Second Kind Predictability in Climate Models

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Limitations of Climate Prediction - Error Amplification

- Data Error
  - Initial Condition (First Kind)
  - Boundary Condition Error (Second Kind)
- Numerical Error
  - Truncation
- Model Parameter Error (Second Kind)

Two dimensional (x, z) atmospheric convection

- Caused by temperature difference at the top and bottom

$$\Delta T = T(0) - T(H)$$

At the top boundary:

$$T(x, H, t) = T(H)$$

At the bottom boundary:

$$T(x, 0, t) = T(0)$$

- When the bottom boundary condition has error:

  $T(x,0,t) = T(0) + \varepsilon \Delta T$,

  What is the model sensitivity to boundary condition error?

Variable Transfer

$$\theta(x,z,t) = T(x,z,t) - \left[T(0) - \frac{T(0)-T(H)}{H}z\right]$$

Saltzman Model (1962)

$$\frac{\partial}{\partial t}\nabla^2\psi = -\frac{\partial(\psi, \nabla^2\psi)}{\partial(x,z)} + \nu\nabla^4\psi + g\alpha\frac{\partial\theta}{\partial x},$$

$$\frac{\partial\theta}{\partial t} = -\frac{\partial(\psi,\theta)}{\partial(x,z)} + \frac{\Delta T}{H}\frac{\partial\psi}{\partial x} + \kappa\nabla^2\theta.$$

With homogeneous boundary conditions

$$\theta(x,0,t) = \theta(x,H,t) = 0$$

Boundary Condition Error Becomes Error in Forcing Term

$$\Delta T / H \partial \psi / \partial x,$$

Three Mode Truncation

$$a(1+a^2)^{-1}\kappa^{-1}\psi = X\sqrt{2}\sin(\pi a H^{-1}x)\sin(\pi H^{-1}z),$$

$$\pi r \Delta T^{-1}\theta = Y\sqrt{2}\cos(\pi a H^{-1}x)\sin(\pi H^{-1}z) - Z\sin(2\pi H^{-1}z)$$

$$r = \frac{R_a}{R_c}, \qquad R_a = g\alpha H^3 \Delta T \nu^{-1}\kappa^{-1}, \qquad R_c = \pi^4 a^{-2}(1+a^2)^3.$$

Boundary Condition Error → Error in the parameter "*r*"

Lorenz System (JAS, 1963)

$$\frac{dX}{d\tau} = -\sigma X + \sigma Y,$$

$$\frac{dY}{d\tau} = -XZ + rX - Y,$$

$$\frac{dZ}{d\tau} = XY - bZ,$$

Two Kinds of Predictability in the Lorenz System (Chu, JAS, 1999)

- Control Run
  - $X(0)=0,\ Y(0)=1,\ Z(0)=0,\ r, b, \sigma$
- Sensitive Run 1 (Initial Condition Error)
  - Same as the Control Run
  - Except $Y(0) = 1+ \varepsilon$
- Sensitive Run 2 (Boundary Condition Error)
  - Same as the Control Run
  - Except $r = r\,[1+ \varepsilon\,]$

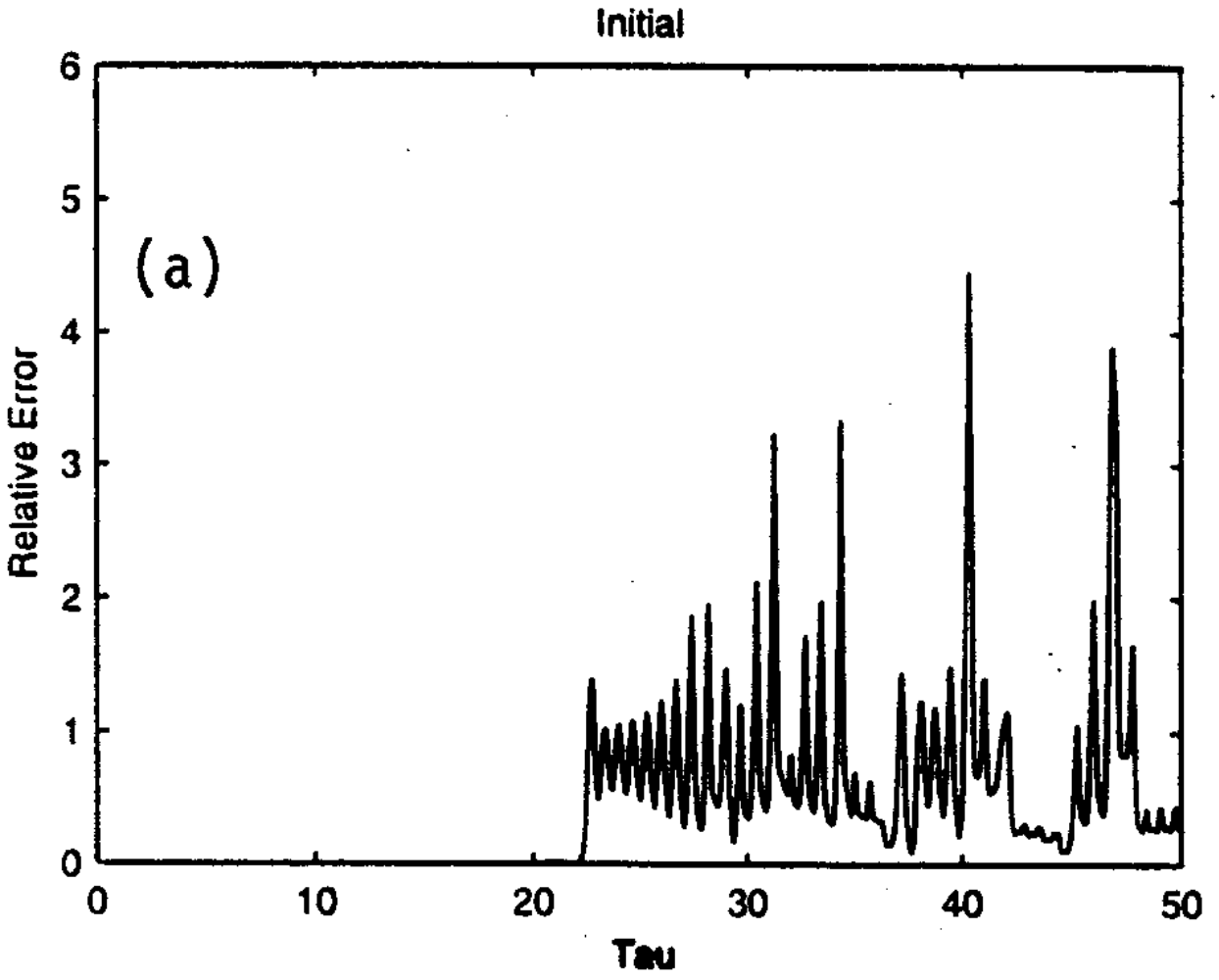
Response to a Tiny Error

- $\Delta T < 100^{o}C$
- Accuracy of Surface Temperature Measurement: $0.1^{o}C$
- 10% of Accuracy: $\varepsilon = 0.0001$

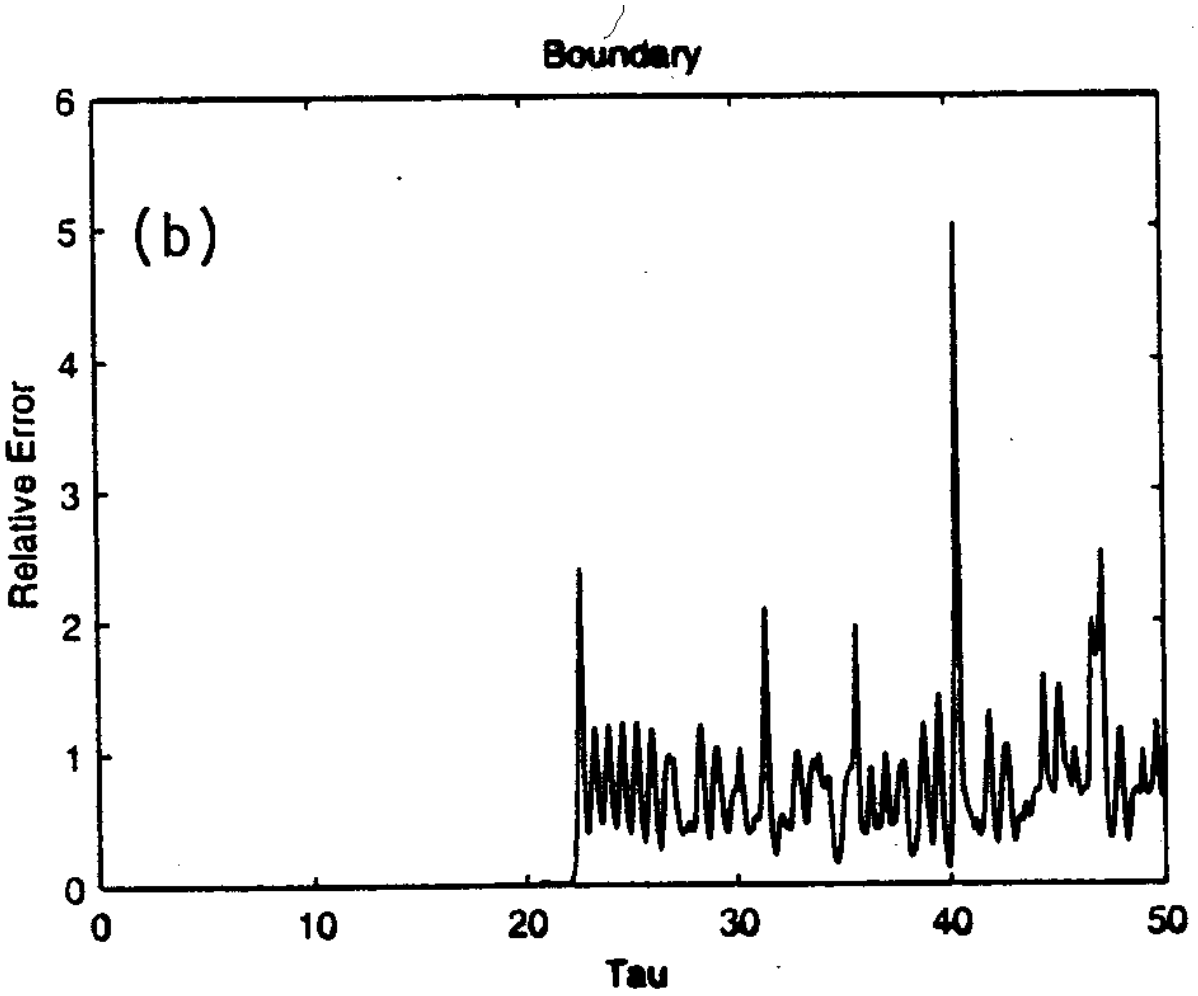
Error Estimation

$$E^{(S)}(\tau) = \frac{\sqrt{[X^{(S)}(\tau) - X^{(C)}(\tau)]^2 + [Y^{(S)}(\tau) - Y^{(C)}(\tau)]^2 + [Z^{(S)}(\tau) - Z^{(C)}(\tau)]^2}}{\sqrt{[X^{(C)}(\tau)]^2 + [Y^{(C)}(\tau)]^2 + [Z^{(C)}(\tau)]^2}}$$

Evolution of model error due to initial error (First Kind)



Evolution of model error due to initial error (Second Kind)



NCAR Community Climate Model (CCM3)

- Spherical harmonics as horizontal basis functions
- 18 vertical levels with a top at 2.917 mb
- Biosphere-Atmosphere Transfer Scheme (BATS)
- CCM3 uses the spectral transform method for the dynamic equations
- CCM3 uses a semi-Lagrangian method for transporting water

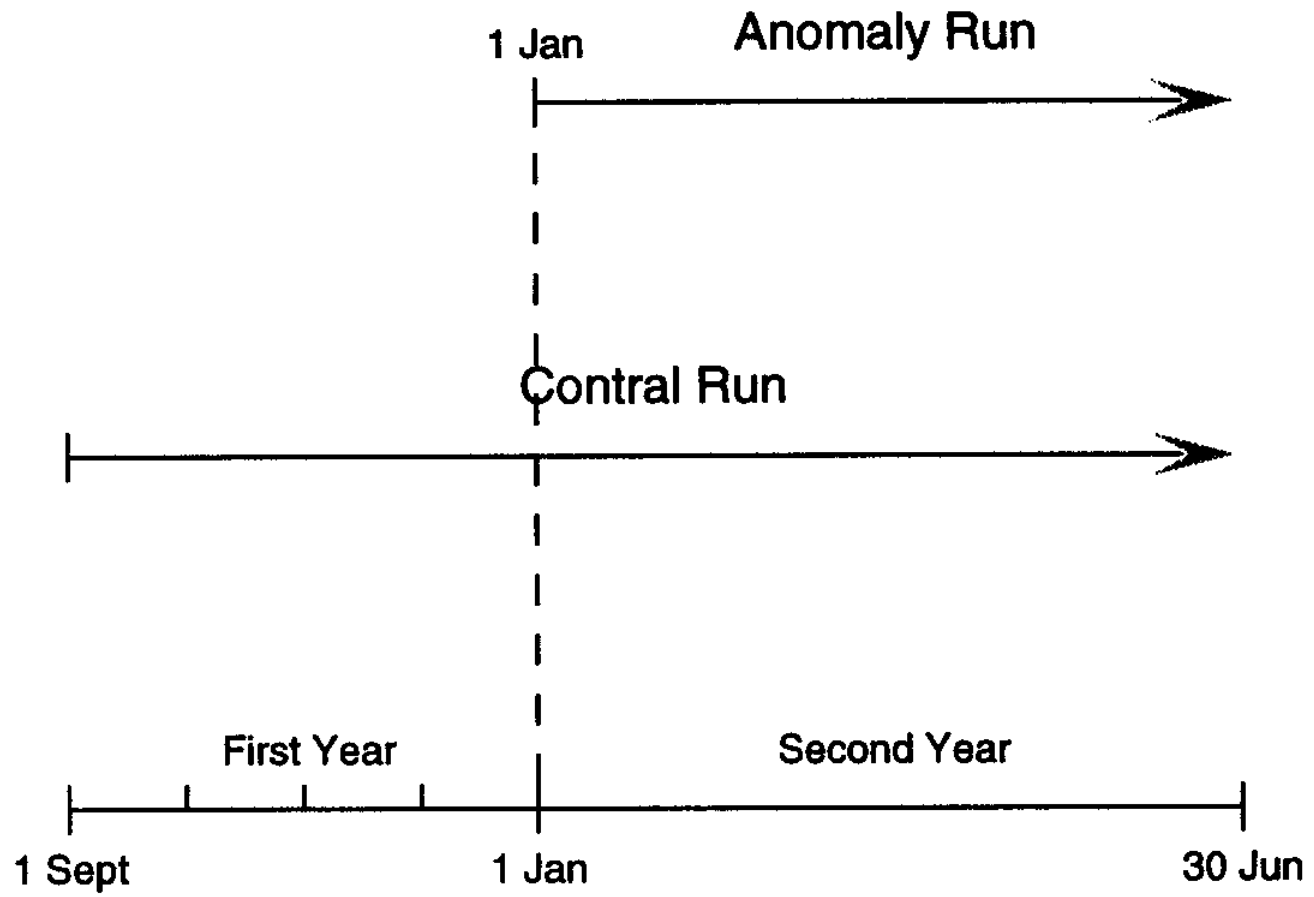
Second Kind Predictability in Climate Models (CCM3)

- Tiny Errors Added to SST (Anomaly Run)

- Mean = 0
- Standard Deviation (σ) = 0.025°C

$$F(\delta T) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(\delta T)^2}{2\sigma^2} \right]$$

Control and Anomaly Runs



Statistical Analysis

- Variability of Control Run

$$SDV_{\psi}(t) = \sqrt{\frac{1}{M-1}\sum_{i}\sum_{j}\left[\psi_c(x_i, y_j, t) - \overline{\psi}_c(t)\right]^2}$$

- RMSD Between Anomaly and Control Runs

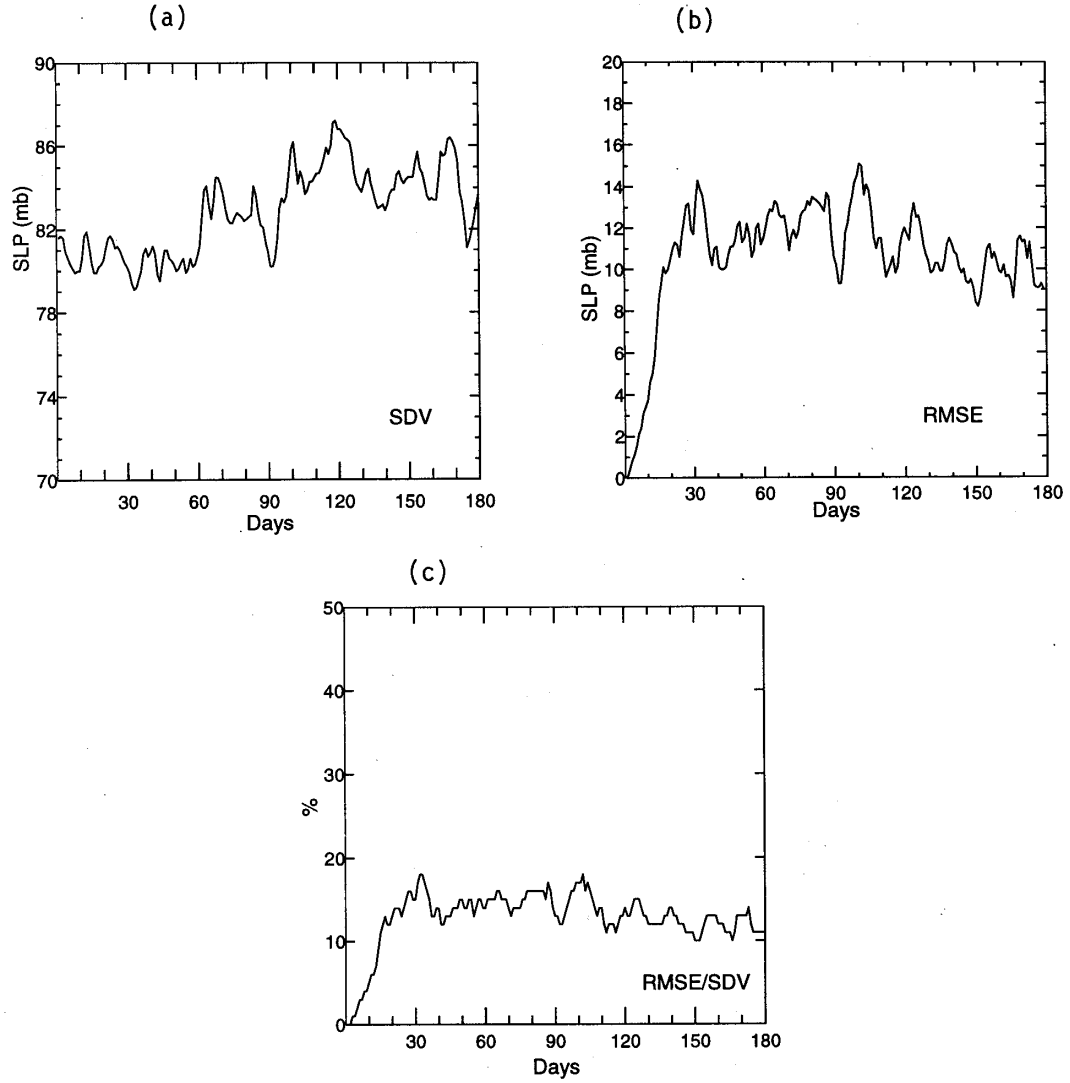
$$RMSE_{\psi}(t) =$$

$$\sqrt{\frac{1}{M-1}\sum_{i}\sum_{j}\left[\psi_a(x_i, y_j, t) - \psi_c(x_i, y_j, t)\right]^2}$$

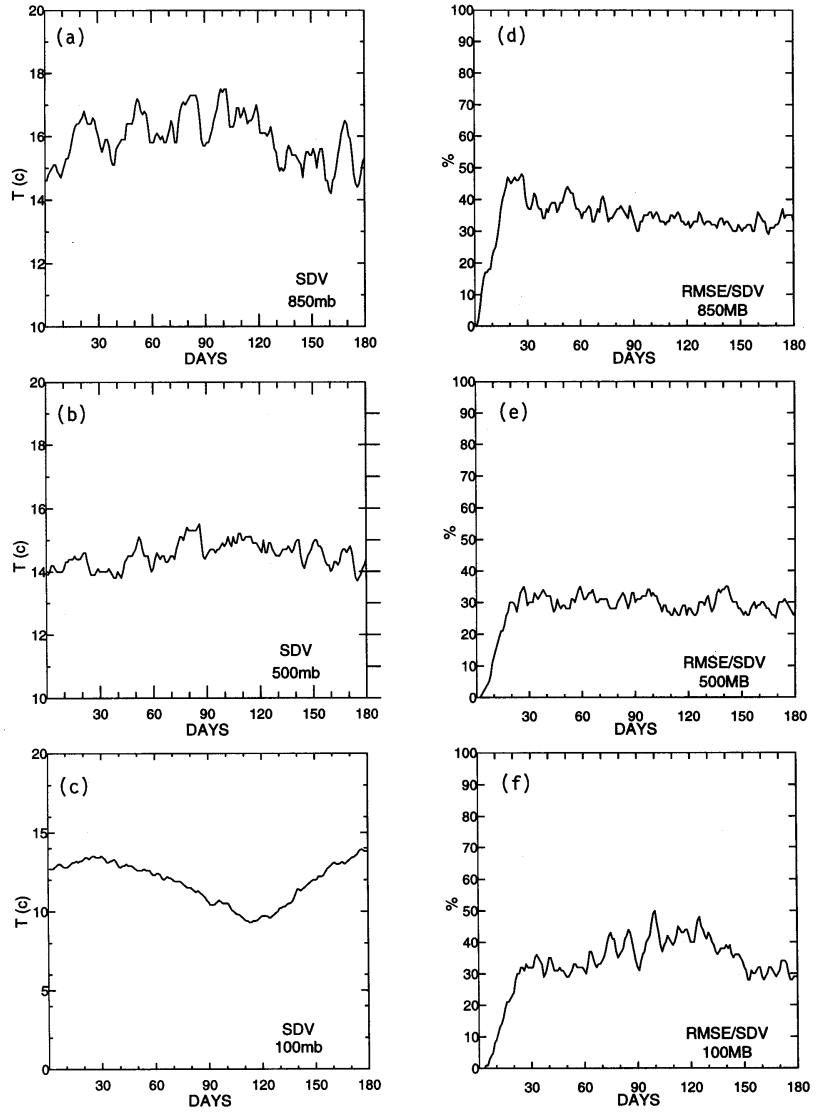
Model Relative Error

$$E_{\psi}(t) = \frac{RMSE_{\psi}(t)}{SDV_{\psi}(t)}$$

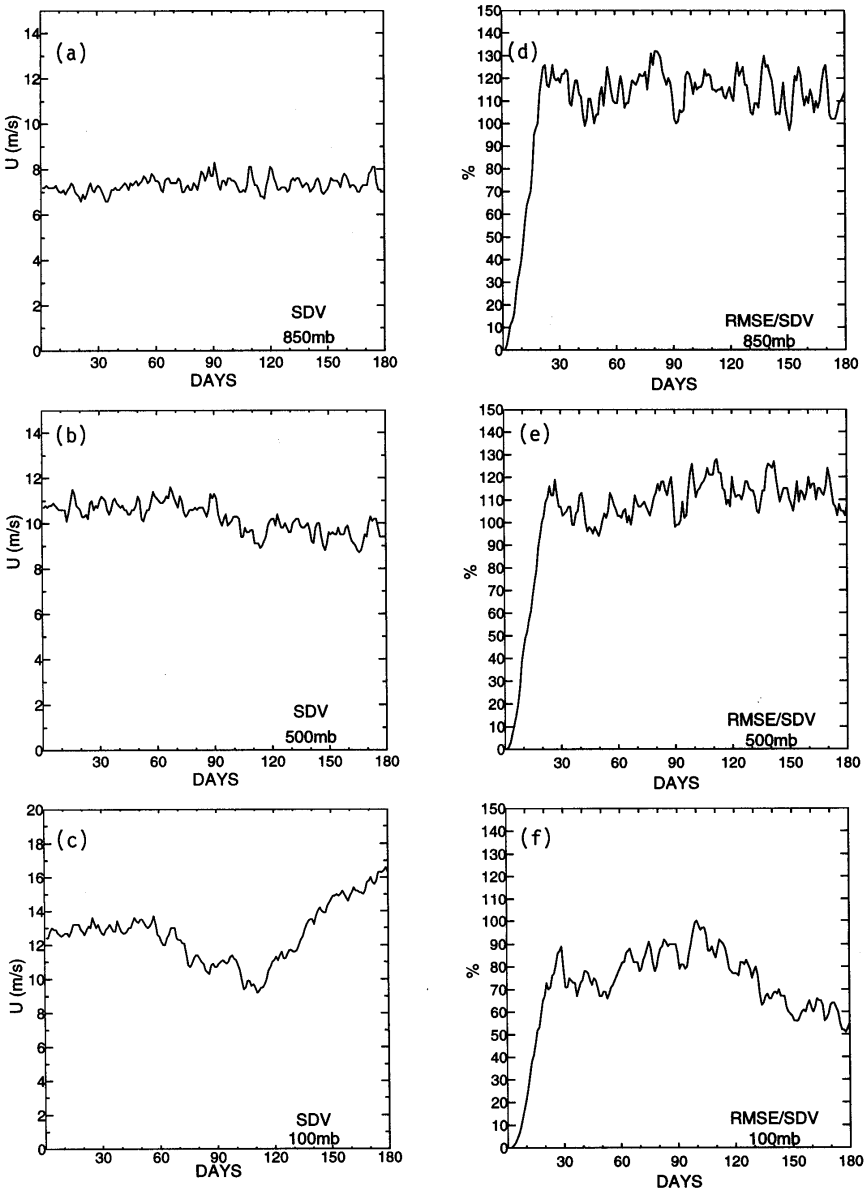
Surface Level Pressure Errors



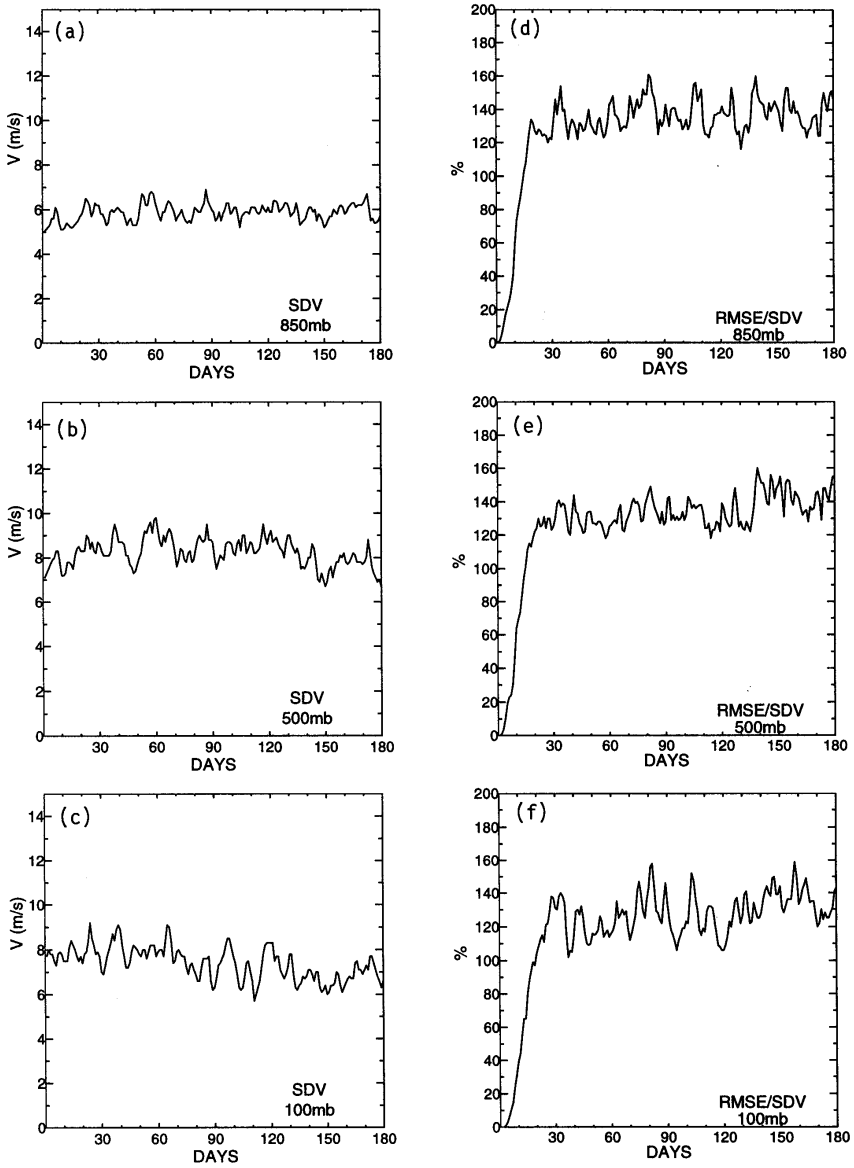
Temperature Errors



Zonal Wind (U) Errors



Latitudinal Wind (V) Errors



Conclusions

- (1) The global atmospheric response to a tiny random SST error (zero mean and 0.025^{o}C standard deviation) is quite strong. For example, at 850 mb the RMS error of the zonal velocity is 1.35 time of the internal variability. This will lead to almost 10 m/s error.
- (2) We estimate that the global atmospheric uncertainty comparing to the internal variability is around 0.3 to 0.5 for temperature, 0.9 to 1.3 for zonal wind, and 1.3 to 1.4 for latitudinal wind.
- (3) Integration of an atmospheric model needs accurate SST data. The noise in the SST data may bring drastic change in the model results.