

EC3400: Computer Project 4

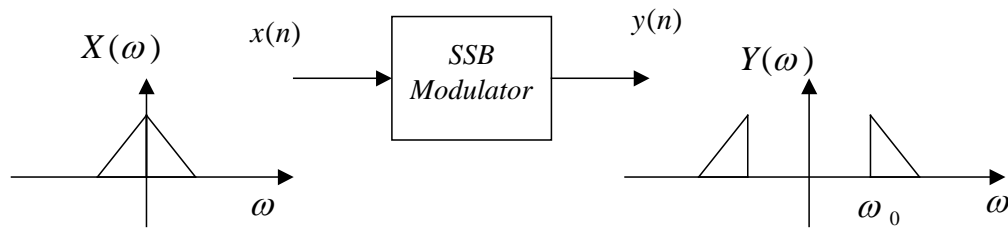
The aim of this project is to show how we can apply filter design techniques to a modulation problem. Apart from being able to design the standard low pass filter, we want to show that any arbitrary filter can be well approximated by an FIR non recursive realization.

Introduction. An important technique in communications is *Amplitude Modulation (AM)*, where we shift the frequency spectrum of a signal, so that we can send it through a channel. The channel is defined by the frequency band associated to it.

In order to make the communication efficient in terms of bandwidth occupancy, various techniques for AM have been developed. In particular Single Side Band (SSB) is an efficient way of modulating a signal without sending redundant information.

The general scheme for SSB modulation is shown below. Notice that we do not shift the whole spectrum, but the two halves only. One of the building blocks of the SSB modulator is the Hilbert Transform.

In this project we address the problem of designing a digital SSB modulator. This would be the case when we combine a number of digital channels (data and voice, for example) into one digital channel, and we transmit it through a medium.



Problem 1. (pencil and paper) Define the *Hilbert Transform* filter $H(\omega)$ by the frequency response

$$H(\omega) = \begin{cases} -j & \text{if } \omega > 0 \\ +j & \text{if } \omega < 0 \\ 0 & \text{if } \omega = 0 \end{cases}$$

with ω being the digital frequency.

Q1: if the signal $x(n)$ has frequency spectrum $X(\omega)$ as shown in the figure below and the Low Pass Filter has bandwidth ω_c (same as the signal) sketch the frequency spectrum of each of the signals $\hat{x}(n)$, $x_+(n)$, $s(n)$ and $y(n)$;

Q2: same as *Q1* replacing $H(\omega)$ with $-H(\omega)$;

Q3: determine the impulse response $h(n)$ of the ideal *Hilbert Transform* filter $H(\omega)$.

Problem 2. The signal SONG.WAV is sampled at CD quality (44.1kHz), and we want to transmit it through a digital channel with the same sampling frequency (44.1kHz) in the frequency band 10.0 to 20.0 kHz.

a) First design a Low Pass Equiripple FIR Filter which limits the bandwidth of the signal SONG.WAV to 10kHz. This filter must have a passband frequency of 10kHz. The stopband must have an attenuation of 40dB. Which *stopband frequency* would you choose?

b) Design the SSB modulator by implementing the Hilbert Transform as an FIR filter with a hamming window of length $N=81$. Also, *use real components only*. In other words all the impulse responses and the signals you use have to be real (no j anywhere).

Check the magnitude of the frequency spectrum of the modulated signal to make sure that it is what you expect.

Note on the implementation: Since the FIR implementation of the Hilbert Transform is shifted in time, in the branch parallel to the Hilbert Transform you have to shift also $x(n)$ by the same amount.

Problem 3: Demodulate the signal by multiplying with $e^{-j\omega_0 n}$ and Low Pass Filtering. Design the system and test it in the data.

Notes on Matlab.

firpm (*remez* in older versions) for equiripple filter

