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False Static Instability Generated by (T, S) Profile Data Analysis and Data Assimilation and a New Convective Adjustment Scheme

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Outline

- (1) False Static Instability
- (2) Adjustment Requirements
- (3) Weakness of Existing Methods
- (4) New Analytical Method

Problems

- (1) False static instability in (T, S) climatological data

- (2) False static instability in (T, S) data assimilation

Criterion of Density Inversion

$$\Delta\rho = \rho_k^a(n+1) - \rho_k^a(n)$$

$n \rightarrow$ z-level $\rightarrow$ z_n

Here, n increases downward.

NODC Static Stability Criterion

- Density inversion → Depth-decrease of density of two consecutive z-levels

$\Delta\rho$ < 0.03 kg m^{-3} (-30 m $\leq$ z < 0)

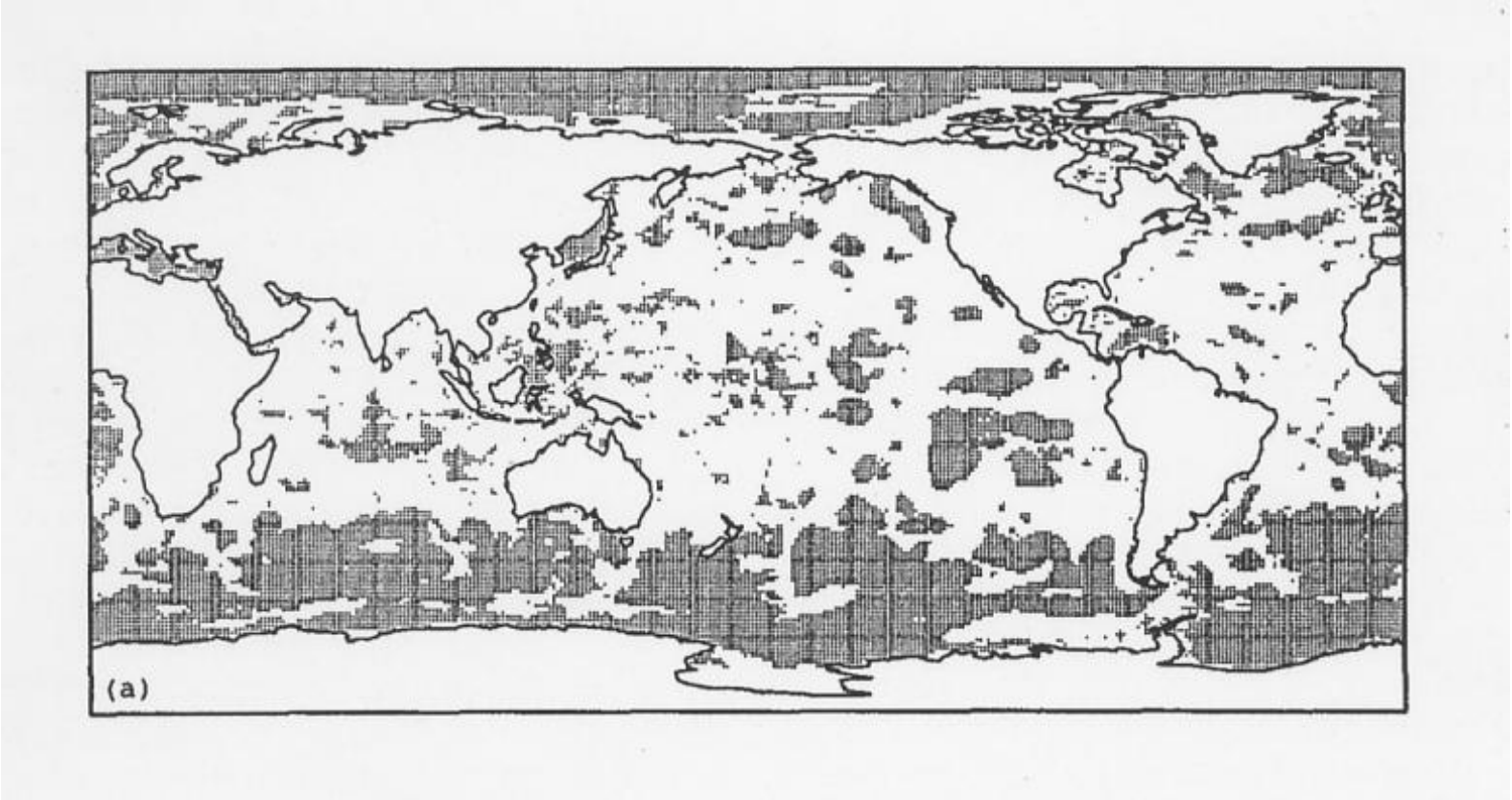
< 0.02 kg m^{-3} (-400 m < z < -30 m)

< 0 kg m^{-3} (z < -400 m)

Observational (*T*, *S*) profiles are usually through static stability check before data analysis/ data assimilation.

Observational (*T*, *S*) profiles are all statically Stable.

Many unstable stations in WOA 82 (Jackett & McDogall, 1995 JTECH)

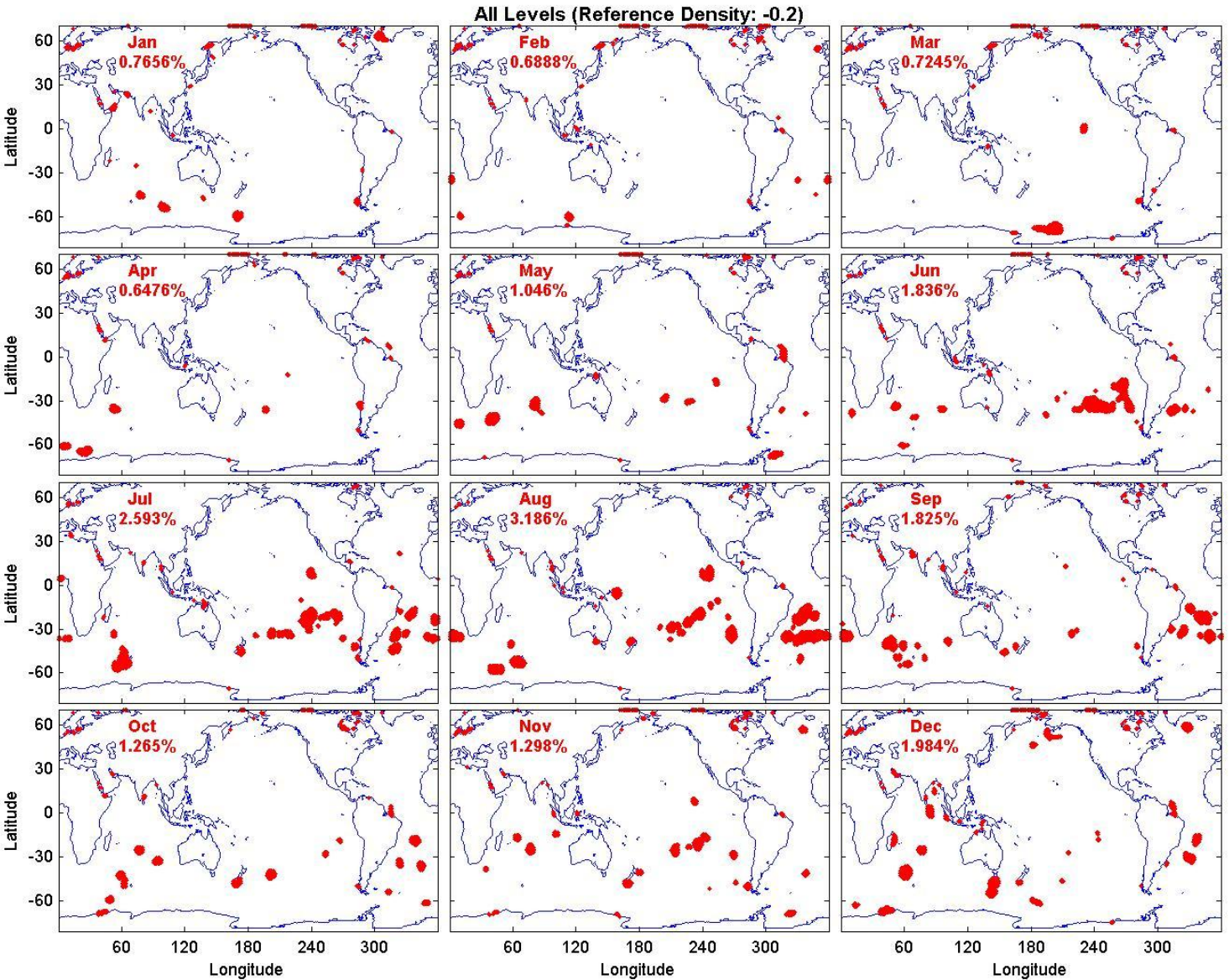


More than 44% of profiles are unstable !

NOAA-WOA94 (Arctic not included)

Max
→ 3.19%
August

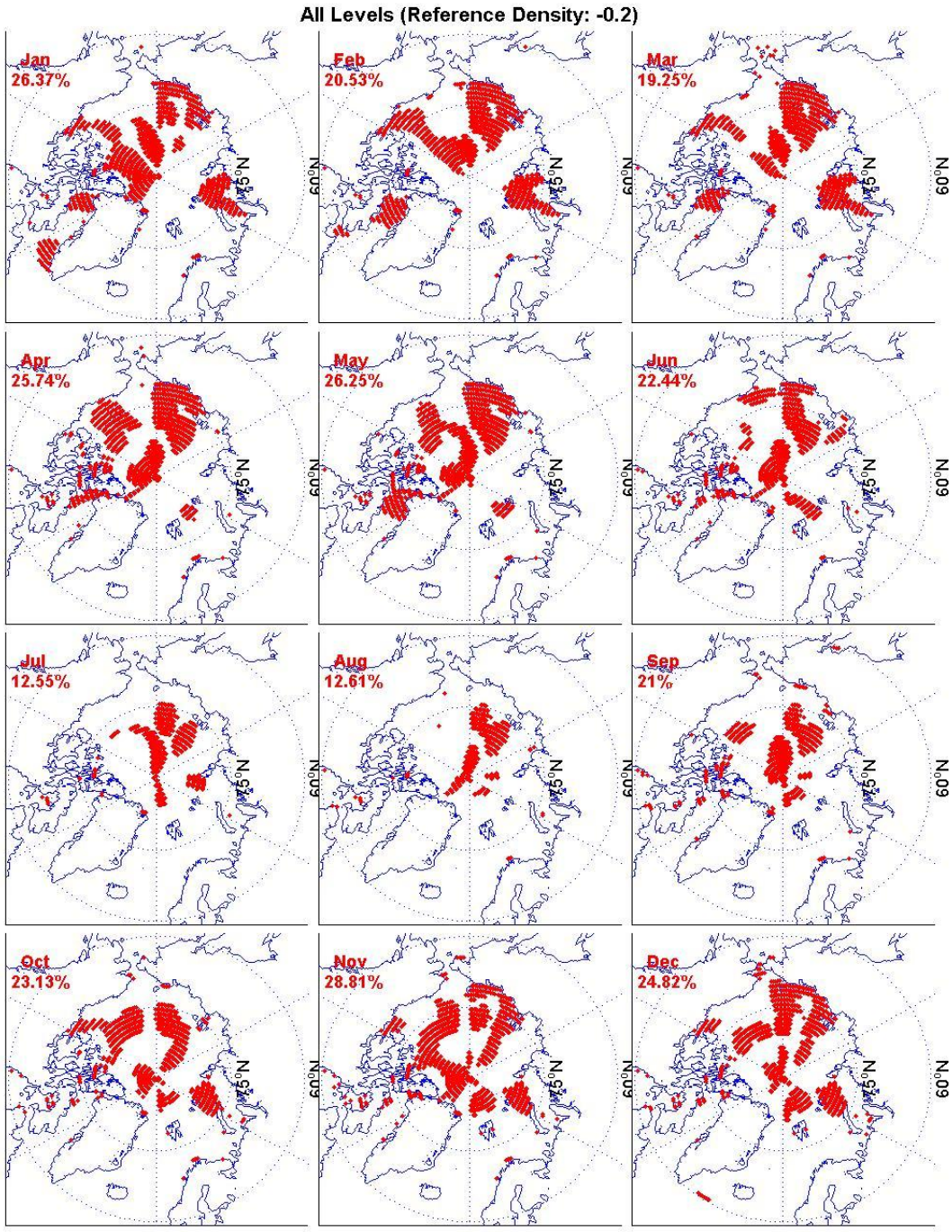
Min
→ 0.65%
April



NOAA-WOA94
Arctic

Max → 28.8% in November

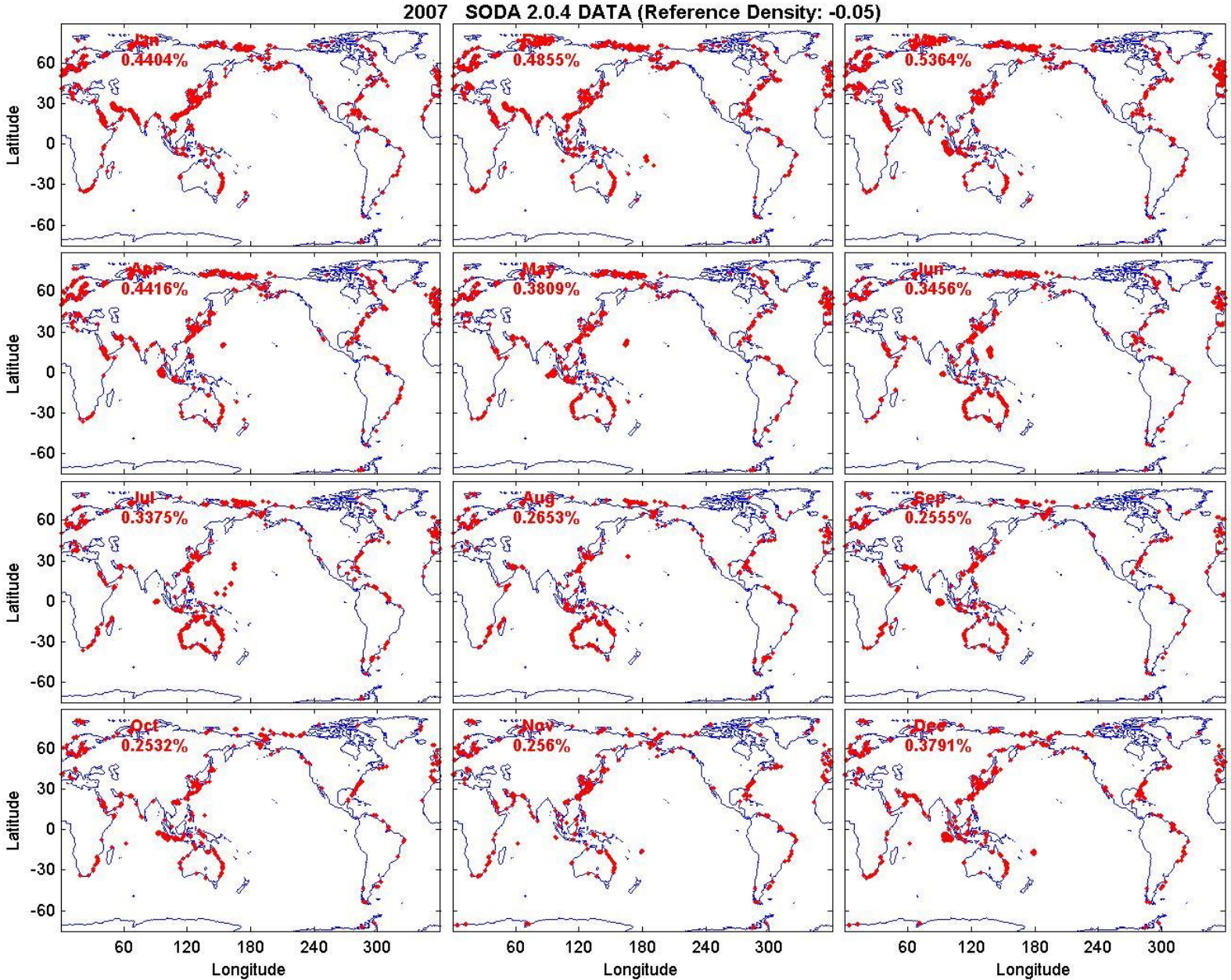
Min → 12.6% in July



Example-1
Simple Ocean Data Assimilation (SODA)
Monthly

GFDL MOM-2b + OI

SODA 2007


2007 SODA 2.0.4 DATA (Reference Density: -0.05)
Jan
0.4404%
Feb
0.4855%
Mar
0.5364%
Apr
0.4416%
May
0.3809%
Jun
0.3456%
Jul
0.3375%
Aug
0.2653%
Sep
0.2555%
Oct
0.2532%
Nov
0.256%
Dec
0.3791%
Latitude
60
30
0
-30
-60
60
120
180
240
300
Longitude

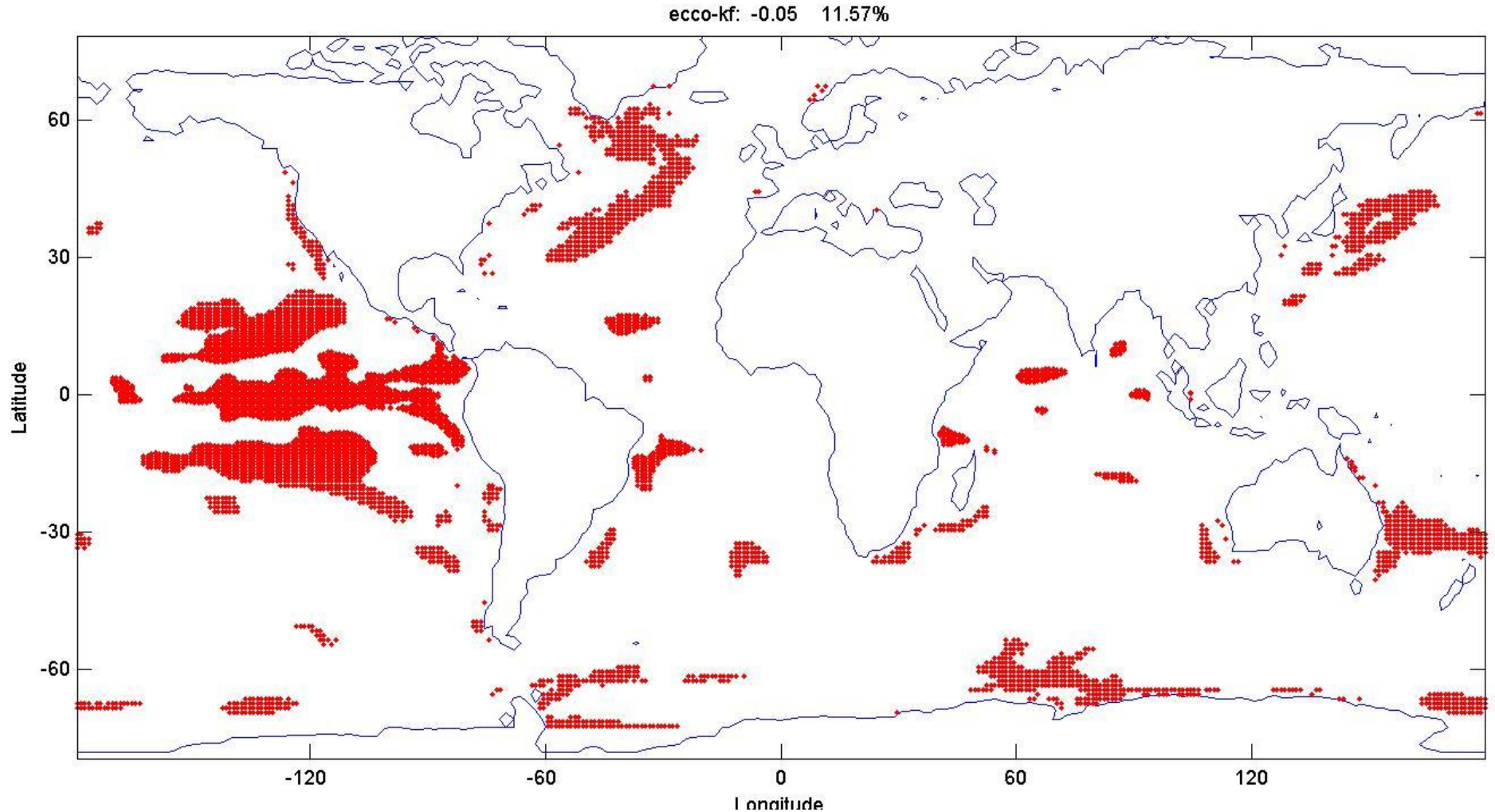
Example-2
JPL Estimating the Circulation and Climate of the Ocean (JPL-ECCO) 10 day

MIT General Circulation Model
+ Kalman Filter

JPL-ECCO 2007 (10-day)

Centered on Dec 31, 2008

Unstable Profiles → 11.57%

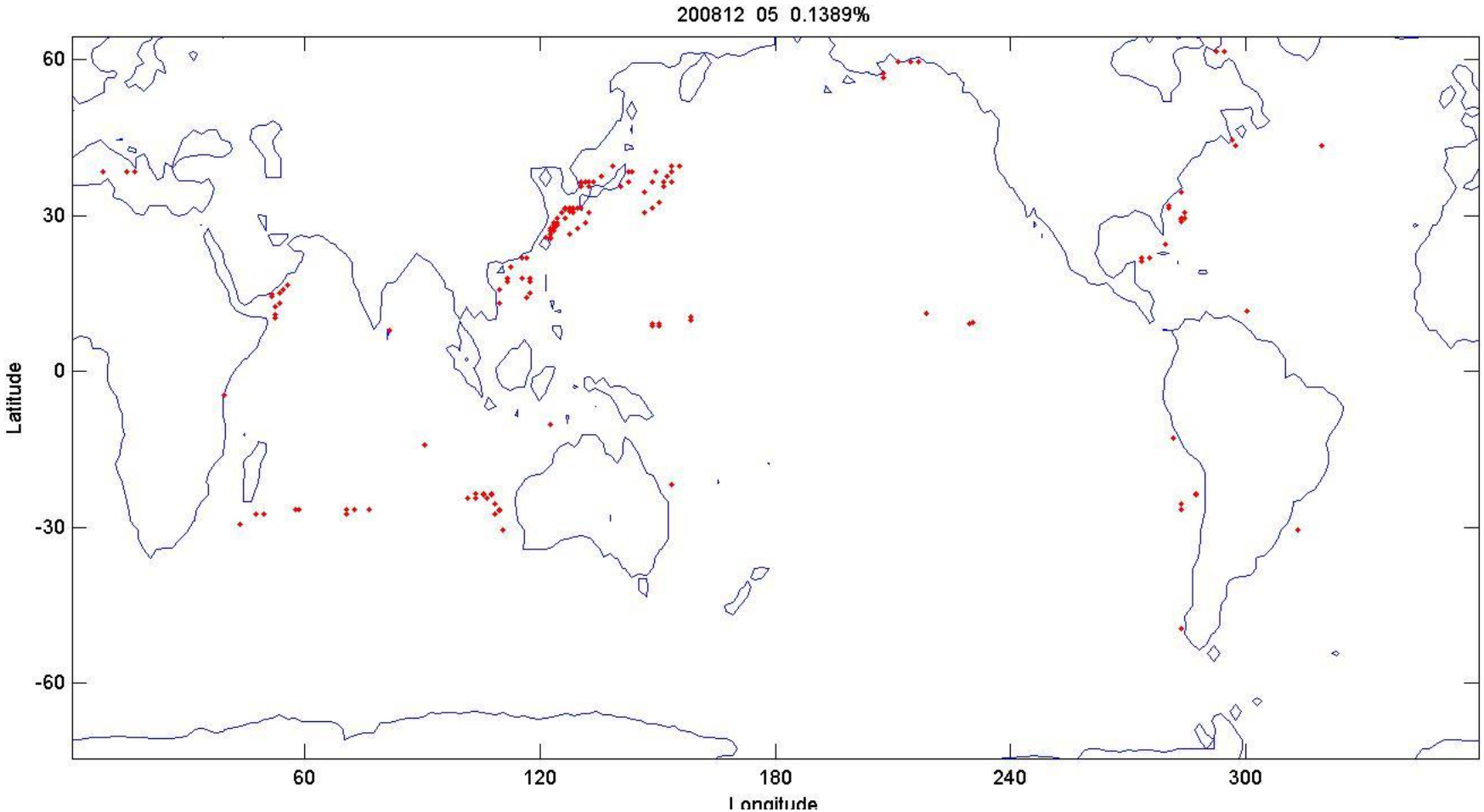


Example-3
NCEP Global Ocean Data Assimilation System (GODAS) Monthly

MOM + 3D-Var

GODAS December 2008

Unstable Profiles → 0.14%

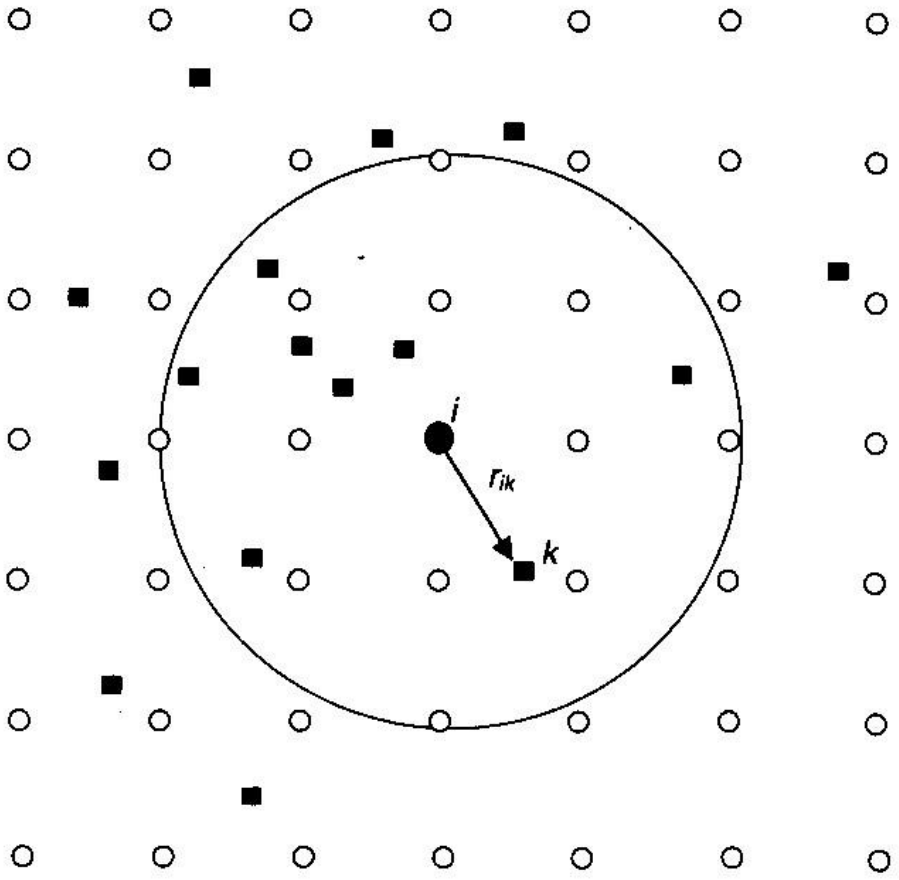


Why?

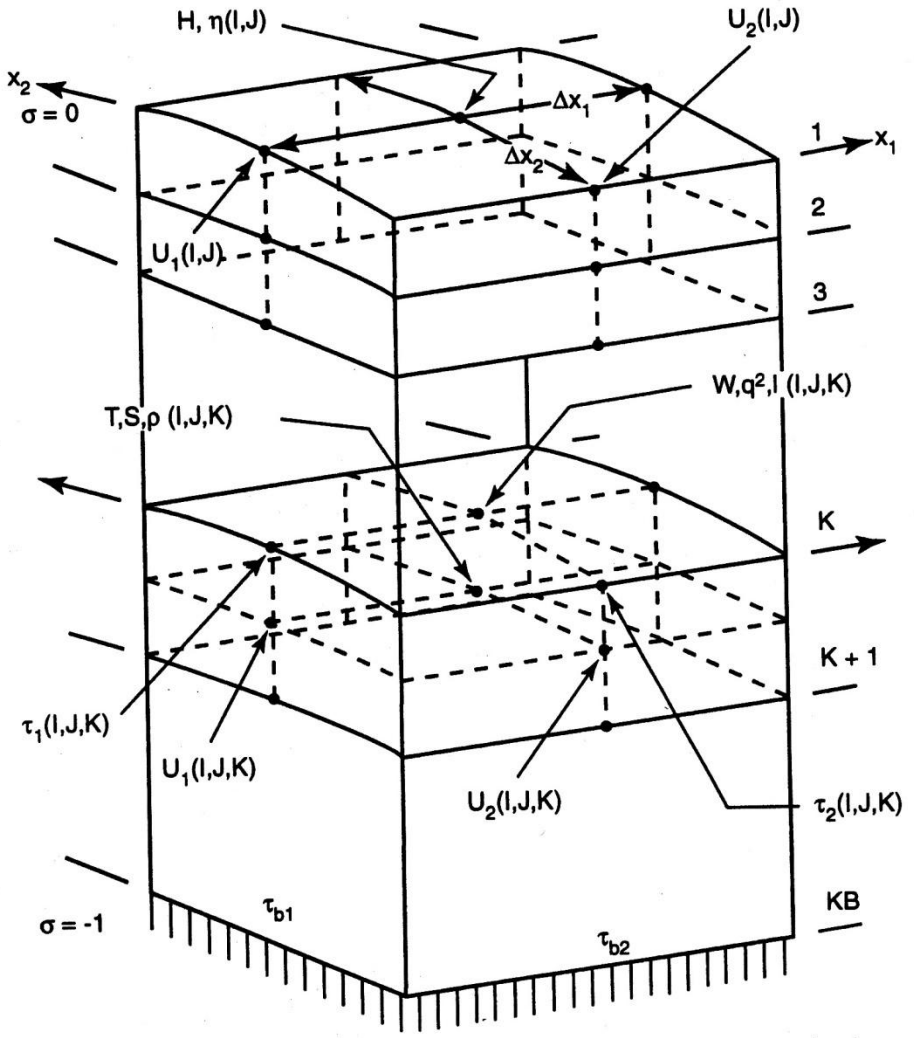
- (1) Data analysis/assimilation → Generalized horizontal average at certain depth (or level)
- (2) Observational profile data → Number of data points varies with depth (or level)
- (3) Stable observational profile data → Unstable analyzed/assimilated profile data

- Ocean observational (*T*, *S*) profile data has different sizes in vertical. The number of observational data may vary with horizontal level, i.e., more data points are assimilated in some levels than others. Due to nonlinearity of the Equation of State, such a treatment may lead to falas static instability.

Grid point (i)
Observational Point (k)



Model Discretization



Data Analysis/Data Assimilation

→ Conducted in the Physical Space (**i**, **j**, **k**)

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{W} \bullet \mathbf{d},$$

Innovation → $\mathbf{d} = \mathbf{y}_o - H(\mathbf{x}_b)$

Key Issue → **W** - Matrix

W → Gain Matrix (OI)

$$\min\left(\boldsymbol{\varepsilon}_a^T \bullet \boldsymbol{\varepsilon}_a\right) \quad \rightarrow$$

$$\mathbf{W} = \mathbf{B}\mathbf{H}^T\left(\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T\right)^{-1}$$

→Depends on **B**

R (usually assumed given)

W – Gain Matrix (3D-Var)

Minimizing Cost Function → $\nabla J(\mathrm{x}) = 0$

$$2J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_b)$$
$$+[\mathbf{y}_o - H(\mathbf{x})]^T \mathbf{R}^{-1}[\mathbf{y}_o - H(\mathbf{x})]$$

$$\mathbf{W} = (\mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H})^{-1}\mathbf{H}^T\mathrm{P}^{-1}$$

→Depends on **B**

R (usually given)

W – Gain Matrix (Kalman Filter)

W – Matrix is called Kalman Gain

$$\mathbf{W}_i = \mathbf{P}^f(t_i)\mathbf{H}^T[\mathbf{R}_i + \mathbf{H}_i\mathbf{P}^f(t_i)\mathbf{H}^T]^{-1}$$

→Depends on forecast error covariance matrix

$$\mathbf{P}^f(t_i)$$

R (known)

Generation of Unrealistic Density Profiles

→ Unrealistic Sound Speed Profiles

→ Affecting ASW

Unstable profiles need to be adjusted before we can use them.

(T, S) Profiles → Adjustment in Data Assimilation with the three Requirements

- (1) Minimal Adjustment
- (2) Conservation of Heat and Salt
- (3) Well-Posed or Analytical (Easy to Get Results)

- All the existing methods do not satisfy these requirements.

Two Existing Methods

- (1) Minimal Adjustment Method (Jackett and McDougal,1995) → JM Method

- (2) Convective Adjustment Method

(1) Minimal Adjustment Method (Jackett and McDougal,1995) JM Method

$$\mathbf{x} = \begin{pmatrix} \mathbf{S} \\ \Theta \end{pmatrix}$$

$$\text{minimize } \|\Delta \mathbf{x}\|^2 \text{ subject to } \mathbf{A}(\mathbf{x} + \Delta \mathbf{x}) \geqslant \mathbf{n}^2,$$

S–θ curve

Weakness → Heat and Salt not Conserved

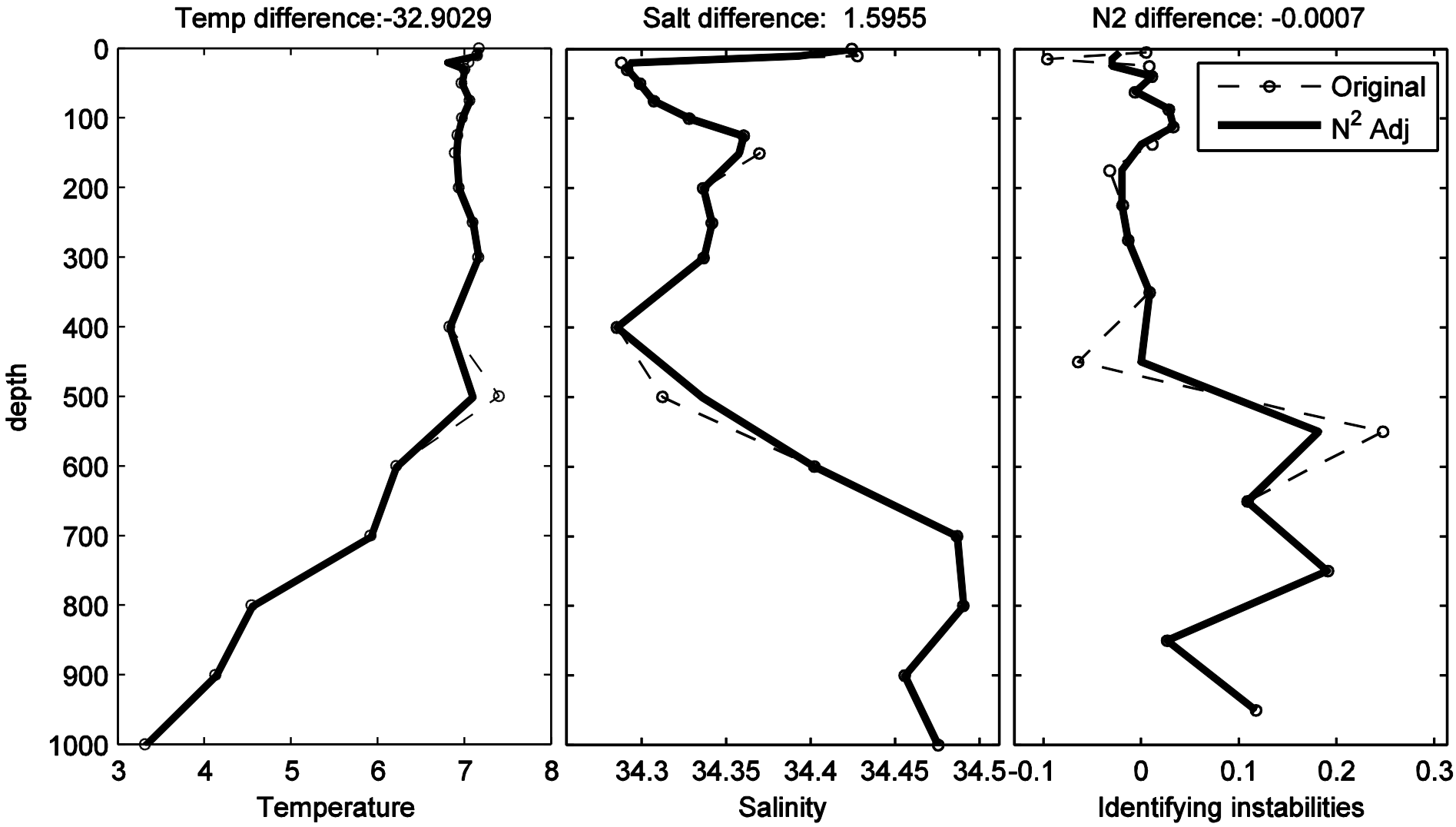
Elements of Matrix **A**

$$
\begin{aligned}
a_{k,k} &= -\beta(S_k^*, \theta_k^*, p_k^*)(p_k - p_{k+1})^{-1} \\
a_{k,k+1} &= -a_{k,k} \\
a_{k,n+k} &= \alpha(S_k^*, \theta_k^*, p_k^*)(p_k - p_{k+1})^{-1} \\
a_{k,n+k+1} &= -a_{k,n+k} \\
a_{k,j} &= 0, \quad j \notin \{k, k+1, n+k, n+k+1\}
\end{aligned}
\tag{7}
$$

for $k = 1, 2, \cdots, n-1$, and the values marked by the asterisk are the midbottle values obtained by linearly interpolating S and θ at $p_k^* = 0.5(p_k + p_{k+1})$.

World Ocean Atlas 2005 (Locarnini et al., 2006)

- Stabilization of (T, S) casts using the JM method



Original (dashed) and adjusted (solid) profiles temperature (T_k), salinity (S_k), and static stability (E_k) at the grid box 171.5°E, 53.5°S using the JM method (Locarnini et al. 2006).

Grid-box 171.5°E, 53.5°S WOA98 profiles before stabilization (from Locarnini et al. 2006, Table B1). Here the symbol '*' in the last column indicates the static instability.

k	Depth (m)	T (°C)	S (ppt)	$\rho(S_{k+1}, T_{k+1}, z_k)$ (kg m^{-3})	$\rho(S_k, T_k, z_k)$ (kg m^{-3})	E_k (kg m^{-3})
1	0	7.1667	34.4243	26.9476	26.9423	0.0054
2	10	7.1489	34.4278	26.8982	26.9939	-0.0957*
3	20	7.0465	34.2880	26.9529	26.9443	0.0085
4	30	7.0050	34.2914	27.0104	26.9990	0.0114
5	50	6.9686	34.2991	27.0967	27.1028	-0.0061*
6	75	7.0604	34.3073	27.2406	27.2120	0.0286
7	100	6.9753	34.3280	27.3892	27.3560	0.0332
8	125	6.9218	34.3604	27.5164	27.5046	0.0117
9	150	6.8919	34.3697	27.6000	27.6316	-0.0316*
10	200	6.9363	34.3364	27.8123	27.8302	-0.0179*
11	250	7.0962	34.3415	28.0295	28.0421	-0.0126*
12	300	7.1622	34.3367	28.2684	28.2593	0.0092
13	400	6.8275	34.2852	28.6664	28.7281	-0.0618*
14	500	7.4001	34.3123	29.3699	29.1238	0.2461
15	600	6.2133	34.4022	29.9386	29.8292	0.1094
16	700	5.9186	34.4868	30.5869	30.3978	0.1891
17	800	4.5426	34.4904	31.0754	31.0488	0.0266
18	900	4.1263	34.4558	31.6539	31.5377	0.1162
19	1000	3.3112	34.4755		32.1176	

Grid-box 171.5° E, 53.5S WOA98 profiles after stabilization using the JM method (from Locarnini et al. 2006, Table B2).

k	Depth (m)	T (°C)	S (ppt)	$\rho(S_{k+1}, T_{k+1}, z_k)$ (kg m^{-3})	$\rho(S_k, T_k, z_k)$ (kg m^{-3})	E_k (kg m^{-3})
1	0	7.1667	34.3096	26.8521	26.8519	0.0002
2	10	7.1489	34.3063	26.8982	26.8982	0.0000
3	20	7.0465	34.2880	26.9529	26.9443	0.0085
4	30	7.0050	34.2914	27.0042	26.9990	0.0051
5	50	7.0132	34.2991	27.0967	27.0967	0.0000
6	75	7.0604	34.3073	27.2361	27.2120	0.0240
7	100	6.9796	34.3228	27.3513	27.3513	0.0000
8	125	6.9897	34.3243	27.4667	27.4667	0.0000
9	150	7.0242	34.3301	27.5820	27.5820	0.0000
10	200	7.0628	34.3364	27.8123	27.8123	0.0000
11	250	7.0962	34.3415	28.0422	28.0421	0.0000
12	300	7.0748	34.3367	28.2719	28.2719	0.0001
13	400	6.8275	34.2894	28.7314	28.7314	0.0000
14	500	6.9604	34.3123	29.3699	29.1899	0.1799
15	600	6.2133	34.4022	29.9386	29.8292	0.1094
16	700	5.9186	34.4868	30.5869	30.3978	0.1891
17	800	4.5426	34.4904	31.0754	31.0488	0.0266
18	900	4.1263	34.4558	31.6539	31.5377	0.1162
19	1000	3.3112	34.4755		32.1176	

Relative root mean adjustment (RRMA)

$$\mathrm{RRMA} =$$

$$\frac{\sqrt{\frac{1}{K}\sum_{k=1}^{K}(\Delta T_k)^2}}{\max(T_k)-\min(T_k)} + \frac{\sqrt{\frac{1}{K}\sum_{k=1}^{K}(\Delta S_k)^2}}{\max(S_k)-\min(S_k)}$$

$$= 0.0712.$$

Weakness of the JM Method → Heat and Salt not Conserved

$$\Delta Q = A\rho_0 c_p \int_{-H}^{0} \Delta T dz$$

$$\Delta(\text{Salt}) = A \int_{-H}^{0} \Delta S dz$$

1° X 1° Area → $A = (\frac{\pi}{180} R)^2 \cos\varphi$

Heat and Salt Change for the (1° X 1°) grid cell

$$\Delta Q = -7.0411 \times 10^{17} \mathrm{J}$$

$$\Delta(\mathrm{salt}) \simeq -0.5443 \times 10^{10} \mathrm{kg}$$

How significant is such an adjustment?

Rough Estimation

- Global Ocean →

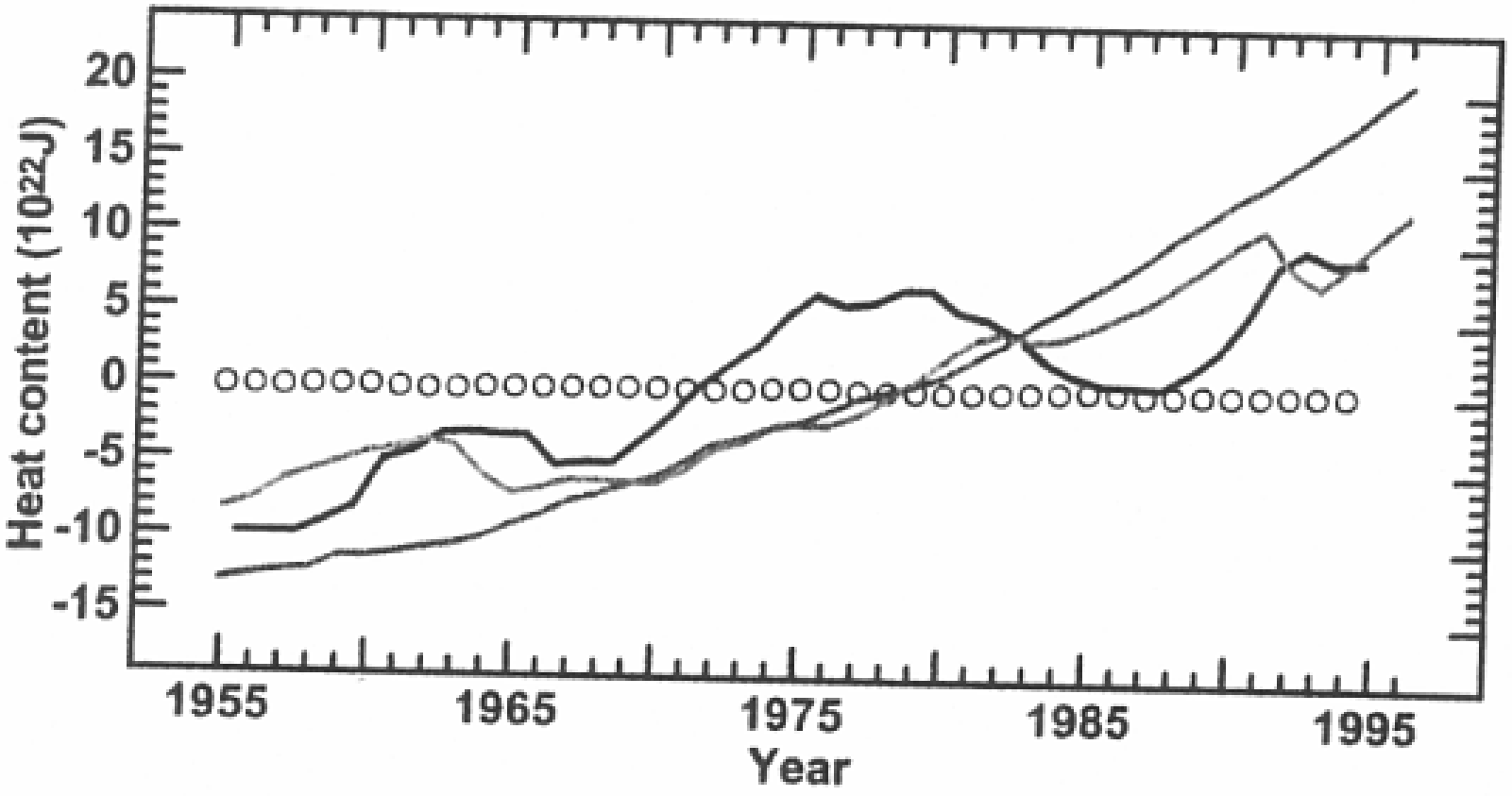
  180X360X70% = 45360

- → 45360X44% → 19958 [1^o X 1^o boxes]

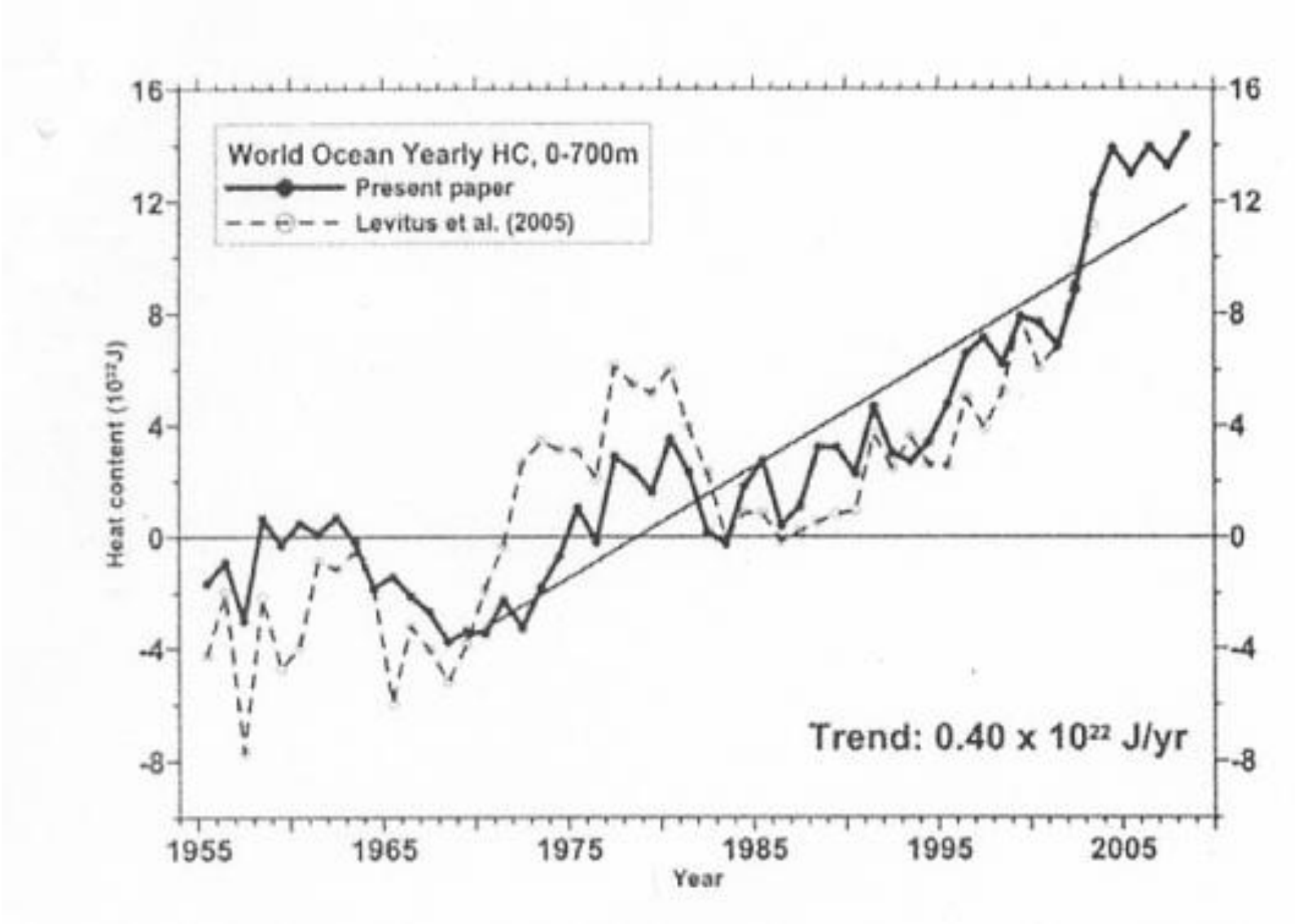
- Error in Heat Change ~ $19958X7.04X10^{17}$ J

  → $1.41X10^{22}$ J

Global Upper Ocean Heat Content (Levitus et al. 2001 Nature)



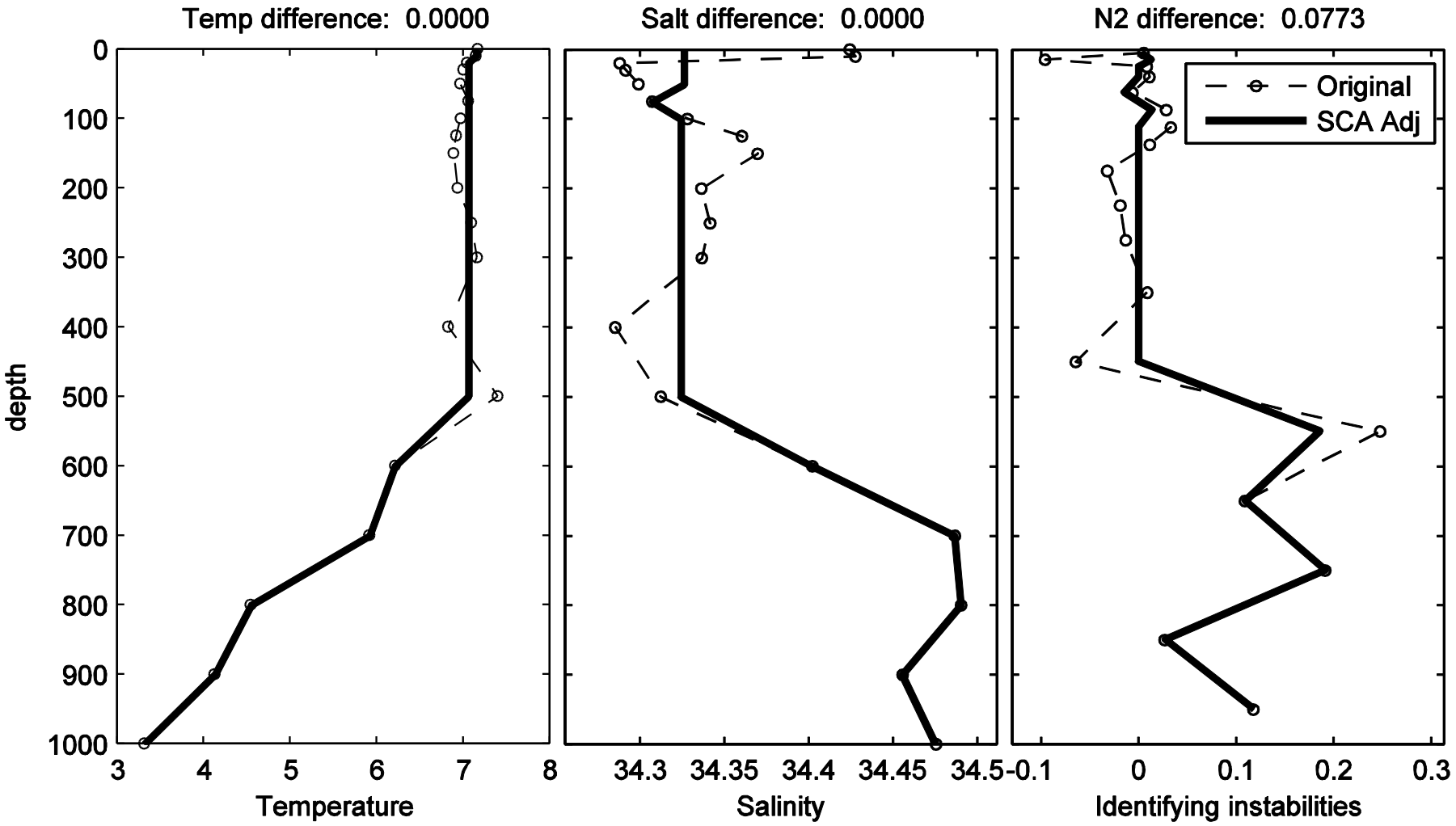
Global Upper Ocean Heat Content (Levitus et al. 2009 GRL)



(2) Convective Adjustment in Ocean Models

- Whenever a water column is statically unstable, temperature and salinity are vertically adjusted to make the water column neutrally stable, with heat and salt conserved in the process.

- For ocean data assimilation, it may over-adjust since there is no convection. Unstable stratification is caused by combination of modeled and observed data.



Original (dashed) and adjusted (solid) profiles temperature (T_k), salinity (S_k), and static stability (E_k) at the grid box 171.5°E, 53.5°S using the complete convective adjustment method (Yin and Sarachik Locarnini 1994).

Performance

- Heat and Salt Conserved

- Large Adjustment

RRMA (Convective Adjustment) = 0.2192 > 0.0712 (JM Method)

(3) A new analytical scheme has been developed satisfying the three requirements

- Chu, P.C., and C.W. Fan, 2010: A conserved minimal adjustment scheme for stabilization of hydrographic profiles. *Journal of Atmospheric and Oceanic Technology*, American Meteorological Society, **27** (6), 1072-1083

Static stability (E)
Lynn and Reid (1968)

- Discrete samples (T_k, S_k) at depth z_k, k = 1, 2, …, K (k increasing downward)

$$E_k = \rho(S_{k+1}, T_{k+1}, z_k) - \rho(S_k, T_k, z_k),$$

$$E_k \geq E_{\min}, \quad k = 1, 2, \ldots, K$$

$E_{\min}$ is the reference value for the minimum static stability

Stabilization

- (a) stability increasing at unstable levels to
- $$E^*_{k_1} = E_{\min} \quad \rightarrow \quad \Delta E_{k_1} = E_{\min} - E_{k_1}$$
- (b) stability decreasing at stable levels

$$E^*_{k_1 \pm n} = \begin{cases} E_{k_1 \pm n} - \Delta E_{k_1} / 2^{m+1} & \text{if } E_{k_1 \pm n} - \Delta E_{k_1} / 2^{m+1} \geq E_{\min} \\ E_{\min} & \text{if } E_{k_1 \pm n} - \Delta E_{k_1} / 2^{m+1} < E_{\min} \end{cases}$$

- (c) normalization for conservation of stability for the cast.

$$I = \sum_{k=1}^{K} E_k, \quad I^* = \sum_{k=1}^{K} E^*_k \qquad E^{**}_k = \frac{I}{I^*} E^*_k$$

2*K* Basic Algebraic Equations for Adjustment

$$\rho(S_{k+1}+\Delta S_{k+1}, T_{k+1}+\Delta T_{k+1}, z_k)-\rho(S_k+\Delta S_k, T_k+\Delta T_k, z_k)=E_k^{**},$$
$$k=1,\ 2,\ \ldots,\ K\text{-}1.$$

Heat Conservation → $$\sum_{k=1}^{K-1}\frac{(\Delta T_k+\Delta T_{k+1})}{2}(z_k-z_{k+1})=0$$

Salt Conservation →

Assigned (T, S) Adjustment Ratio y_k → $$\Delta T_k+y_k\Delta S_k=0$$

k = 1, 2, … *K*-1

Various Methods to Determine γ_k

- (1) Changing in angle in the T-S diagram
- (2) Variability of (T,S) profile

$$\gamma_k = \gamma \equiv \frac{\max(T_k) - \min(T_k)}{\max(S_k) - \min(S_k)},$$

$$\mathbf{P} \equiv \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ \vdots \\ \vdots \\ \vdots \\ p_{M-1} \\ p_M \end{bmatrix} = \begin{bmatrix} \Delta T_1 \\ \Delta S_1 \\ \Delta T_2 \\ \Delta S_2 \\ \vdots \\ \vdots \\ \vdots \\ \Delta T_K \\ \Delta S_K \end{bmatrix}, \qquad M = 2K.$$

2K Basic Algebraic Equations

$$\mathbf{F}(\mathbf{P}) = \mathbf{0} \qquad (1)$$

F has the dimension of 2*K*

Combination of (*K* -1) Nonlinear

and (*K*+1) Linear Equations

Newton Method to Solve (1)

$$\mathbf{P}^{(j+1)} = \mathbf{P}^{(j)} - \mathbf{J}_F^{-1}(\mathbf{P}^{(j)})\mathbf{F}(\mathbf{P}^{(j)})$$

$$j = 0,\ 1,\ 2,...$$

$$\mathbf{P}^{(j+1)} = \mathbf{P}^{(j)} + \mathbf{d}^{(j)}$$

$$\mathbf{J}_F(\mathbf{P}^{(j)}) \bullet \mathbf{d}^{(j)} = \mathbf{b}^{(j)}$$

$$\mathbf{b}^{(j)} \equiv -\mathbf{F}(\mathbf{P}^{(j)})$$

$$\mathbf{J}_F(\mathbf{P}^{(j)}) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1M} \\ a_{21} & a_{22} & \cdots & a_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ a_{M1} & a_{M2} & \cdots & a_{MM} \end{bmatrix}$$

$$\mathbf{J}_F(\mathbf{P}^{(j)}) = \begin{bmatrix} * & 0 & * & 0 & * & 0 & \dots & \dots & * & 0 \\ 0 & * & 0 & * & 0 & * & \dots & \dots & 0 & * \\ * & * & 0 & 0 & 0 & 0 & \dots & \dots & 0 & 0 \\ * & * & * & * & 0 & 0 & \dots & \dots & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 & \dots & \dots & 0 & 0 \\ 0 & 0 & * & * & * & * & \dots & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & \dots & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & \dots & * & * & * & * \end{bmatrix}$$

$$a_{11} = \frac{\Delta z_1}{2}, a_{13} = \frac{\Delta z_1 + \Delta z_2}{2}, \ldots, a_{1,M-3} = \frac{\Delta z_{N-2} + \Delta z_{N-1}}{2}, a_{1,M-1} = \frac{\Delta z_{N-1}}{2},$$

$$a_{12} = a_{14} = a_{16} = \ldots\ldots = a_{1M} = 0,$$

$$a_{22} = \frac{\Delta z_1}{2}, a_{24} = \frac{\Delta z_1 + \Delta z_2}{2}, \ldots, a_{2,M-2} = \frac{\Delta z_{N-2} + \Delta z_{N-1}}{2}, a_{2M} = \frac{\Delta z_{N-1}}{2}$$

$$a_{21} = a_{23} = a_{25} = \ldots = a_{2,M-1} = 0,$$

$$a_{31} = 1,\ a_{32} = \gamma_1,\ \ a_{33} = a_{34} = a_{35} = \ldots = a_{3M} = 0$$

$$a_{41} = \frac{\partial \rho\left(S_1 + \Delta S_1^{(j)}, T_1 + T_1^{(j)}, z_2\right)}{\partial T}$$

$$= -\rho\left(S_1 + \Delta S_1^{(j)}, T_1 + \Delta T_1^{(j)}, z_2\right)\alpha\left(S_1 + \Delta S_1^{(j)}, T_1 + \Delta T_1^{(j)}, z_2\right),$$

$$a_{42} = \frac{\partial \rho\left(S_1 + \Delta S_1^{(j)}, T_1 + \Delta T_1^{(j)}, z_2\right)}{\partial S}$$

$$= \rho\left(S_1 + \Delta S_1^{(j)}, T_1 + \Delta T_1^{(j)}, z_2\right)\beta\left(S_1 + \Delta S_1^{(j)}, T_1 + \Delta T_1^{(j)}, z_2\right),$$

$$a_{43} = -\frac{\partial \rho\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_2\right)}{\partial T}$$

$$= \rho\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_2\right)\alpha\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_2\right),$$

$$a_{44} = -\frac{\partial \rho\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_2\right)}{\partial S}$$

$$= -\rho\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_2\right)\beta\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_2\right),$$

$$a_{45} = a_{46} = ... = a_{4M} = 0,$$

$$a_{53} = 1, \quad a_{54} = \gamma_2,$$

$$a_{51} = a_{52} = a_{55} = a_{56} = a_{57} = ... = a_{5M} = 0,$$

$$a_{63} = \frac{\partial \rho\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_3\right)}{\partial T}$$

$$= -\rho\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_3\right)\alpha\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_3\right),$$

$$a_{64} = \frac{\partial \rho\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_3\right)}{\partial S}$$

$$= \rho\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_3\right)\beta\left(S_2 + \Delta S_2^{(j)}, T_2 + \Delta T_2^{(j)}, z_3\right),$$

$$a_{65} = -\frac{\partial \rho\left(S_3 + \Delta S_3^{(j)}, T_3 + \Delta T_3^{(j)}, z_3\right)}{\partial T}$$

$$= \rho\left(S_3 + \Delta S_3^{(j)}, T_3 + \Delta T_3^{(j)}, z_3\right)\alpha\left(S_3 + \Delta S_3^{(j)}, T_3 + \Delta T_3^{(j)}, z_3\right),$$

$$a_{66} = -\frac{\partial \rho\left(S_3 + \Delta S_3^{(j)}, T_3 + \Delta T_3^{(j)}, z_3\right)}{\partial S}$$

$$= -\rho\left(S_3 + \Delta S_3^{(j)}, T_3 + \Delta T_3^{(j)}, z_3\right)\beta\left(S_3 + \Delta S_3^{(j)}, T_3 + \Delta T_3^{(j)}, z_3\right),$$

$$a_{61} = a_{62} = a_{67} = a_{68} = \ldots\ldots = a_{6M} = 0,$$

……

$$a_{M-1,M-3}=1,\quad a_{M-1,M-2}=y_{N-1},$$

$$a_{M-1,1}=a_{M-1,2}=\cdots\cdots=a_{M-1,M-4}=a_{M-1,M-1}=a_{M-1,M}=0,$$

$$a_{M,M-3}=\frac{\partial\rho\left(S_{K-1}+\Delta S_{K-1}^{(j)},T_{K-1}+\Delta T_{K-1}^{(j)},z_K\right)}{\partial T}$$

$$=-\rho\left(S_{K-1}+\Delta S_{K-1}^{(j)},T_{K-1}+\Delta T_{K-1}^{(j)},z_K\right)\alpha\left(S_{K-1}+\Delta S_{K-1}^{(j)},T_{K-1}+\Delta T_{K-1}^{(j)},z_K\right),$$

$$a_{M,M-2}=\frac{\partial\rho\left(S_{K-1}+\Delta S_{K-1}^{(j)},T_{K-1}+\Delta T_{K-1}^{(j)},z_K\right)}{\partial S}$$

$$=\rho\left(S_{K-1}+\Delta S_{K-1}^{(j)},T_{K-1}+\Delta T_{K-1}^{(j)},z_K\right)\beta\left(S_{K-1}+\Delta S_{K-1}^{(j)},T_{K-1}+\Delta T_{K-1}^{(j)},z_K\right),$$

$$a_{M,M-1}=-\frac{\partial\rho\left(S_K+\Delta S_K^{(j)},T_K+\Delta T_K^{(j)},z_K\right)}{\partial T}$$

$$=\rho\left(S_K+\Delta S_K^{(j)},T_K+\Delta T_K^{(j)},z_K\right)\alpha\left(S_K+\Delta S_K^{(j)},T_K+\Delta T_K^{(j)},z_K\right),$$

$$a_{MM}=-\frac{\partial\rho\left(S_K+\Delta S_K^{(j)},T_K+\Delta T_K^{(j)},z_K\right)}{\partial S}$$

$$=-\rho\left(S_K+\Delta S_K^{(j)},T_K+\Delta T_K^{(j)},z_K\right)\beta\left(S_K+\Delta S_K^{(j)},T_K+\Delta T_K^{(j)},z_K\right),$$

$$b_1 = 0, \quad b_2 = 0, \quad b_3 = 0,$$

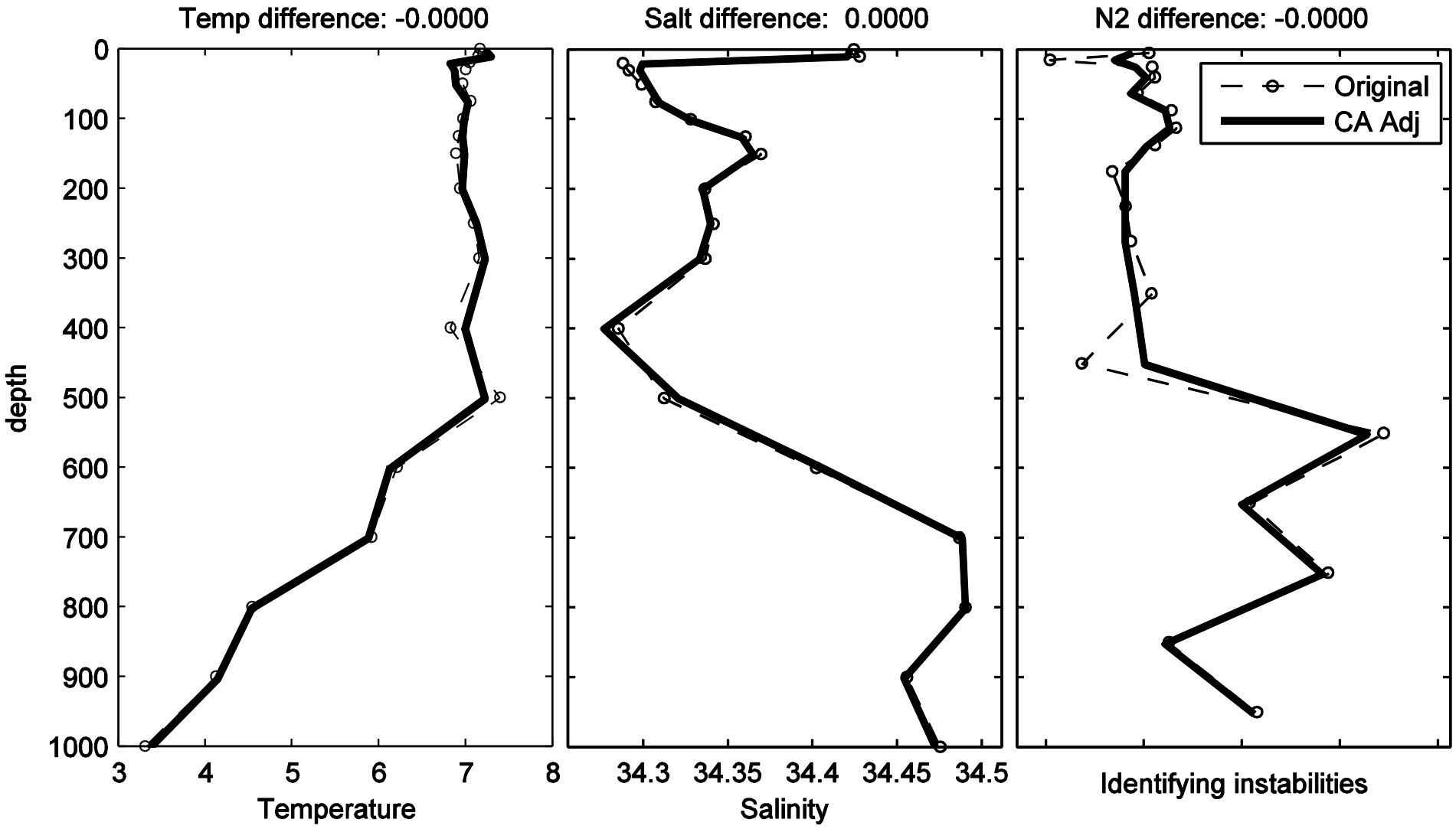
$$b_4 = E_1^{**} - \rho\left(S_1 + \Delta S_1^{(J)}, T_1 + \Delta T_1^{(J)}, z_1\right) + \rho\left(S_2 + \Delta S_2^{(J)}, T_2 + \Delta T_2^{(J)}, z_2\right),$$

$$b_5 = 0,$$

$$b_6 = E_2^{**} - \rho\left(S_2 + \Delta S_2^{(J)}, T_2 + \Delta T_2^{(J)}, z_2\right) + \rho\left(S_3 + \Delta S_3^{(J)}, T_3 + \Delta T_3^{(J)}, z_2\right), \ldots,$$

$$b_{M-1} = 0,$$

$$b_M = E_{K-1}^{**} - \rho\left(S_{K-1} + \Delta S_{K-1}^{(J)}, T_K + \Delta T_{K-1}^{(J)}, z_{K-1}\right) + \rho\left(S_K + \Delta S_K^{(J)}, T_K + \Delta T_K^{(J)}, z_{K-1}\right).$$



Original (dashed) and adjusted (solid) profiles temperature (T_k), salinity (S_k), and static stability (E_k) at the grid box 171.5°E, 53.5°S using the analytical conserved method.

Performance

- Heat and Salt Conserved
- Minimal Adjustment

RRMA = 0.0482 < 0.0712 (JM Method)

- Analytical → Well-Posed

Conclusions

- False static instability is generated by data analysis and data assimilation.
- Minimal conserved adjustment is needed after (T, S) data analysis/assimilation
- The proposed method has the following features:
  - (1) Heat and salt conservation
  - (2) Removal of static instabilities with small (T, S) adjustments
  - (3) Analytical form