
let $y[n] = 0.6\,y[n-1] + x[n] + 2x[n-1]$

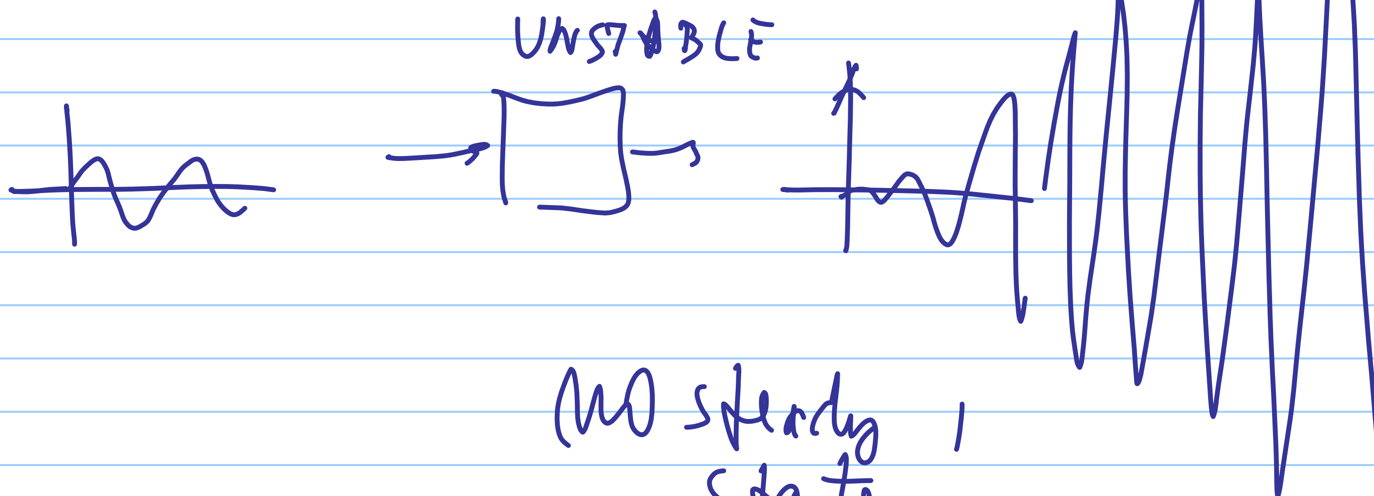
$$H(z) = \frac{1+2z^{-1}}{1-0.6z^{-1}} = \frac{z+2}{z-0.6}$$ ✓

stable ?



stable





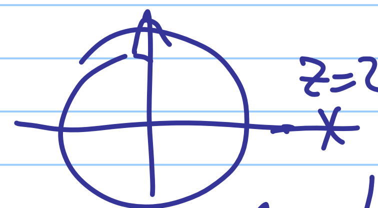
NO steady state!

Ex:

$$y[n] = 2y[n-1] + x[n-1]$$

$$H(z) = \frac{z^{-1}}{1-2z^{-1}} = \frac{1}{z-2}$$

unstable



unstable

Freq. Response ?

$$\cancel{H(\omega) = \frac{1}{e^{j\omega}-2}}$$

it does not exist