Suppression of Enemy Air Defenses (SEAD) as an Information Duel

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Abstract: Blue strike aircraft enter region $\mathcal{R}$ to attack Red targets. In Case 1, Blue conducts (preplanned) SEAD to establish air superiority. In the (reactive) SEAD scenario, which is Case 2, such superiority is already in place, but is jeopardized by prohibitive interference from Red, which threatens Blue's ability to conduct missions. We utilize both deterministic and stochastic models to explore optimal tactics for Red in such engagements. Policies are developed which will guide *both* Red's determination of the modes of operation of his engagement radar, *and* his choice of Blue opponent to target next. An index in the form of a simple transaction kill ratio plays a major role throughout. © 2002 Wiley Periodicals, Inc. Naval Research Logistics 49: 723–742, 2002; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/nav.10046*

1. INTRODUCTION: SUPPRESSION OF ENEMY AIR DEFENSES (SEAD)

In examining SEAD operations it is useful to determine the tactics that maximize the effectiveness of either the attacking aircraft or the defending integrated air defense system (IADS). This paper develops models based on two opposing sides: Blue and Red. Blue strike aircraft (the attackers) are entering a region, denoted as $\mathcal{R}$, to attack Red targets. Two cases are considered. First Blue conducts SEAD—either concurrently with other air missions or prior to other missions—to establish air superiority. In the second case Blue has already established air superiority, but then Red introduces prohibitive interference that threatens Blue's ability to conduct its missions.

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The models are illustrated based on U.S. Marine Corps SEAD doctrine (U.S. Marine Corps [13]). Joint doctrine provides a general description of SEAD operations, but does not address specific tactics. Marine Corps SEAD doctrine is well developed and clearly situates the two cases modeled. The models and their results, however, are not limited to Marine Corps air operations. The Red and Blue tactics described by the models can be applied to any side of any conflict involving attacking aircraft opposed by an IADS. Appendix A provides SEAD-related definitions paraphrased from doctrinal publications.

1.1. Pre-Planned SEAD

The first case is described by Marine Corps SEAD doctrine as *pre-planned SEAD.* Joint doctrine (Joint Chiefs of Staff [6]) describes this type of SEAD as *localized.* In preplanned SEAD, Blue aircraft are specifically assigned the task of suppressing air defenses. These efforts are conducted either concurrently with other air missions (air interdiction, aerial reconnaissance, etc.)—to create a window of air superiority, or they are conducted prior to other air missions—to roll back the effective coverage of Red's IADS.

Blue attackers must enter the engagement envelopes of Red surface-to-air missiles (SAMs) to reach their targets in region $\mathcal{R}$. Red SAMs within $\mathcal{R}$ attempt to prevent the Blue attackers from reaching the target area. Blue attackers are supported by Blue SEAD aircraft.

1.2. Reactive SEAD

The second case involves *Reactive SEAD* (RSEAD). Joint doctrine describes these types of missions as *opportune SEAD* (Joint Chiefs of Staff [6]). In this case, Blue has previously established air superiority in $\mathcal{R}$ and is conducting air operations such as close air support (CAS). Red has introduced prohibitive interference (a threat to Blue's air superiority) with a mobile SAM. Red is attempting to maximize the number of Blue aircraft killed before the SAM is suppressed.

Blue is forced to eliminate the prohibitive interference. Several levels of RSEAD are available: electronic warfare (EW) in support of CAS (Posadas [12]), reactive anti-radiation missile (ARM) attacks, or a coordinated strike against Red's SAM.

1.3. SEAD as an Information Duel

The Red IADS gathers and processes targeting information on Blue attackers. The IADS must complete five functions: detect, identify, track, assign the target, and control of weapons (U.S. Marine Corps [13]). In the pre-planned SEAD case, Red early warning (EW) radars first detect Blue attackers, and then identify them as hostile. Next, Red fuses data from multiple sensors to correlate Blue attackers as target tracks. Blue aircraft are tracked until they can be assigned as targets to engagement radars co-located with Red SAM batteries. Finally, Red uses engagement radars to control its missiles through target intercepts. The first four functions (detect, identify, track, assign the target) are conducted by a command node typically co-located with the EW radar. Weapons control is executed locally by the SAM.

In the RSEAD case, the Red IADS studied here consists of a single mobile SAM system. All five functions are conducted by the mobile SAM system. The Red SAM has placed itself amidst Blue air operations and is operating autonomously (Posadas [12]). While passive measures (such as optical and infrared measures) are typically used to track Blue aircraft, Red must still employ engagement radars for missile guidance once a SAM is fired.

The Red SAMs in both models employ command guidance or semi-active guidance (MAWTS-1 [10]). These types of SAM guidance require engagement radars (target tracking, missile guidance, and illumination radars) to radiate throughout the SAM engagement until missile impact. Blue's SEAD aircraft remain outside Red's engagement envelopes and employ ARMs against Red's engagement radars. The Blue ARMs passively home on Red engagement radar emissions. On impact, ARMs inflict damage on the radar and its surroundings.

Red engagement radars can either operate in a continuous emission mode or in an intermittent (blinking) mode. The intermittent mode is a form of emissions control used by Red for electronic protection (MAWTS-1 [10]). Continuous mode maximizes the kill probability of Red SAMs on Blue aircraft. However, continuous mode also maximizes the effectiveness of Blue ARMs against Red SAM engagement radars. Intermittent mode reduces Red's ability to kill Blue attackers, but also diminishes the effectiveness of Blue's ARMs.

Red's engagement radar tactics drives the information exchange in these engagements. Continuous radiation by Red maximizes targeting information for both sides. Red requires precise position information on Blue attackers for successful SAM engagements. Blue requires Red emissions to provide guidance information for the ARMs. Intermittent mode presents a tradeoff option for Red by which more Red SAMs survive, but fewer Blue attackers are killed.

The missile duel (SAM vs. ARM) draws its results from the information duel (Red IADS targeting vs. Blue SEAD targeting). Red controls the information exchange rate and must establish a tactic that maximizes the number of surviving Red SAMs, while simultaneously maximizing the number of Blue kills. In the case of multiple kinds of Blue aircraft, Red can also decide the order in which it engages the aircraft. In the case of pre-planned SEAD, Blue must develop a tactic that properly sizes its attack force to succeed in its mission. In the RSEAD case, Blue seeks to determine the lethality of the Red SAM: How many Blue aircraft would be killed before the Red SAM is killed? Blue also attempts to determine the best aircraft type to use when attacking the Red SAM.

1.4. Modeling the Information Duel

Models 1 and 2 describe the pre-planned and reactive SEAD cases in terms of simple state-space models. Both deterministic and stochastic modeling techniques are used. The stochastic models are Markov processes and Markov decision processes that can be solved explicitly in the present circumstances. These models may also be implemented in object-oriented simulations with stochastic and adaptive features, but this is not carried out in the present paper.

In each model, more attempted information acquisition exposes the defender to greater risk. Both models share a common feature: Information acquisition can be both beneficial and harmful. Both models share a common result: It is possible to derive a tactic that balances the benefit and disadvantage of information acquisition. Because of the uncomplicated approach, it is possible to explicitly characterize "optimal" strategies in simple form.

Other analyses of SEAD include Macfadzean [8], Keaney and Cohen [7], and Bailey [1]. High-resolution Monte Carlo simulation is most often used to study suppression of enemy air defense. An example is *Suppressor*—a Monte Carlo simulation developed for the Air Force and maintained at Wright-Patterson AFB in Dayton, Ohio. Bailey [1] presents an analytical model to study the effects of jamming. The two basic models presented in this paper are limited to SEAD using ARMs.

2. MODEL 1: AN EXPLORATORY DETERMINISTIC MODEL FOR PREPLANNED SEAD

2.1. Assumptions

The following assumptions apply to Model 1:

1. Pre-planned SEAD is being employed by Blue in region $\mathcal{R}$, against a fully operational Red IADS.
2. Blue attackers must enter Red SAM engagement envelopes to reach their targets. The time spent inside Red engagement zone is Blue's vulnerability window (measured in time).
3. Red is defending $\mathcal{R}$ with several SAMs, using command or semi-active guidance. These SAMs have overlapping engagement envelopes that create a single engagement zone.
4. Red engagement radars are co-located with Red SAM batteries.
5. Red engagement radars may operate in either continuous or intermittent mode; all Red engagement radars operate in the same mode.
6. Red does not present a significant fighter threat to Blue.
7. Blue attackers start beyond the radar horizon (undetected).
8. Blue SEAD aircraft are separated from the Blue attackers, and remain outside Red engagement envelopes.
9. Blue SEAD aircraft fire ARMs on a pre-emptive timeline and have sufficient ARMs to cover the Blue attackers' vulnerability window.
10. Red has sufficient SAMs to attack throughout the vulnerability window.

2.2. Scenario

Red's IADS includes a constant total of R_{EW} early warning radars and $R_A(t)$ engagement radars at time t. The IADS functions are represented by ξ_{EW}—the rate at which the IADS detects, identifies, correlates, tracks and assigns targets, and ν_{RB}—the rate at which the SAMs fire and complete engagements (weapons control).

At time $t = 0$, a force of $B_U(0)$ Blue attackers crosses the radar horizon of the R_{EW} early warning radars. The number of Blues present, but undetected, at t (undetected state) is $B_U(t)$ for any $t > 0$. Blue attackers are modeled as detected at a rate in time proportional to the number of undetected Blue attackers at time $t > 0$; (this assumption is easily relaxed, e.g., to essentially simultaneous detection). The number of Blue attackers detected at time t (detected state) is denoted by $B_D(t)$. The Blue force also includes B_S standoff SEAD aircraft that fire ARMs at Red engagement radars. These aircraft are not subject to attrition by the Red IADS.

Blues detected by Red are processed through the first four IADS functions. Once Blue target tracks are assigned to SAM batteries, they are processed by individual engagement radars. The Red IADS passing of Blue target tracks to Red engagement radars is modeled as a service/queuing system with $R_A(t)$ "servers" (i.e., engagement radars). A *service time* is the time for an engagement to be completed—from missile firing to an impact or a miss.

2.3. Dynamic Equations

The following dynamic equations describe Model 1.

$$\frac{dB_U(t)}{dt} = \underbrace{\lambda(t)}_{\substack{\text{Blue arrival} \\ \text{rate at } t}} - \underbrace{\xi_{EW} B_U(t) R_{EW}}_{\substack{\text{detection rate of Blue} \\ \text{by Red early warning}}}. \tag{1}$$

Equation (1) describes the decline of $B_U(t)$—the undetected Blue attacker population in $\mathcal{R}$ at t. It does not model saturation of the Red early warning system. Undetected Blues become detected in proportion to their number.

$$\frac{dB_D(t)}{dt} = \underbrace{\xi_{EW} B_U(t) R_{EW}}_{\substack{\text{detection rate of Blue} \\ \text{by Red early warning}}} - \underbrace{\nu_{RB} R_A(t) \frac{B_D(t)}{1 + B_D(t)} (\theta_{RI} P_{RI} + \theta_{RQ} P_{RQ})}_{\substack{\text{attrition rate of detected Blues resulting from shooters cued by} \\ \text{acquisition radar, either emitting continuously or intermittently}}}. \tag{2}$$

The complete Blue attrition term $\nu_{RB} R_A(t)(B_D(t)/(1 + B_D(t)))$ represents the rate at which Red SAMs engage detected Blues; the component term $(B_D(t)/(1 + B_D(t)))$ represents saturation of the Red engagement radars by Blue attackers in queue (on the target list). If $B_D(t)$ significantly exceeds unity, then Red SAMs can only complete target engagement at a rate proportional to their own (current) force size (see Filipiak [2], and Gaver and Jacobs [3]). Saturability at a larger value can be adjusted by adding a parameter, and provisions for Red's loss of Blue target tracks could likewise be made. The effect of Blue decoys can also be added.

The term $(\theta_{RI} P_{RI} + \theta_{RQ} P_{RQ})$ represents the kill probability of a Red SAM that *either* chooses to engage a target using continuous emission (the probability of this choice is θ_{RI}), in which case the resulting kill probability is P_{RI}; *or* uses intermittent mode (the probability of this choice is $\theta_{RQ} = 1 - \theta_{RI}$), with kill probability P_{RQ}. P_{RI} is greater than P_{RQ}. Although continuous mode leads to higher kill probability, it also exposes the Red SAMs to more risk from Blue ARMS. The parameters θ_{RI} and θ_{RQ} are Red decision variables. Simple rules are next developed for setting these decision variables.

The dynamic equation for Red engagement radars, $R_A(t)$, is:

$$\frac{dR_A(t)}{dt} = -\underbrace{\left(\nu_{RB} B_S R_A(t) \frac{B_D(t)\theta_{RI}}{1 + B_D(t)}\right) \cdot \nu_{BR} P_{BRI}}_{\substack{\text{rate at which continuously emitting Red acquisition} \\ \text{radars lead to retaliatory kills by Blue standoff weapon}}} - \underbrace{\left(\nu_{RB} B_S R_A(t) \frac{B_D(t)\theta_{RQ}}{1 + B_D(t)}\right) \cdot \nu_{BR} P_{BRQ}}_{\substack{\text{rate at which intermittently emitting Red acquisition} \\ \text{radars lead to retaliatory kills by Blue standoff weapon}}}. \tag{3}$$

The first term, $(\nu_{RB} R_A(t)(B_D(t)\theta_{RI}/(1 + B_D(t))))$, represents the (saturable) rate at which the current Red SAMs engage Blue attackers while in continuous emission mode (probability θ_{RI}). This rate translates into a rate of attrition of the Red engagement radars in continuous emission mode by Blue SEAD aircraft: $B_S \nu_{BR} P_{BRI}$. The subsequent term is the same as the last, but accounts for the occasions on which Red engagement radars emit intermittently, and hence are killed at a smaller rate: $B_S \nu_{BR} P_{BRQ}$. Although a mixed policy is available, Red is assumed to employ a pure strategy (set θ_{RI} and θ_{RQ} to *either* 1 or 0).

2.4. Analysis

Suppose the "combat clock" is started at $t = 0$, with all Blue attackers initially present but undetected at that time (no continuous arrival of Blue attackers in the present example). Equations (1)–(3) can be explicitly analyzed in closed form if $\lambda(t) \equiv 0$ for all t (all Blues initially present). The solution to (1) is

$$B_U(t) = B_U(0)\exp\{-\xi R_{EW} t\}. \tag{4}$$

Let

$$\bar{\theta}_R = (\theta_{RI} P_{RI} + \theta_{RQ} P_{RQ})$$

and

$$\bar{\theta}_B = (\theta_{RI} P_{BRI} + \theta_{RQ} P_{BRQ}).$$

Adding Eqs. (1) and (2) results in

$$\frac{d\bar{B}(t)}{dt} = -R_A(t)\frac{B_D(t)}{1 + B_D(t)}\bar{\theta}_R \nu_{RB}, \tag{5}$$

where $\bar{B}(t) = B_U(t) + B_D(t)$; of course, $\bar{B}(0) = B_U(0)$.

Equation (3) can be rewritten as

$$\frac{1}{\bar{\theta}_B}\frac{dR_A(t)}{dt} = -\nu_{RB}\nu_{BR}B_S R_A(t)\frac{B_D(t)}{1 + B_D(t)}. \tag{6}$$

Divide equation (6) by (5), which results in

$$\frac{dR_A(t)}{d\bar{B}(t)} = \nu_{BR} B_S \frac{\bar{\theta}_B}{\bar{\theta}_R}. \tag{7}$$

Integrating results in

$$[\bar{B}(t) - \bar{B}(0)] = \frac{1}{\nu_{BR} B_S}\frac{\bar{\theta}_R}{\bar{\theta}_B}[R_A(t) - R_A(0)]. \tag{8}$$

Notice that the solution is essentially parameterized by an exchange ratio, $\bar{\theta}_R/\bar{\theta}_B$, and the rate at which SEAD Blue aircraft fire at Red engagement radars, ν_{BR}.

Since $\lambda(t) = 0$ for all t, if $t \to \infty$, then either $\lim_{t\to\infty} \bar{B}(t) = 0$ or $\lim_{t\to\infty} R_A(t) = 0$. In fact, if $\bar{B}(0) > (1/\nu_{BR} B_S)(\bar{\theta}_R/\bar{\theta}_B) R_A(0)$, then

$$\bar{B}(\infty) = \bar{B}(0) - \frac{1}{\nu_{BR} B_S}\frac{\bar{\theta}_R}{\bar{\theta}_B} R_A(0) \qquad \text{and} \qquad R_A(\infty) = 0; \tag{9a}$$

i.e., Blue kills all Red SAM engagement radars; while if $\bar{B}(0) < (1/\nu_{BR}B_S)(\bar{\theta}_R/\bar{\theta}_B)R_A(0)$, then $\bar{B}(\infty) = 0$ otherwise, and

$$R_A(\infty) = R_A(0) - \nu_{BR}B_S\frac{\bar{\theta}_B}{\bar{\theta}_R}\bar{B}(0). \tag{9b}$$

So some Red engagement radars survive and are available for countering later attacks.

For a fixed rate at which Blue SEAD aircraft fire at Red engagement radars, ν_{BR}, it is always to the advantage of Red to maximize the exchange ratio $\bar{\theta}_R/\bar{\theta}_B$. Doing so either maximizes Blue's losses if $\bar{B}(0)$ is sufficiently large, or maximizes Red's survivors. Differentiation of the exchange ratio shows that

$$\textit{Red should emit continuously } (\theta_{RI} = 1) \qquad \text{if} \qquad \frac{P_{RI}}{P_{BRI}} > \frac{P_{RQ}}{P_{BRQ}}$$

$$\textit{or, equivalently,} \qquad \text{if} \qquad \frac{P_{RI}}{P_{RQ}} > \frac{P_{BRI}}{P_{BRQ}} \tag{10}$$

Otherwise, Red should employ intermittent mode.

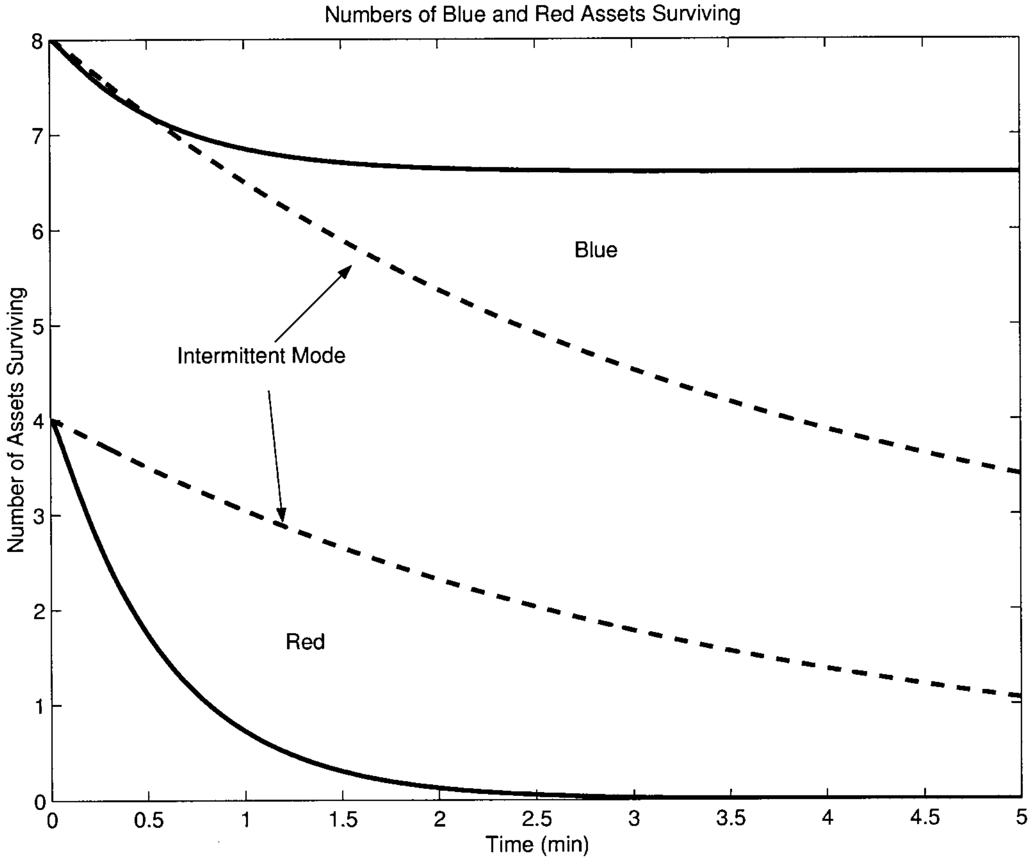
Therefore, Red should emit continuously if Red's relative advantage from so doing *exceeds* the relative advantage to Blue. It is noteworthy that in this model the optimal strategy for Red holds, regardless of the value of ν_{RB}, Red's attrition rate on Blue. Nor does the result depend on ξ_{EW}, the completion rate of the first four Red IADS functions. Section 3 results in the reappearance of this tactic in the context of a seemingly quite different stochastic model. Blue planners can use these results to size an attacking force.

Figures 1 and 2 display the numbers of Red and Blue assets remaining as a function of time. Parameters for each figure are shown. In Figure 1, there exists one Red early warning radar ($R_{EW} = 1$), and Blue target tracks are processed through the first four IADS functions at the rate of 10 per minute ($\xi_{EW} = 10$). Each Red SAM can complete one engagement per minute ($\nu_{RB} = 1$, the Red service rate). The probability of a Red engagement radar, in continuous mode, killing a Blue attacker, P_{RI}, is 0.7. In intermittent mode, this kill probability, P_{RQ}, is 0.5. Each Blue SEAD aircraft is capable of firing ARMs at the rate of 1 per minute ($\nu_{RB} = 1$, the Blue service rate). The probability of a Blue ARM killing a Red engagement radar in continuous mode, P_{BRI}, is 0.5. Against engagement radars in intermittent mode, the kill probability, P_{BRQ}, is 0.08.

Figures 1 and 2 involve eight attacking Blue aircraft and four Blue SEAD aircraft against four Red SAMs. The vulnerability window for Blue attackers is 5 min.

Figure 1 compares the numbers of remaining Red and Blue assets as a function of Red's engagement radar mode (continuous or intermittent). Intermittent mode results in fewer Red casualties and more kills on Blue attackers. More Reds survive and a specified number of Blues is killed sooner in intermittent mode for the Figure 1 case.

In Figure 2, P_{BRI} is decreased from 0.5 to 0.1. As a result, Blue's relative advantage (P_{BRI}/P_{BRQ}) decreases from 6.25 to 1.25. Red's relative advantage is 1.4 in both cases. In the case of Figure 2, continuous mode results in more kills for both sides. However, a more significant increase is observed in number of Blues killed than in the number of Reds killed, when Red operates in continuous mode in Figure 2.



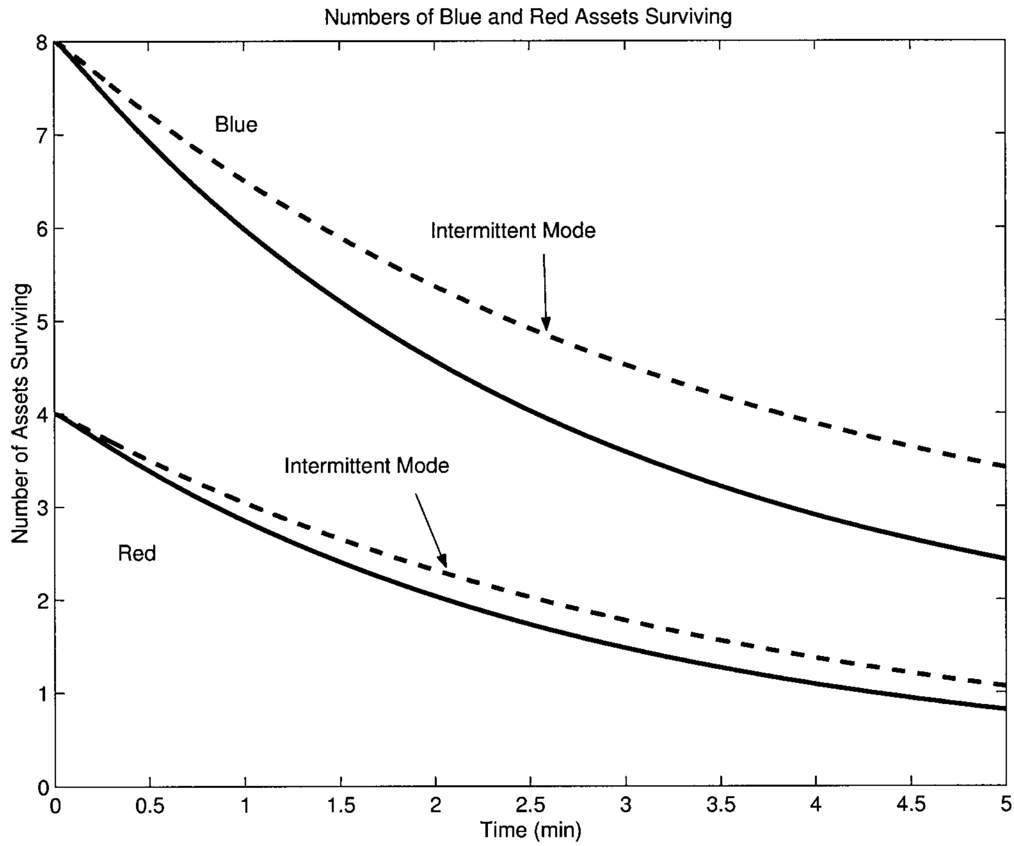
$\xi_{EW} = 10,\ \nu_{RB} = 1,\ \nu_{BR} = 1,\ P_{RI} = 0.7,\ P_{RQ} = 0.7,\ P_{BRI} = 0.5,\ P_{BRQ} = 0.08$	
$R_A(0) = 4,\ R_{EW} = 1,\ B_U(0) = 8,\ B_S = 4,\ P_{RI}/P_{RQ} = 1.4,\ P_{BRI}/P_{BRQ} = 6.25$	

Figure 1. Numbers of surviving Red SAMS and Blue attackers are displayed for two cases: Red engagement radar using continuous mode and Red engagement radar using intermittent mode. The exchange ratios favor Red's use of intermittent mode.

In Figure 1, Red's relative advantage is less than Blue's; Red has a higher relative advantage than Blue in Figure 2. According to the derived tactic, Red should use intermittent mode in the Figure 1 case and continuous mode for the case in Figure 2. The graphical results support this tactic for maximizing Blue kills and Red survivors.

3. MODEL 2: RSEAD—ELEMENTARY STOCHASTIC DUELS

In Model 2, Blue is employing RSEAD against a "pop-up" Red mobile SAM in region $\mathfrak{R}$. Blue is in the midst of CAS operations, in $\mathfrak{R}$, which require air superiority. The Red SAM has interfered with CAS by engaging Blue CAS attackers. Blue has SEAD aircraft with ARMs in $\mathfrak{R}$. Blue has decided to employ deliberate RSEAD to attack the Red SAM directly with aircraft diverted from their CAS mission. If the Red SAM has been accurately located, a deliberate



$\xi_{EW} = 10$,	$\nu_{RB} = 1$,	$\nu_{BR} = 1$,	$P_{RI} = 0.7$,	$P_{RQ} = 0.5$,	$P_{BRI} = 0.1$,	$P_{BRQ} = 0.08$
$R_A(0) = 4$,	$R_{EW} = 1$,	$B_U(0) = 8$,	$B_S = 4$,	$P_{RI}/P_{RQ} = 1.4$,	$P_{BRI}/P_{BRQ} = 1.25$	

Figure 2. Numbers of surviving Red SAMS and Blue attackers are displayed for two cases: Red engagement radar using continuous mode and Red engagement radar using intermittent mode. The exchange ratios favor Red's use of continuous mode.

RSEAD strike can proceed, and Blue SEAD aircraft will support the strike with pre-emptive ARMs shots. Otherwise, Blue SEAD aircraft will fire reactive ARM shots every time the Red SAM engages Blue CAS aircraft.

3.1. Assumptions

The following assumptions apply to Model 2:

1. RSEAD is being employed by Blue in region $\mathfrak{R}$ against a single, autonomous Red mobile SAM.
2. Blue attackers must enter Red SAM engagement envelopes to reach their targets

during CAS. The time spent inside Red's engagement zone is Blue's vulnerability window (measured in time).
3. After Blue attained air superiority in $\mathcal{R}$, Red has introduced prohibitive interference with a mobile SAM using command or semi-active guidance.
4. The Red engagement radar is co-located with the Red SAM battery.
5. Red's engagement radar operates in either continuous or intermittent mode.
6. Red does not present a significant fighter threat to Blue.
7. Blue SEAD aircraft are separate from Blue attackers and remain outside the Red engagement envelope.
8. EW in support of CAS is not sufficient to suppress the Red SAM's attacks. Deliberate RSEAD is required.
9. Blue SEAD aircraft fire ARMs in a reactive mode against an unlocated (localized along a line of bearing) Red SAM, and Blue fires ARMs on a pre-emptive timeline against a located (to within 100 m^2) Red SAM.
10. Shots are exchanged one at a time. Unlocated SAM case: Red fires at a Blue CAS aircraft; then a Blue SEAD aircraft fires a reactive ARM shot. Located SAM case: Red fires at attacking Blue aircraft, and a preemptive ARM is fired by Blue SEAD aircraft.
11. Blue may employ one or more of several platform types in the deliberate RSEAD strike.

3.2. Model 2a: Unlocated SAM/Reactive ARM

Model 2a is the case where the Red SAM has not been located accurately enough for a deliberate RSEAD strike. Instead, reactive ARM shots are employed by Blue each time Red engages Blue CAS aircraft. This model attempts to answer the following questions:

1. How many shots does Red complete, and how many Blues are killed, before the Red engagement radar is eliminated?
2. What is a good ("optimal") tactic for the Red SAM to follow so as to maximize possible Blue attrition before being eliminated itself?

Let K_B be the random number of Blues killed before Red is killed. Then

$$K_B = \begin{cases} 0 & \text{with probability } \theta_{RQ}(1 - P_{RQ})P_{BRQ} + \theta_{RI}(1 - P_{RI})P_{BRI}, \\ 1 & \text{with probability } \theta_{RQ}P_{RQ}P_{BRQ} + \theta_{RI}P_{RI}P_{BRI}, \\ 0 + K'_B & \text{with probability } \theta_{RQ}(1 - P_{RQ})(1 - P_{BRQ}) + \theta_{RI}(1 - P_{RI})(1 - P_{BRI}), \\ 1 + K'_B & \text{with probability } \theta_{RQ}P_{RQ}(1 - P_{BRQ}) + \theta_{RI}P_{RI}(1 - P_{BRI}), \end{cases} \tag{11}$$

where K'_B is a random variable having the same (unconditional) distribution as K_B: It is the result of "starting over" (Red engages new CAS aircraft).

Now take conditional expectations to find

$$E[K_B] = 1 \cdot [\theta_{RQ}P_{RQ} + \theta_{RI}P_{RI}] + [\theta_{RQ}(1 - P_{BRQ}) + \theta_{RI}(1 - P_{BRI})]E[K'_B]$$

or

$$E[K_B] = \frac{[\theta_{RQ}P_{RQ} + \theta_{RI}P_{RI}]}{1 - [\theta_{RQ}(1 - P_{BRQ}) + \theta_{RI}(1 - P_{BRI})]} = \frac{\theta_{RQ}P_{RQ} + \theta_{RI}P_{RI}}{\theta_{RQ}P_{BRQ} + \theta_{RI}P_{BRI}} \tag{12}$$

since $E[K'_B] = E[K_B]$.

For Red, the tactic that maximizes the expected number of Blues killed before it is eliminated is determined by the simple transaction kill ratio relationship

$$\begin{aligned} &\textit{emit continuously} \quad &\text{if} \quad &\frac{P_{RI}}{P_{BRI}} > \frac{P_{RQ}}{P_{BRQ}}, \\ &\textit{emit intermittently} \quad &\text{if} \quad &\frac{P_{RQ}}{P_{BRQ}} > \frac{P_{RI}}{P_{BRI}}. \end{aligned} \tag{13}$$

Continuous and intermittent modes are equally effective if equality holds. This can be shown by examining the derivative on θ_{RQ} of (12), and is exactly the condition (11) found for the deterministic model.

3.3. Model 2b: Located SAM/Deliberate RSEAD Allowing for Red Misclassification of Blue Target Types

In this model, Blue has located the Red SAM with sufficient accuracy to launch a deliberate RSEAD strike. Several different types of Blue platforms are available for the RSEAD strike. Assume there are i types of Blue aircraft: $i = 1, \ldots, I$. Let $P_{RQ}(i)$ or $P_{RI}(i)$ be the probability of an intermittently or continuously emitting Red engagement radar resulting in a kill of a type i Blue aircraft. Let α_i be the probability a Blue target is of type i, $i = 1, \ldots, I$. There is the quite realistic possibility of Red misclassifying the type of a Blue aircraft. Let γ_{ij} be the probability Red classifies a Blue type i target as a type j target; $\gamma_{ii} < 1$ is a distant possibility.

$$\begin{aligned} E[K_B] &= \sum_i \sum_j \alpha_i \gamma_{ij} \{\theta_{RQ}(j) P_{RQ}(i) + \theta_{RI}(j) P_{RI}(i) \\ &\quad + [\theta_{RQ}(j)(1 - P_{BRQ}) + \theta_{RI}(j)(1 - P_{BRI})] E[K_B]\} \\ &= \frac{\sum_i \sum_j \alpha_i \gamma_{ij} [\theta_{RQ}(j) P_{RQ}(i) + \theta_{RI}(j) P_{RI}(i)]}{1 - \sum_i \sum_j \alpha_i \gamma_{ij} [\theta_{RQ}(j)(1 - P_{BRQ}) + \theta_{RI}(j)(1 - P_{BRI})]} \\ &= \frac{\sum_i \sum_j \alpha_i \gamma_{ij} [\theta_{RQ}(j) P_{RQ}(i) + \theta_{RI}(j) P_{RI}(i)]}{\sum_i \sum_j \alpha_i \gamma_{ij} [\theta_{RQ}(j) P_{BRQ} + \theta_{RI}(j) P_{BRI}]} \end{aligned} \tag{14}$$

To find the values of $\theta_{RQ}(j)$ that maximize $E[K_B]$, note that the value of $\theta_{RQ}(j)$ can be determined for each j independent of the other values. Fix the values of $\theta_{RQ}(i)$ $i \neq j$, then $E[K_B]$ can be rewritten as

$$\begin{aligned} g(\theta_{RQ}(j)) &= \frac{c_1(0) + \sum_i \alpha_i \gamma_{ij} [\theta_{RQ}(j) P_{RQ}(i) + \theta_{RI}(j) P_{RI}(i)]}{c_2(0) + \sum_i \alpha_i \gamma_{ij} [\theta_{RQ}(j) P_{BRQ} + \theta_{RI}(j) P_{BRI}]} \\ &= \frac{c_1(1) + \sum_i \pi(i \mid j) [\theta_{RQ}(j) P_{RQ}(i) + \theta_{RI}(j) P_{RI}(i)]}{c_2(1) + \sum_i \pi(i \mid j) [\theta_{RQ}(j) P_{BRQ} + \theta_{RI}(j) P_{BRI}]}, \end{aligned} \tag{15}$$

where

$$\pi(i \mid j) = \frac{\alpha_i \gamma_{ij}}{\sum_k \alpha_k \gamma_{kj}} \tag{16}$$

the conditional probability the target is of type i given it is classified as type j; $c_1(0)$, $c_2(0)$, $c_1(1)$, and $c_2(1)$ are constants not involving $\theta_{RQ}(j)$.

Let

$$f_Q(j) = \sum_i \pi(i \mid j) \frac{P_{RQ}(i)}{P_{BRQ}},$$

$$f_I(j) = \sum_i \pi(i \mid j) \frac{P_{RI}(i)}{P_{BRI}}, \tag{17}$$

and

$$M(j) = \max(f_Q(j), f_I(j)). \tag{18}$$

Since

$$\frac{\sum_i \pi(i \mid j)[\theta_{RQ}(j)P_{RQ}(i) + \theta_{RI}(j)P_{RI}(i)]}{\sum_i \pi(i \mid j)[\theta_{RQ}(j)P_{BRQ} + \theta_{RI}(j)P_{BRI}]} \le M(j)$$

for $j = 1, \ldots, I$, $0 \le \theta_{RQ}(i) \le 1$, it follows that the heuristic/approximate tactic to maximize the expected number of Blue kills for Red is:

For a target that is *classified* as type j

$$\begin{aligned} &\textit{emit continuously} \quad \text{if} \quad \sum_{i=1}^{I} \pi(i \mid j) \frac{P_{RI}(i)}{P_{BRI}} > \sum_{i=1}^{I} \pi(i \mid j) \frac{P_{RQ}(i)}{P_{BRQ}}, \\ &\textit{emit intermittently} \quad \text{if} \quad \sum_i \pi(i \mid j) \frac{P_{RQ}(i)}{P_{BRQ}} > \sum_i \pi(i \mid j) \frac{P_{RI}(i)}{P_{BRI}}, \end{aligned} \tag{19}$$

where (16) gives $\pi(i \mid j)$. In practice, both α_i and $\{\gamma_{ij}\}$ would have to be estimated or inferred.

3.4. Model 2c: Allowing for Red to Sequence Shots at Blue Target Types

We modify and develop Models 2a–2b as follows: A single Red SAM engages a fixed number N of Blue CAS attackers. As in Model 2a, every time that Red fires at a Blue aircraft, a Blue SEAD aircraft fires a reactive ARM shot. As in Model 2b, each Blue can be one of I types and Red has imperfect information concerning the Blue types he faces. At each stage in the shooting, Blue will summarize his beliefs about the Blues he faces via a collection of (posterior) probability distributions. These distributions will be derived using data from Red's classification of the Blues he faces, as in Model 2b, and possibly also data obtained during the engagement itself. On the basis of these beliefs, Red has to decide at each stage (a) which Blue attacker should he shoot at next, and (b) in support of this attack, whether Red's engagement radar should operate in continuous or intermittent mode.

Red's objective in developing a strategy for shooting is to maximize the expected number of Blue attackers destroyed before he is eliminated. We shall consider two scenarios.

3.4.1. Scenario 1

We suppose first that Red shoots at each Blue target (at most) once. Hence, Red's shooting strategy involves (a) a sequencing of the N Blues, together with (b) a decision for each Blue regarding whether Red's radar should operate in continuous or intermittent mode whenever that Blue is shot.

Consider (b) first and assume that probability distribution $\pi^n = \{\pi_i^n, 1 \le i \le I\}$, $1 \le n \le N$, describes Red's beliefs about the type (i) of Blue n before shooting begins. Following the analysis of Model 2b to (19), we suppose that Red adopts the following heuristic: Emit continuously when shooting at Blue n if

$$\sum_{i=1}^{I} \pi_i^n \frac{P_{RI}(i)}{P_{BRI}} > \sum_{i=1}^{I} \pi_i^n \frac{P_{RQ}(i)}{P_{BRQ}} \tag{20}$$

and emits intermittently otherwise. We use $P_R^n(i)$ and P_{BR}^n to denote the kill probabilities for Red and Blue, respectively, which result from this decision. Hence, for example, when inequality (20) holds we have $P_R^n(i) = P_{RI}(i)$ and $P_{BR}^n = P_{BRI}$. Having determined the level of Red's radar operation for each Blue target we now proceed to determine (a), the best ordering of the Blue targets for Red. Renumber the Blues such that

$$\sum_{i=1}^{I} \pi_i^1 \frac{P_R^1(i)}{P_{BR}^1} \ge \sum_{i=1}^{I} \pi_i^2 \frac{P_R^2(i)}{P_{BR}^2} \ge \cdots \ge \sum_{i=1}^{I} \pi_i^N \frac{P_R^N(i)}{P_{BR}^N}. \tag{21}$$

LEMMA 1: Given that Red makes decisions about radar levels according to (20), then shooting at the Blue targets according to the sequence $\{1, 2, \ldots, N\}$ will maximize the expected number of Blues destroyed before Red is eliminated.

PROOF: We use induction on N, the number of Blue targets. The result is trivial when $N = 1$. Suppose it to hold for $N = k$ and consider a problem in which $N = k + 1$ with targets $\{1, 2, \ldots, k + 1\}$ numbered according to (21). Hypothesize that a sequence in which $n \ne 1$ is the first Blue target is uniquely optimal, and obtain a contradiction. Invoking the inductive hypothesis, the first two targets in the uniquely optimal sequence must be $\{n, 1\}$ and the expected number of Blues destroyed before Red is eliminated may be written

$$\sum_{i=1}^{I} \pi_i^n P_R^n(i) + (1 - P_{BR}^n)\left\{\sum_{i=1}^{I} \pi_i^1 P_R^1(i)\right\} + (1 - P_{BR}^n)(1 - P_{BR}^1)V, \tag{22}$$

where V is the value of the objective for the sequence $\{2, \ldots, n - 1, n + 1, \ldots N\}$. However, the value associated with the sequence $\{1, n, 2, \ldots, n - 1, n + 1, \ldots, N\}$, in which the ordering of targets 1 and n is now reversed, may be written

$$\sum_{i=1}^{I} \pi_i^1 P_R^1(i) + (1 - P_{BR}^1)\left\{\sum_{i=1}^{I} \pi_i^n P_R^n(i)\right\} + (1 - P_{BR}^1)(1 - P_{BR}^n)V. \tag{23}$$

However, from (21) we have that

$$\sum_{i=1}^{I} \pi_i^1 \frac{P_R^1(i)}{P_{BR}^1} \geq \sum_{i=1}^{I} \pi_i^n \frac{P_R^n(i)}{P_{BR}^n}, \tag{24}$$

from which it follows easily that the expression in (23) is no less than that in (22). This contradicts the unique optimality of the proposed sequence which begins with n and the result follows.

Hence from Lemma 1 and the material preceding, a sound heuristic policy is for Red to shoot at the Blues according to the ordering in (21). When Red shoots at Blue n his engagement radar operates continuously if (20) holds, and intermittently otherwise. The reader should note that the same form of target index arises in the radar decisions in (20) as in the sequencing decisions in (21).

3.4.2. Scenario 2

Scenario 1 is now developed in such a way that Red is assumed to have access to perfect intelligence regarding whether his earlier shots at Blue were successful. Hence, at each decision epoch $t = 0, 1, 2, \ldots,$ Red shoots at any of the Blue targets still alive at t, and continues to do so until he himself is eliminated. An additional feature of such a setup is that Red's beliefs about the still-alive Blue targets will be modified in light of information received about earlier shots being unsuccessful. As Red's beliefs about the Blues he is facing change, so will his decisions about the mode in which his engagement radars should be operated and also the order in which Blues should be targeted.

At each time $t = 0, 1, 2, \ldots$ let $A(t)$ be the set of Blue targets alive at t. If $n \in A(t)$, we write $n(t)$ for the number of times Blue n has been shot at (unsuccessfully) up to t. Further, for $r = 1, 2, \ldots, n(t)$ we write $M(n, r)$ for the mode in which Red's radar was operating when Blue n was shot at on the rth occasion. We use the notations $P_R^{n,r}(i)$, $P_{BR}^{n,r}$ to denote probabilities pertaining to Red's rth shot at target n, i.e.,

$$M(n, r) = I \Rightarrow P_R^{n,r}(i) = P_{RI}(i) \qquad \text{and} \qquad P_{BR}^{n,r} = P_{BRI}. \tag{25}$$

We also write $\pi^{n,n(t)} = \{\pi_i^{n,n(t)}, 1 \leq i \leq I\}$ for the posterior distribution describing Red's beliefs about Blue $n \in A(t)$ at time t. By Bayes' theorem we have, for all values of n, $n(t)$ that

$$\pi_i^{n,n(t)} \propto \pi_i^{n,0} \prod_{r=1}^{n(t)} \{1 - P_R^{n,r}(i)\}, \qquad 1 \leq i \leq I. \tag{26}$$

At each time t, Red will need to decide which Blue from $A(t)$ to target next, and in which mode his engagement radar should operate in support of that shot. These decisions will be based upon the collection of independent posterior distributions $\{\pi^{n,n(t)}, n \in A(t)\}$.

As with our analysis of Scenario 1, we first develop a heuristic approach to the radar decisions. For each $n \in N$, we require a sequence $\{M(n, r), r \in \mathbf{Z}^+\}$ of radar level determinations. We obtain a sequence for each Blue in isolation though a dynamic program (DP), whose goal will be to maximize Red's probability of killing the individual Blue by a consecutive sequence of shots. Let $V(\pi)$ denote the maximal such probability for a target with prior distribution $\pi = \{\pi_i, 1 \le i \le I\}$. $V(\pi)$ satisfies the optimality equation

$$V(\pi) = \max\left(\begin{array}{l} \sum_{i=1}^{I} \pi_i P_{RI}(i) + (1 - P_{BRI})\left[\sum_{i=1}^{I} \pi_i\{1 - P_{RI}(i)\}\right] V(\pi^I); \\ \sum_{i=1}^{I} \pi_i P_{RQ}(i) + (1 - P_{BRQ})\left[\sum_{i=1}^{I} \pi_i\{1 - P_{RQ}(i)\}\right] V(\pi^Q) \end{array}\right) \quad (27)$$

where the first term in (;), on the rhs of (27), relates to the choice of continuous radar operation and the second term to that of intermittent operation. In (27) we use π^I (π^Q) for the posterior distribution following a single unsuccessful shot supported by radar operating in continuous (intermittent) mode, i.e.,

$$\pi_i^I \propto \pi_i\{1 - P_{RI}(i)\}, \qquad 1 \le i \le I,$$

and similarly for π_i^Q, $1 \le i \le I$. Use $a(\pi)$ to denote the maximizing action in (27). The maximizing sequence $\{M(n, r), r \in \mathbf{Z}^+\}$ for Blue n will be given by the relations

$$M(n, r) = a(\pi^{n,r-1}), \qquad n \in I, \quad r \in \mathbf{Z}^+, \quad (28)$$

where $\pi^{n,r-1}$ is the posterior distribution resulting from (unsuccessful) utilization of the first $r - 1$ terms in the radar sequence. Please note that, from Model 2b, and in particular (19) and (20), a simpler approach which will provide a good approximation to the above, develops the sequence $\{M(n, r), r \in \mathbf{Z}^+\}$ from the recursion

$$\sum_{i=1}^{I} \pi_i^{n,r-1} \frac{P_{RI}(i)}{P_{BRI}} > \sum_{i=1}^{I} \pi_i^{n,r-1} \frac{P_{RQ}(i)}{P_{BRQ}} \Rightarrow M(n, r) = I, \qquad r \in \mathbf{Z}^+, \quad (29)$$

with $M(n, r) = Q$ otherwise.

Now consider the problem of how Red should choose a Blue target from $A(t)$ at time t under the assumption that radar levels in support of shots at each target n will be in accord with the sequence $\{M(n, r), r \in \mathbf{Z}^+\}$ determined from (28). We require an appropriate development of the simple index-based sequencing rule given in (21) and develop an index $G_n\{\pi^{n,n(t)}; n(t)\}$ for target $n \in A(t)$ at time t which is a measure of the reward rate available to Red from further shots (numbered $n(t) + 1, n(t) + 2, \ldots, T$) on Blue n. The index is given by

$$G_n\{\pi^{n,n(t)}; n(t)\} = \max_{r\geq 1}\left[\frac{\sum_{i=1}^{I} \pi_i^{n,n(t)}(\sum_{s=1}^{r} P_R^{n,n(t)+s}(i)[\prod_{\nu=1}^{s-1}\{1 - P_R^{n,n(t)+\nu}\}\{1 - P_{BR}^{n,n(t)+\nu}\}])}{\sum_{i=1}^{I} \pi_i^{n,n(t)}\{1 - A_{1i}(r) - A_{2i}(r)\}}\right] \tag{30}$$

where

$$A_{1i}(r) = \sum_{s=1}^{r} P_R^{n,n(t)+s}(i)\{1 - P_{BR}^{n,n(t)+s}\}\left[\prod_{\nu=1}^{s-1}\{1 - P_R^{n,n(t)+\nu}(i)\}\{1 - P_{BR}^{n,n(t)+\nu}\}\right]$$

and

$$A_{2i}(r) = \prod_{s=1}^{r}\{1 - P_R^{n,n(t)+s}(i)\}\{1 - P_{BR}^{n,n(t)+s}\}.$$

LEMMA 2: Given that Red makes decisions about radar levels according to (28), then the policy which at each decision epoch $t = 0, 1, 2, \ldots$ shoots at any target $n \in A(t)$, such that

$$G_n\{\pi^{n,n(t)}; n(t)\} = \max_{m\in A(t)} G_m\{\pi^{m,m(t)}; m(t)\} \tag{31}$$

maximizes the expected number of Blues destroyed before Red is eliminated.

COMMENTS:

1. Glazebrook, Gaver, and Jacobs [5] show that Red's problem of maximizing the expected number of Blues to be killed before he is himself eliminated may be modeled as a generalized bandit problem once the radar levels have been established. From Nash [11], an index policy is optimal. See Glazebrook, Gaver, and Jacobs [5] for details of the derivation of the index in (30) and (31).
2. The reader may easily check that upon substitution of $n(t) = 0$, $r = 1$ in the expression on the rhs of (30) we obtain

$$\sum_{i=1}^{I} \pi_i^{n,0}\frac{P_R^{n,1}(i)}{P_{BR}^{n,1}}.$$

   Hence the indices in (21) may be viewed as special cases of those in (30), which are appropriate when Red is restricted to (at most) one shot at each target.
3. Manor and Kress [9] argue the importance of scheduling problems of the kind we have discussed above. In a scenario which is, in most respects, simpler than that considered here, they develop an index policy for (optimal) shooting.
4. Although the index policies in Lemmas 1 and 2 are developed for static environments in which the Blue target population is fixed, the developed indices will continue to be an effective decision support tool for situations in which the targets in range are continually changing. See Gaver, Glazebrook, and Pilnick [4] for comments on the robustness of index policies in the context of a ship replenishment problem.

Table 1. The mean number of Blues killed by Red prior to Red's own destruction under (i) an index policy and (ii) a random shooting policy.

ϕ	(b_1, b_2)	(2, 8)	(4, 6)	(6, 4)	(8, 2)
1	(i)	2.0 (0.06)	2.7 (0.07)	3.0 (0.09)	3.3 (0.09)
	(ii)	1.3 (0.05)	1.7 (0.06)	2.1 (0.07)	2.6 (0.09)
0.95	(i)	1.9 (0.05)	2.5 (0.07)	2.9 (0.08)	3.3 (0.09)
	(ii)	1.5 (0.06)	1.6 (0.06)	2.0 (0.07)	2.7 (0.09)
0.9	(i)	2.0 (0.06)	2.4 (0.07)	2.9 (0.08)	3.1 (0.09)
	(ii)	1.3 (0.05)	1.7 (0.06)	2.1 (0.07)	2.6 (0.09)
0.85	(i)	1.8 (0.06)	2.3 (0.07)	3.0 (0.09)	3.1 (0.09)
	(ii)	1.4 (0.06)	1.7 (0.06)	2.0 (0.07)	2.6 (0.09)
0.8	(i)	1.7 (0.06)	2.1 (0.07)	2.9 (0.08)	3.0 (0.09)
	(ii)	1.4 (0.06)	1.7 (0.07)	2.2 (0.08)	2.5 (0.09)
0.7	(i)	1.5 (0.06)	2.1 (0.07)	2.5 (0.08)	2.9 (0.09)
	(ii)	1.4 (0.05)	1.8 (0.06)	2.0 (0.07)	2.6 (0.08)
0.6	(i)	1.4 (0.05)	1.9 (0.07)	2.2 (0.07)	2.8 (0.09)
	(ii)	1.4 (0.06)	1.7 (0.06)	2.0 (0.07)	2.6 (0.09)
0.5	(i)	1.3 (0.05)	1.7 (0.06)	2.1 (0.08)	2.7 (0.09)
	(ii)	1.4 (0.06)	1.7 (0.07)	2.1 (0.08)	2.6 (0.08)

Blue types more alike: $P_R(1) = 0.6$, $P_{BR}(1) = 0.4$; $P_R(2) = 0.4$, $P_{BR}(2) = 0.6$.

5. We conducted a numerical investigation of the index policy in Lemma 2 in a situation in which the radar was assumed to be operating at a single level (continuous emission, say) throughout, but where the retaliation kill probabilities were assumed to be Blue-type dependent. Lemma 2 may easily be extended to take account of the latter complication. The scenario investigated envisaged the Blue targets being of two types, with b_1 being of type 1 and b_2 of type 2. Red uses a sensor to initially estimate the type of each Blue target. The probability that Red classifies a type i target as type i is ϕ, $i = 1, 2$; otherwise it is classified (wrongly) as the other type. Under an assumption that Red attaches a probability of 0.5 to each Blue type before receiving the classification information, we infer from an application of Bayes' theorem that Red's prior for each Blue classified as type 1 is $(\phi, 1 - \phi)$, while his prior for those classified as type 2 is $(1 - \phi, \phi)$.

A simulation model implemented two shooting policies for Red: (i) an (optimal) index policy; and (ii) random shooting in which, at each decision epoch, Red chooses to engage one of the remaining Blues chosen at random (with equal probabilities). Results are presented in Tables 1 and 2. In each cell of both tables we report the estimated mean number of Blues killed prior to Red's destruction, with the corresponding standard error in brackets. The upper figures in each cell correspond to the index policy and the lower figures to the random shooting policy. All entries are based on 1000 replications.

As might be expected, the index policy outperforms the random shooting policy other than at $\phi = 0.5$, where the sensor does no better than the flip of a fair coin and the two policies are statistically virtually identical. The level of excess number of Blues killed achieved by the index policy is remarkably high when Red receives high quality information from the sensor assets (i.e., ϕ is high). However, even rather mediocre information ($\phi = 0.6$, say) can be put to very good use by Red. The value of the information to Red is unsurprisingly greater when the Blue types are more distinct.

Table 2. The mean number of Blues killed by Red prior to Red's own destruction under (i) an index policy and (ii) a random shooting policy.

ϕ	(b_1, b_2)	(2, 8)	(4, 6)	(6, 4)	(8, 2)
1	(i)	2.1 (0.04)	3.3 (0.06)	4.2 (0.08)	5.0 (0.10)
	(ii)	0.8 (0.04)	1.3 (0.05)	2.0 (0.07)	3.1 (0.09)
0.95	(i)	1.9 (0.05)	3.1 (0.06)	4.0 (0.08)	4.8 (0.10)
	(ii)	0.9 (0.04)	1.4 (0.05)	2.0 (0.07)	3.1 (0.09)
0.9	(i)	1.8 (0.05)	2.7 (0.06)	3.7 (0.08)	4.7 (0.10)
	(ii)	0.9 (0.04)	1.4 (0.05)	1.9 (0.07)	3.0 (0.09)
0.85	(i)	1.6 (0.05)	2.5 (0.06)	3.6 (0.08)	4.4 (0.10)
	(ii)	1.0 (0.04)	1.3 (0.05)	1.9 (0.07)	3.1 (0.09)
0.8	(i)	1.4 (0.05)	2.3 (0.07)	3.1 (0.08)	4.3 (0.10)
	(ii)	0.9 (0.04)	1.2 (0.05)	2.0 (0.07)	3.1 (0.09)
0.7	(i)	1.2 (0.05)	2.0 (0.06)	2.7 (0.08)	3.9 (0.10)
	(ii)	1.0 (0.05)	1.3 (0.05)	2.0 (0.07)	3.0 (0.09)
0.6	(i)	1.1 (0.04)	1.5 (0.05)	2.3 (0.07)	3.4 (0.09)
	(ii)	0.9 (0.04)	1.2 (0.05)	2.0 (0.07)	3.0 (0.09)
0.5	(i)	0.9 (0.04)	1.3 (0.05)	2.0 (0.07)	3.1 (0.09)
	(ii)	0.9 (0.04)	1.4 (0.05)	1.9 (0.07)	3.2 (0.09)

Blue types less alike: $P_R(1) = 0.7$, $P_{BR}(1) = 0.3$; $P_R(2) = 0.3$, $P_{BR}(2) = 0.7$.

4. SUMMARY

Red's IADS and Blue's SEAD were modeled as service systems where SAMs and ARMs duel using engagement radar emissions as information. The IADS service system has a finite lifetime. The assignment of a system is to kill incoming strike aircraft (Red), *or* impact defending engagement radars (Blue). The service systems' lifetimes and the amount of work accomplished during the lifetime depend on the level of effort (emission mode) of the servers (engagement radars). It also depends on the quality of information available to Red and Blue.

For Red, a high level of effort (continuous mode) may result in more work completed (Blue attackers killed per unit time), but a shorter lifetime. Lower effort (intermittent mode) by Red may result in less work accomplished, but a longer lifetime. However, there are cases where lower effort by Red maximizes the work accomplished because more Red SAMs survive to continue working. Decision rules (tactics) were derived to maximize the amount of work (Blue attackers killed) accomplished by the service system (Red's IADS) during its lifetime. The decision rules pertain both to the level of effort and the type of Blue target to be engaged.

An analysis of Red's IADS by Blue can provide detailed information on the sizing of an attack force. These models apply to both pre-emptive SEAD against a full IADS, and to reactive SEAD against a single, autonomous SAM. It is also possible to extend the model to make the effect of Blue SEAD efforts more robust by including decoys, jamming, and saturation of Red EW radars. The latter situations are subjects for later research.

APPENDIX A: GLOSSARY

Anti-Air Warfare (AAW)—Action required to gain and maintain air superiority.

Prohibitive Interference—Enemy influence that denies previously established air superiority.

Suppression of Enemy Air Defenses (SEAD)—Action that neutralizes, destroys, or temporarily degrades surface-based enemy air defenses by destructive or disruptive means.

- **Concurrent SEAD**—Destructive or disruptive efforts simultaneous with other missions such as air interdiction, armed reconnaissance, or close air support.

- **Pre-Planned SEAD**—Destructive or disruptive efforts allocated or apportioned through the normal air tasking order cycle to target strategic surface-to-air missiles early warning and ground controlled intercept radar sites, command and control nodes, and other integrated air defense components.
- **Sequential SEAD**—Preemptive destructive or disruptive efforts that precede other missions to introduce a "window of air superiority."

Reactive SEAD (RSEAD)—SEAD for reestablishment of air superiority after enemy introduces prohibitive interference with a surface-to-air threat.

- **Alert RSEAD**—Dedicated RSEAD Assets on Air or Strip Alert prepared to strike an enemy surface to air threat once it has been located.
- **Deliberate RSEAD**—Coordinated response to strike an enemy surface-to-air threat with assets diverted from other missions.
- **Immediate RSEAD**—Self-defense attack on an enemy surface to air threat by a platform that locates the enemy air defense asset targets while conducting another mission.

APPENDIX B: INDEX OF SYMBOLS

Section 2.3

$B_U(t)$ = number of undetected Blue attackers at time t,
$B_D(t)$ = number of detected Blue attackers at time t,
B_S = number of Blue standoff SEAD aircraft,
$R_A(t)$ = number of Red engagement radars at time t,
$\lambda(t)$ = arrival rate of Blue attackers at time t,
ξ_{EW} = rate at which integrated air defense system (IADS) detects, identifies, tracks, and assigns a Blue attacker,
ν_{RB} = rate at which surface-to-air missiles (SAMs) fire and complete an engagement against an assigned Blue attacker,
ν_{BR} = rate at which a standoff Blue SEAD aircraft fires at a Red engagement radar,
θ_{RI} = probability Red engagement radar emits continuously while guiding a surface-to-air missile (SAM),
θ_{RQ} = probability Red engagement radar emits intermittently while guiding a surface-to-air missile,
P_{RI} = probability a Blue attacker that is engaged by a SAM guided by Red engagement radar emitting continuously is killed,
P_{RQ} = probability a Blue attacker that is engaged by a SAM guided by Red engagement radar emitting intermittently is killed,
P_{BRI} = probability a Blue standoff SEAD aircraft kills a Red engagement radar emitting continuously,
P_{BRQ} = probability a Blue standoff SEAD aircraft kills a Red engagement radar emitting intermittently.

Section 3.2

K_B = random number of Blue attackers killed before a Red engagement radar is killed.

Section 3.3

I = number of types of Blue attackers,
$\theta_{RI}(i)$ = probability Red engagement radar emits continuously while guiding a SAM toward a Blue attacker of perceived type i,
$\theta_{RQ}(i)$ = probability Red engagement radar emits intermittently while guiding a SAM toward a Blue attacker of perceived type i,
$P_{RI}(i)$ = probability a Blue attacker of type i that is engaged by a SAM guided by a Red engagement radar emitting continuously is killed,
$P_{RQ}(i)$ = probability a Blue attacker of type i that is engaged by a SAM guided by a Red engagement radar emitting intermittently is killed,

α_i = probability a Blue attacker is of type i,
γ_{ij} = probability Red classifies a Blue attacker of type i as an attacker of type j,
$\pi(i|j)$ = conditional probability a Blue attacker classified as a type j is a type i.

Section 3.4

N = number of Blue attackers,
π_i^n = prior probability Blue attacker n is of type i,
$P_R^n(i)$ = probability Red kills Blue attacker n which is of type i using the best heuristic policy for engagement radar mode,
$A(t)$ = set of Blue attackers alive at time t,
$n(t)$ = number of times Red has shot at Blue attacker n during the first t shots,
$M(n, r)$ = mode (continuously or intermittently) Red's engagement radar is operating in for the rth engagement against Blue attacker n,
$P_R^{n,r}(i)$ = conditional probability Blue attacker n is killed as a result of the rth engagement which uses the best heuristic for engagement radar mode given the attacker is a type i target,
$P_{BR}^{n,r}$ = probability Blue SEAD aircraft kills the Red engagement radar after the rth engagement of Blue attacker n when the engagement radar is using the best heuristic engagement mode,
$\pi_i^{n,n(t)}$ = posterior probability Blue attacker n that has been unsuccessfully engaged $n(t)$ times is of type i,
π_i^I = posterior probability Blue attacker is of type i following an unsuccessful engagement using a Red engagement radar operating in continuous mode,
π_i^Q = posterior probability Blue attacker is of type i following an unsuccessful engagement using a Red engagement radar operating in intermittent mode,
$\pi^{n,r-1}$ = posterior distribution of the type of Blue attacker n given the attacker was unsuccessfully engaged $r - 1$ times,
$G_n\{\pi^{n,n(t)}; n(t)\}$ = index for Blue attacker n that has been unsuccessfully engaged $n(t)$ times using the best policy for engagement radar modes.

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