

PROJECT 1 - PROB 1

Example:

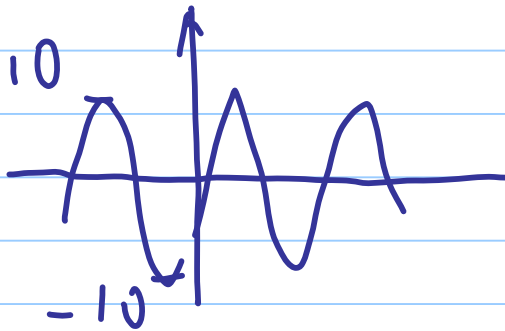
Note Title

3/31/2011

$$x(t) = 10 \cos(\underline{8000\pi t} + 0.7) , t = \text{sec.}$$

↑
Amplitude

↑
phase (in radians)



Frequency $8000\pi t = 2\pi F_0 t$

$$F_0 = 4000 \text{ Hz}$$

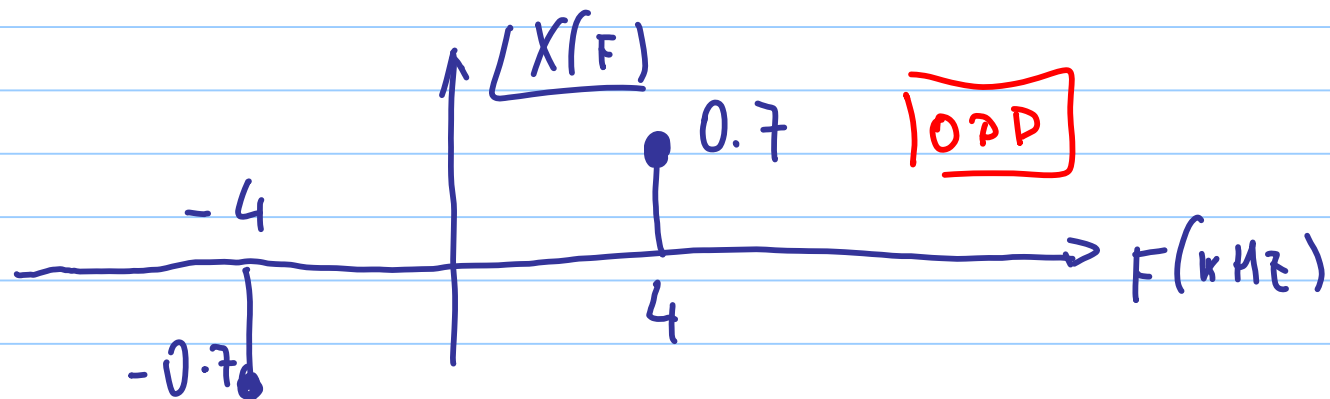
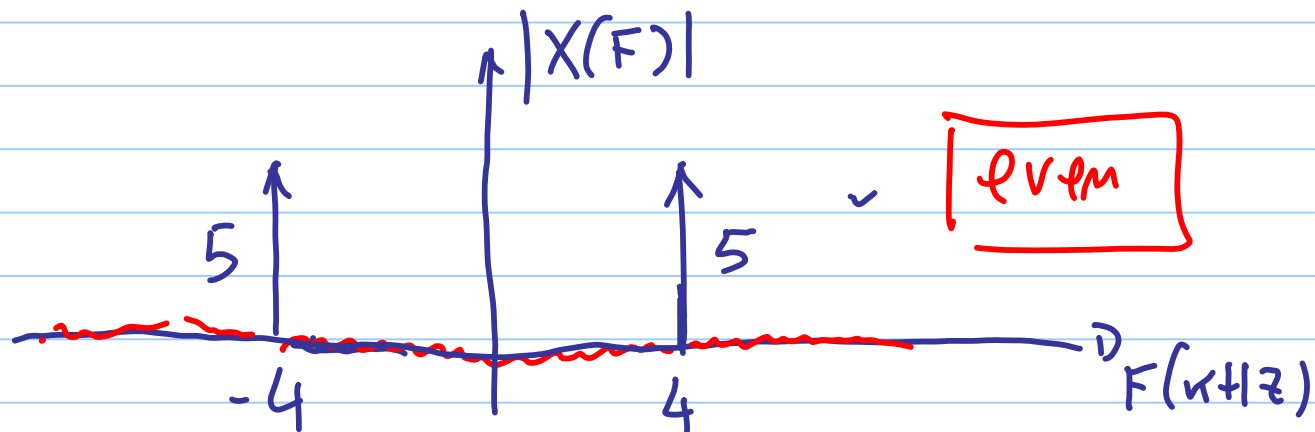
Recall:

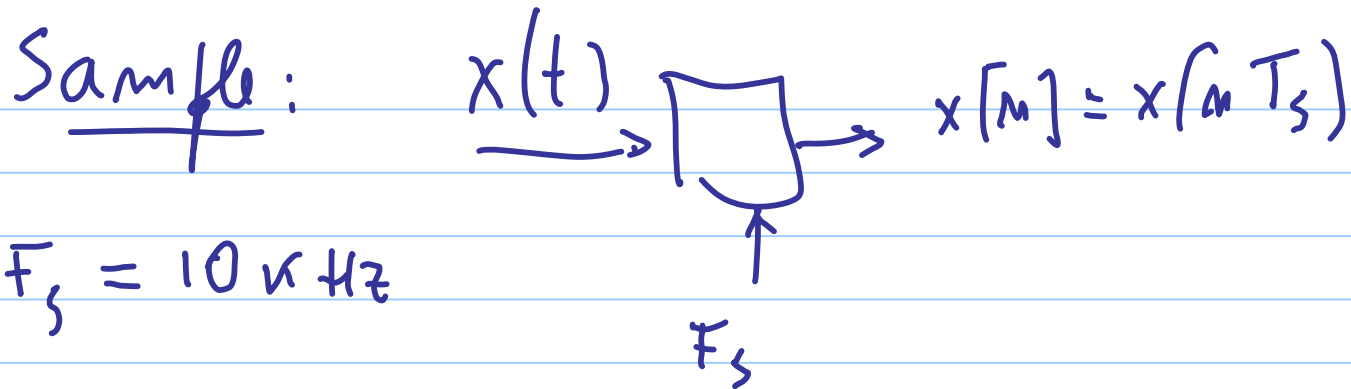
$$x(t) = \frac{10}{2} \left(e^{j0.7} e^{j8000\pi t} + e^{-j0.7} e^{-j8000\pi t} \right)$$

$$= \underbrace{(5e^{j0.7})}_{\substack{\uparrow \\ \text{mag.}}} \underbrace{e^{j8000\pi t}}_{\text{phase}} + \underbrace{(5e^{-j0.7})}_{\substack{\uparrow \\ \text{mag.}}} \underbrace{e^{-j8000\pi t}}_{\text{phase}}$$

$$X(F) = \text{FT}\{x(t)\}$$

$$= 5e^{j0.7} \delta(F-4000) + 5e^{-j0.7} \delta(F+4000) \quad \checkmark$$

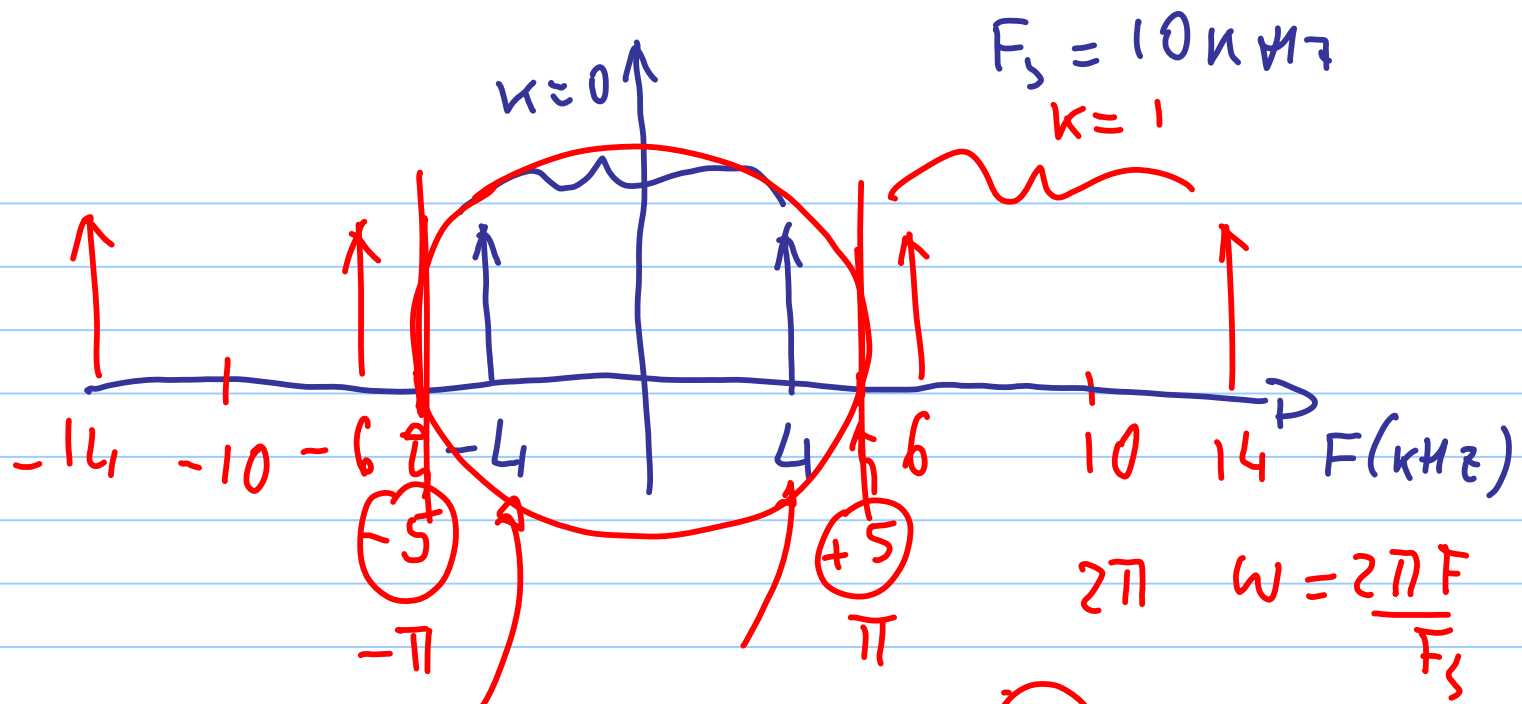




Recall: $X(\omega) = \text{DTFT}\{x[n]\}$

$$X_{\text{DTFT}}(\omega) = F_s \sum_{k=-\infty}^{\infty} X(F - kF_s), \quad \omega = \frac{2\pi F}{F_s}$$

repetition of $X(F)$ rescaling of freq.



No aliasing:

$$X_{\text{DTFT}}(\omega) = F_s X_{\text{FT}}(F) \Big|_{F = \frac{\omega F_s}{2\pi}} \quad -\pi \leq \omega < \pi$$

$$= F_s \left(5e^{j0.7} \delta\left(\frac{\omega F_s}{2\pi} - F_0\right) + \right.$$

$$\left. 5e^{-j0.7} \delta\left(\frac{\omega F_s}{2\pi} + F_0\right) \right)$$

$$\delta\left(\frac{\omega F_s}{2\pi} - F_0\right) = \delta\left(\frac{F_s}{2\pi}\right)(\omega - \omega_0) =$$

$$\omega_0 = 2\pi \frac{F_0}{F_s}$$

$$= \frac{2\pi}{F_s} \delta(\omega - \omega_0)$$

Very simple algebra \longrightarrow

$$X(\omega) = 2\pi 5 e^{j0.7} \delta\left(\omega - \frac{4\pi}{5}\right) +$$

$$+ 2\pi 5 e^{-j0.7} \delta\left(\omega + \frac{4\pi}{5}\right)$$

$$-\pi \leq \omega < \pi$$



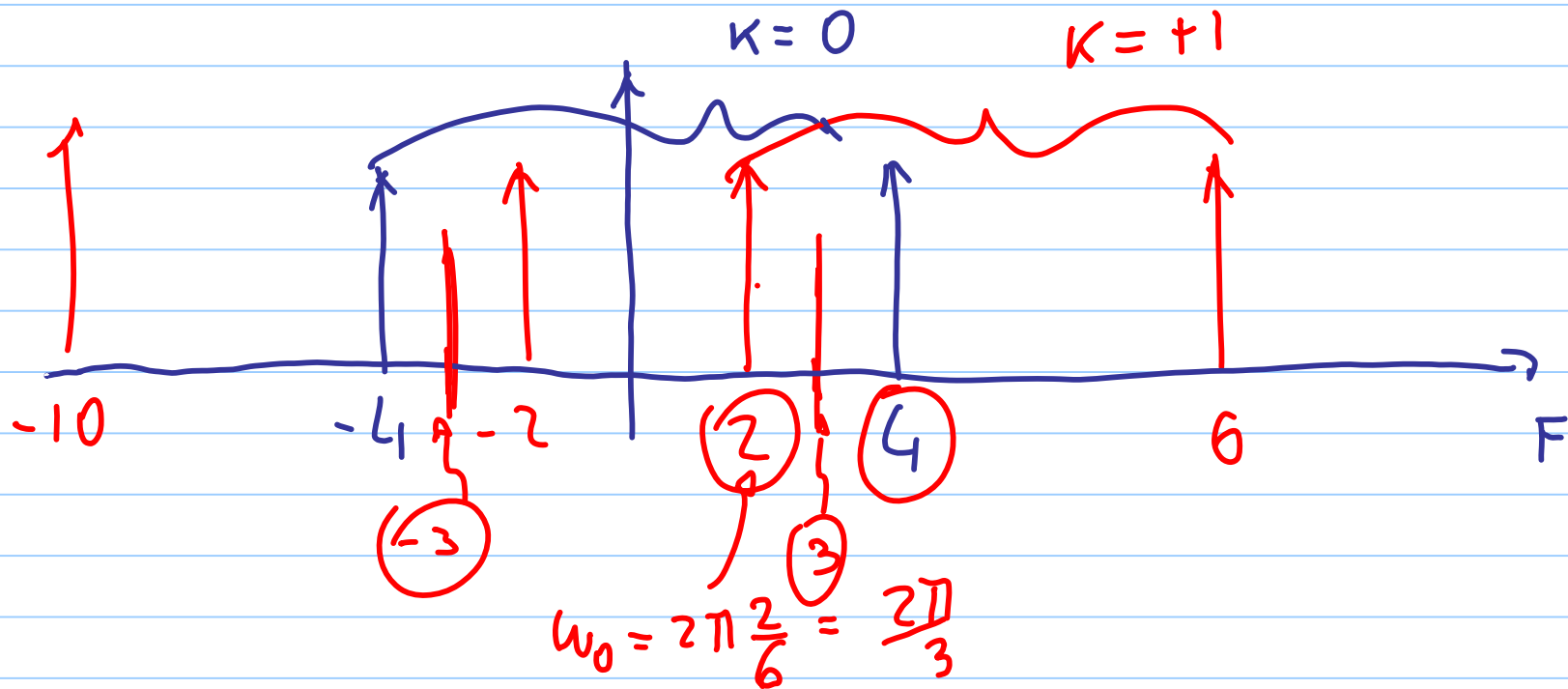
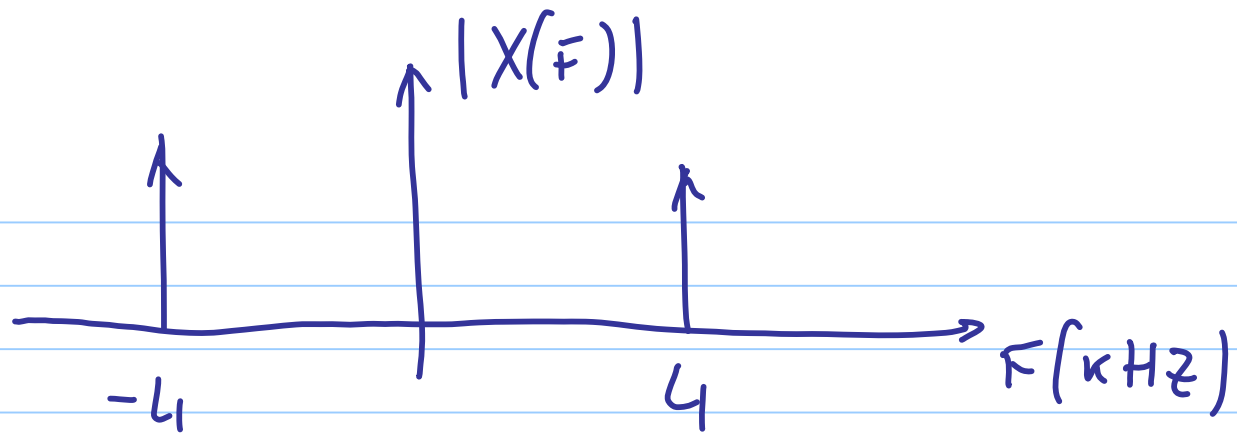
Ex. with aliasing:

$$F_s = 6 \text{ kHz}$$

aliasing since

$$F_0 = 4 \text{ kHz}$$

$$\frac{F_s}{2} = 3 \text{ kHz} < F_0$$



$$X(\omega) = 2\pi \times 5 e^{-j0.7} \delta\left(\omega - \frac{2\pi}{3}\right) +$$

$$+ 2\pi \times 5 e^{j0.7} \delta\left(\omega + \frac{2\pi}{3}\right)$$

$$-\pi \leq \omega < \pi$$

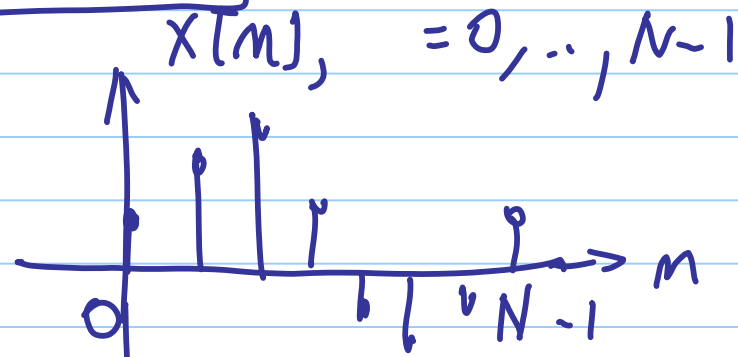
PROBLEM 2 (MATLAB):

FFT = FAST FT

DFT = Discrete FT

Definition:

same length.



$$X[k] = \text{DFT} \{ x[m] \} = \sum_{m=0}^{N-1} x[m] e^{-j\omega m} \quad \left| \quad \omega = k \frac{2\pi}{N} \right.$$

$k = 0, \dots, N-1$