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A Dynamic Model for Posttraumatic Stress Disorder Among U.S. Troops in Operation Iraqi Freedom

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We develop a dynamic model in which Operation Iraqi Freedom (OIF) servicemembers incur a random amount of combat stress during each month of deployment, develop posttraumatic stress disorder (PTSD) if their cumulative stress exceeds a servicemember-specific threshold, and then develop symptoms of PTSD after an additional time lag. Using Department of Defense deployment data and Mental Health Advisory Team PTSD survey data to calibrate the model, we predict that—because of the long time lags and the fact that some surveyed servicemembers experience additional combat after being surveyed—the fraction of Army soldiers and Marines who eventually suffer from PTSD will be approximately twice as large as in the raw survey data. We cannot put a confidence interval around this estimate, but there is considerable uncertainty (perhaps ±30%). The estimated PTSD rate translates into ≈300,000 PTSD cases among all Army soldiers and Marines in OIF, with ≈20,000 new cases each year the war is prolonged. The heterogeneity of threshold levels among servicemembers suggests that although multiple deployments raise an individual's risk of PTSD, in aggregate, multiple deployments lower the total number of PTSD cases by ≈30% relative to a hypothetical case in which the war was fought with many more servicemembers (i.e., a draft) deploying only once. The time lag dynamics suggest that, in aggregate, reserve servicemembers show symptoms ≈1–2 years before active servicemembers and predict that >75% of OIF servicemembers who self-reported symptoms during their second deployment were exposed to the PTSD-generating stress during their first deployment.

Key words: health care; military; reliability; failure models
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1. Introduction

Posttraumatic stress disorder (PTSD) is an often persistent (Kessler et al. 1995) and sometimes debilitating (Zatzick et al. 1997) condition that is common among veterans of past (Centers for Disease Control 1988, Schlenger et al. 1992) and current (Hoge et al. 2004) wars and is strongly associated with the amount of combat exposure (Hoge et al. 2004, Office of the Surgeon Multinational Force (OSMF) 2006b). The tempo of the deployment cycles in Operation Iraqi Freedom (OIF) is higher than for any war since World War II, with many troops on multiple deployments (OSMF 2006b) and some Army soldiers experiencing 15-month deployments (Tyson and White 2007). To assure ample mental health resources to care for returning troops, it is important for the Department of Veterans Affairs (VA) to forecast the timing and number of new PTSD cases over the coming years, which is complicated by the fact that many cases have delayed onset (Wolfe et al. 1999).

We introduce a dynamic mathematical model, which is described in §2, that uses OIF data to predict the incidence of symptomatic PTSD cases for OIF troops over the next several years and to gain an understanding of the relationship between deployment tempo, combat stress, and PTSD prevalence. The model contains a deployment model, which uses Department of Defense (DOD) data to construct a monthly deployment schedule for individual servicemembers in OIF and a PTSD model, which determines whether each servicemember develops PTSD. The PTSD model is a variant of the strength-stress models used in the reliability literature (Johnson 1988). Each servicemember has a random strength and accumulates stress according to a stochastic process: stress increases during deployments according to a nonhomogeneous compound Poisson process whose mean is proportional to the average number of monthly casualties in OIF, and the stress decreases during periods between deployments. If a servicemember's maximum stress level does not exceed his strength, then he does not develop PTSD. Otherwise, he develops PTSD during the first month in which his stress exceeds his strength and then develops symptoms after a random time lag that depends on whether he is still in the military or has returned to civilian life. The parameters for the PTSD model are estimated from data from several PTSD surveys carried out by the Army's Mental Health Advisory Team (MHAT).

With more data, this model could be used by the DOD to evaluate different deployment scenarios. However, with current data, we envision this model being most useful as a tool the VA could use to help estimate the demand for PTSD treatment by servicemembers returning from OIF in the coming years. In §3, we use this model to estimate the total number of PTSD cases under several different withdrawal scenarios and also perform various sensitivity analyses. These results are discussed in §4. We are unaware of any other modeling efforts related to PTSD.

2. Model Overview

The model has two parts: a deployment model and a PTSD model. We summarize the deployment model in §2.1, but the detailed formulation is in §1 of the online appendix (provided in the e-companion).[1] The PTSD model is described in detail in §2.2. The parameter values for the PTSD model are reported in §2.3, and the parameter estimation procedure appears in §2 of the online appendix.

2.1. Deployment Model

The U.S. military attempts to adhere to a unit rotation policy in OIF, where ideally an entire unit is moved into a theater, stays in place for a specified duration, and is then replaced by another unit when it returns home for a brief rest and further training before its next deployment (Congressional Budget Office (CBO) 2005). The lengths of the deployment and rest periods differ for active Army, reserve Army (e.g., National Guard and Army Reserve), and Marine personnel. We do not include Navy or Air Force personnel because they see much less combat than the Army and Marines (Statistical Information Analysis Division (SIAD) 2008a) and because we do not have any PTSD data for them. There are relatively few reserve Marines, and they are combined with the active Marines in our model.

[1] An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal. informs.org/.

For major combat troops and their attached combat support troops in the active Army, National Guard, and Marines, deployment schedules are constructed (§1.1 of the online appendix) from published data at the brigade level (for active Army and National Guard) or the more detailed battalion level (for Marines), where there are typically three–four battalions per brigade. Data are insufficient to model different occupational categories of servicemembers in more detail, and the survey data are conflicting, with some suggesting small differences in PTSD rates among different occupational categories in the Army (Smith et al. 2008) and some suggesting significant differences (OSMF 2005); our model captures the variation in risk among different categories at an aggregate level by incorporating a very bursty (compound Poisson) stochastic process for combat exposure. Because detailed deployment histories of unattached support troops (e.g., Army Reserve) are unavailable, we estimate their deployment schedules by assuming that they follow a cyclic deployment and rest schedule (§1.2 of the online appendix). The initial deployment dates for the unattached troops are chosen so that total monthly deployments (SIAD 2008b, GlobalSecurity.org 2008) and certain deployment characteristics (e.g., the fraction of servicemembers on their first deployment) at several snapshots in time (OSMF 2006a, b) are accurately predicted by the model (§1.2.3 of the online appendix). These deployment schedules, as well as other data for our model, are through September 2008.

We assume that every servicemember who starts a deployment stays until the end of the deployment. A fraction of troops—based on annual continuation rate data for the various military branches (CBO 2006)—in a particular deployment leave the military before the next planned deployment, and separated servicemembers are replaced by new servicemembers to maintain constant troop strength in each unit.

2.2. PTSD Model

The PTSD model can be viewed as a variant of the strength-stress models used in the reliability of manufactured items (Johnson 1988), in which both the strength and the stress of a servicemember are random (and, in our case, where the stress varies over time according to a stochastic process). The model has four main components. The first characterizes the precise sequence of months that each servicemember deploys, the second models a servicemember's exposure to and recovery from stress, the third defines the relationship between stress and developing PTSD, and the final part describes the delay between when a servicemember develops PTSD and when symptoms manifest themselves.

2.2.1. Deployment History. In our model $j = 1, 2, 3$ denotes active Army, reserve Army (which includes Army Reserve and National Guard), and (active plus reserve) Marines, respectively. The kth servicemember of type j has an indicator process $\{C_{kj}(t), t = 1, 2, \ldots\}$ that characterizes his or her deployment history, where $C_{kj}(t) = 1$ if servicemember k was deployed during month t and $C_{kj}(t) = 0$ otherwise. There are many potential deployment history vectors because different servicemembers are first deployed at different times, separate from the military at different times, and may be attached or unattached. There are cohorts of troops with the same deployment schedules and hence the same $C_{kj}(t)$ vector. For example, in our model the 1st Brigade, 1st Armored Division consists of 5,000 servicemembers and deploys twice to OIF (see §1.1 and Table 1 in the online appendix). This brigade has three associated cohorts: 1,879 troops who deploy only during the first tour of duty, 3,121 troops who deploy for both tours of duty, and 1,879 troops who deploy for only the second tour. Although we define these cohorts deterministically for computational tractability, in reality the number of servicemembers in each cohort will be related to a multinomial random variable with a very small (e.g., ≈0.01) coefficient of variation. All troops within the same cohort have the same $C_{kj}(t)$ vector, and the $C_{kj}(t)$ vector associated with one cohort is different from the $C_{kj}(t)$ vector associated with another cohort. In the following subsections, we calculate whether (and when) servicemember k of type j develops PTSD as a function of $\{C_{kj}(t), t = 1, 2, \ldots\}$.

2.2.2. Stress Exposure and Recovery. Let $D_{kj}(t)$ be the random cumulative stress of the kth servicemember of type j at the end of month t. We assume that the initial stress just before the first month of deployment is an independent and identically distributed (i.i.d.) exponential random variable with mean α^{-1}, regardless of troop type. This reflects the heterogeneity of servicemembers' precombat experiences. We let $\{E_{kj}(t), t = 1, 2, \ldots\}$ be independent (but not identically distributed) random variables that represent the random stress that the kth servicemember of type j incurs during month t if he or she is deployed during that month. To maintain computational tractability when we analyze the sum of these random variables, and to allow flexibility in the amount of inter-servicemember variability in combat exposure, we model $E_{kj}(t)$ as a compound Poisson random variable with mean $\lambda_j(t)$ and constant batch size b (independent of j and t); hence, the mean of the underlying Poisson random variable is $\lambda_j(t)/b$ and the variance of $E_{kj}(t)$ is $b\lambda_j(t)$. The constant batch size does not vary by military branch or month, which allows the probability mass function of the cumulative stress to be easily derived while allowing flexibility in the coefficient of variation of the monthly stress among deployed troops.

Traumatic experience is causally related to PTSD (Fontana and Rosenheck 1998), and we assume that the average monthly stress, $\lambda_j(t)$, is related to the total number of OIF casualties per servicemember. The correlation between the number of fatalities and the number wounded in each month is 0.75 for the Army and 0.85 for the Marines (§2 of the online appendix), and there have been 6.99 times as many wounded as fatalities for Army and 8.51 times as many wounded as fatalities for Marines (SIAD 2008a). We model stress so that the fatalities and wounded are equally represented (although our quantitative results change little if we equate monthly stress to either fatalities or wounded). That is, the mean amount of stress for Army soldiers in a particular month is set equal to $(6.99 \times \text{fatalities} + \text{wounded})$ for that month divided by the total deployment for that month; for Marines the mean amount of stress in a particular month is $(8.51 \times \text{fatalities} + \text{wounded})$ for that month divided by the total deployment for that month. The average monthly stress $\lambda_j(t)$ is illustrated in Figure 6 in the online appendix.

To model the partial recuperation of servicemembers when they are not deployed, we assume there is a geometric decay at monthly rate θ during months when $C_{kj}(t) = 0$, where $\theta \in [0, 1]$; i.e., the cumulative stress level decreases from $D_{kj}(t)$ to $\theta D_{kj}(t)$ after a month of no deployment. If a servicemember first deploys in month t, he does not undergo recuperation during months $0, \ldots, t-1$. Let $\tau_{kj}(t)$ be the month during which the current deployment started if $C_{kj}(t) = 1$, and let $\tau_{kj}(t)$ be the month during which the current break started if $C_{kj}(t) = 0$. Then for $t = 1, 2, \ldots$, the stress dynamics are given by

$$D_{kj}(t) = D_{kj}(\tau_{kj}(t) - 1) + \sum_{s=\tau_{kj}(t)}^{t} E_{kj}(s) \quad \text{if } C_{kj}(t) = 1, \qquad (1)$$

$$D_{kj}(t) = D_{kj}(\tau_{kj}(t) - 1)\theta^{t-\tau_{kj}(t)+1} \quad \text{if } C_{kj}(t) = 0. \qquad (2)$$

In §2.3, we calibrate our model with data from the MHAT studies (OSMF 2006a, b). The MHAT surveys have been administered annually since late 2003 (OSMF 2003, 2005, 2006a, b, 2008). Their purpose is to assess the behavioral health of servicemembers deployed to OIF (OSMF 2006a). A sample of servicemembers deployed to OIF are given a survey with questions on environmental factors, individual and unit characteristics, and behavioral health status (OSMF 2006a). One of the key findings of these reports is that there is a positive correlation between the probability of developing PTSD and combat intensity, number of deployments, and length of deployments (OSMF 2006a, b). In our model the probability of developing PTSD is an increasing function of the stress level $D_{kj}(t)$ (see Equation (5)). Therefore, because $\lambda_j(t)$ should be a reasonable representation of the combat intensity and the stress level increases with the length and number of deployments (see Equations (1) and (2)), our formulation is consistent with the findings in the MHAT studies.

2.2.3. Stress Threshold. Each servicemember has a different random threshold for stress, which represents his or her strength in the stress-strength model. We denote this stress threshold by $\bar{D}_{kj}$ and assume a servicemember gets PTSD if $\max_t D_{kj}(t) \geq \bar{D}_{kj}$. In particular, if his cumulative stress exceeds $\bar{D}_{kj}$ at some point, then he develops PTSD, even if subsequent rest periods bring the cumulative stress level below $\bar{D}_{kj}$. Hence, our model is a variant of strength-stress models (Johnson 1988), in which both the strength ($\bar{D}_{kj}$) and stress ($\{D_{kj}(t), t = 1, 2, \ldots\}$) are random. Servicemember k of type j develops PTSD in month $\bar{t}_{kj}$, which satisfies

$$\bar{t}_{kj} = \min\{t \mid D_{kj}(t) \geq \bar{D}_{kj}\}. \tag{3}$$

The number of servicemembers of type j who have developed PTSD by month t, which we denote by $Y_j(t)$, is

$$Y_j(t) = \sum_k I_{\{\bar{t}_{kj} \leq t\}}, \tag{4}$$

where $I_{\{x\}}$ is the indicator function of the event x.

We assume that $\bar{D}_{kj}$ has an exponential distribution with mean γ^{-1}. The thresholds have the same distribution regardless of the troop type, which is consistent with empirical studies (OSMF 2005, 2006a; Milliken et al. 2007) comparing active Army and reserve servicemembers. However, PTSD rates can vary significantly across different unit types (e.g., transportation versus Medical; see annex A of OSMF 2005), which suggests that the threshold can depend on unit type. A more detailed analysis would further divide troops into specific unit types (e.g., transportation, combat, military police, engineering, medical, etc.), with each unit type having its own distribution for the threshold value and the stress process. Unfortunately, such refined data are not available to perform this detailed analysis.

The motivation for choosing an exponential distribution for $\bar{D}_{kj}$ is based on dose-response functions in the infectious disease literature. The distribution of the stress threshold has a one-to-one correspondence to a dose-response function, where the response is the likelihood of PTSD and the dose is the maximum cumulative stress. There are two standard dose-response models in the infectious disease literature: the Poisson model, which is used for some respirable diseases (Wells 1955), and a sigmoid (e.g., probit or logit) model, where the response is a sigmoid function of the logarithm of the dose (Finney 1971). Though PTSD is not an infectious disease, there are general similarities between the development of PTSD and the progression of infectious diseases, and we use the Poisson model in the base case (which yields the exponential distribution for $\bar{D}_{kj}$) and perform a sensitivity analysis using the probit model.

The Poisson model is consistent with a "one-hit" model, where the size of the dose is Poisson with mean γD and a single unit of dose that "hits" the target (e.g., enters the lungs, or in this case is sufficiently stressful) is sufficient to cause infection. For the Poisson model, the relationship between the response (i.e., the likelihood of PTSD) and the dose (i.e., cumulative stress) is given by

$$1 - e^{-\gamma D}, \tag{5}$$

where D is the cumulative stress and γ gives a measure of how much stress is required to cause PTSD. Consequently, the probability of developing PTSD is a concave function of the maximum cumulative stress level experienced by a servicemember (see also Figure 2 in §3.2), and the first traumatic event a soldier is exposed to during a deployment will have the largest marginal impact on his risk of developing PTSD.

To determine the distribution that corresponds to Equation (5), we assume there is an i.i.d. $U[0, 1]$ random variable u_{kj} associated with each servicemember. Inverting Equation (5) yields each servicemember's random stress threshold $\bar{D}_{kj}$, given by

$$\bar{D}_{kj} = -\frac{1}{\gamma}\ln(1 - u_{kj}). \tag{6}$$

This value is an exponential random variable with mean γ^{-1}.

The probability that a servicemember of type j develops PTSD is given by $P(\max_t D_{kj}(t) > \bar{D}_{kj})$. The cumulative stress $D_{kj}(t)$ is nondecreasing in time if there is no recuperation ($\theta = 1$), and in this case we can make a back-of-the-envelope estimate of this probability as a function of the total number of months deployed, m. We assume that t_m is the final month that this servicemember deploys, and to facilitate this estimation we ignore the initial stress. This, combined with the assumption of no recuperation, implies that $D_{kj}(t_m)$ is a compound Poisson random variable with batch size b and mean $\sum_{t:\, C_{kj}(t)=1} \lambda_j(t)$. Recalling that the threshold $\bar{D}_{kj}$ has an exponential distribution with mean γ^{-1}, we have that the probability of developing PTSD is given by

$$P\Big(\max_t D_{kj}(t) > \bar{D}_{kj}\Big)$$
$$= P(D_{kj}(t_m) > \bar{D}_{kj}), \quad (7a)$$
$$= \mathbf{E}[P(D_{kj}(t_m) > \bar{D}_{kj} \mid D_{kj}(t_m))], \quad (7b)$$
$$= \mathbf{E}[1 - e^{-\gamma D_{kj}(t_m)}], \quad (7c)$$
$$= 1 - \exp\Big(-\sum_{t:\, C_{kj}(t)=1} \frac{\lambda_j(t)}{b}(1 - e^{-\gamma b})\Big). \quad (7d)$$

The step from (7c) to (7d) makes use of the moment generating function of a Poisson random variable. If we assume $\lambda_j(t) = \lambda_j$ is constant, then the expression in Equation (7d) simplifies to

$$P\Big(\max_t D_{kj}(t) > \bar{D}_{kj}\Big) = 1 - e^{-\kappa_j m}, \quad (8)$$

where m is the total number of months a servicemember deploys, and $\kappa_j = (\lambda_j/b)(1 - e^{-\gamma b})$. Substituting average values for λ_j and the base-case values of b and γ, all of which are estimated in §2 of the online appendix, we find that $\kappa_1 = 0.028$ for the Army and $\kappa_3 = 0.040$ for the Marines.

Before turning to the time lag before symptom onset, we note that another conceivable distribution for $\bar{D}_{kj}$ would be the distribution corresponding to the probit dose-response model. Many dose-response curves have a sigmoid (e.g., probit or logit) behavior between the response and the logarithm of the dose (Finney 1971). The probit version states that the probability that a servicemember with cumulative stress D develops PTSD is $\Phi(\beta \ln(D/\mathrm{ID}_{50}))$, where $\Phi(\cdot)$ is the cumulative distribution function (cdf) of the standard normal distribution, ID_{50} is the cumulative stress that causes PTSD in half the population, and the probit slope β determines the population heterogeneity. For the probit model, Equation (6) is replaced by $\bar{D}_{kj} = \mathrm{ID}_{50} \exp(\Phi^{-1}(u_{kj})/\beta)$. When we need to distinguish between the exponential distribution corresponding to the Poisson dose-response function and the threshold distribution corresponding to the probit dose-response function, we will refer to the dose-response function (i.e., the Poisson model or the probit model).

2.2.4. Time Lag Dynamics. A servicemember with PTSD experiences a lognormal time lag between the first time his cumulative stress level exceeds $\bar{D}$ and the time at which he first develops symptoms. We choose a lognormal random variable because the time lag is qualitatively similar to the latent periods of infectious diseases, which often fit lognormal distributions well (Limpert et al. 2001). In this analysis we assume that someone "develops" symptoms not when he first physically exhibits symptoms, but when he first reports or admits to symptoms. Hence, there are two components to the time lag: the lag between the traumatic event and the physical manifestation of symptoms, and the delay between the onset of symptoms and the reporting of symptoms. Because the studies we use to calibrate our model collect data from self-reported surveys (Wolfe et al. 1999, Milliken et al. 2007), we cannot disentangle these two factors. It is important that both factors are embedded in the time lag, because both contribute to the underreporting of PTSD. In addition, the great majority of servicemembers and veterans would not receive treatment until they self-reported symptoms, which is consistent with our model's goal of helping to estimate the demand for mental health resources.

Recent data suggest that the time lag depends strongly on whether a servicemember is physically in the military (in our model, $j = 1$ or $j = 3$ and the servicemember has not discontinued service, or $j = 2$ and $C_{k2}(t) = 1$) or has returned to civilian life ($j = 1$ or $j = 3$ and the servicemember has discontinued service, or $j = 2$ and $C_{k2}(t) = 0$) (Milliken et al. 2007). The difference may be caused by organizational barriers to mental health care (Hoge et al. 2004, Figure 11 in OSMF 2006b), health-care benefits, the perceived stigma (Figure 10 in OSMF 2006b) associated with mental health problems (note that the survey results in Milliken et al. 2007 become part of each servicemember's personal record), the military support system, the possibility of delayed discharge after symptoms are revealed, the expectation while in the military that mental health will improve on return to civilian life, and the stress involved with readjustment to civilian life (Milliken et al. 2007, Tanielian et al. 2008). To account for this dependence, we model both a military time lag and a civilian time lag. We assume that a servicemember's time lag is given by the random variable T_1 when he is physically in the military (even if he is not deployed), but switches to the random variable T_2 when he returns to civilian life. We assume that this switch occurs in a memoryless manner, so that the time lag while in civilian life is independent of how long the servicemember was symptomless while physically in the military. However, if a Reserve Army servicemember ($j = 2$) serves multiple deployments, we assume that history is maintained across consecutive periods while the servicemember is physically in the military and consecutive periods while he or she is in civilian life; i.e., T_1 and T_2 apply to the cumulative amount of time physically in the military and in civilian life, respectively. For $i = 1, 2$, we let $f_i(t)$ and $F_i(t)$ denote the pdf and cdf of T_i, which is lognormal with median e^{μ_i} and dispersion factor e^{s_i}.

We let $S_j(t)$ denote the cumulative number of servicemembers of type j who have developed PTSD symptoms by time t. Defining X_{kj} as the amount of time between when servicemember k of type j develops PTSD and the onset of symptoms, we have

$$S_j(t) = \sum_k I_{\{\bar{t}_{kj}+X_{kj}\le t\}}. \tag{9}$$

Although the X_{kj} are independent, they are not identically distributed. The X_{kj} depend on several factors, including whether a servicemember is active or reserve, the specific deployment schedule, and when the servicemember separates from the military. Thus, the X_{kj} cannot be written easily as a function of T_1 and T_2.

We do not model the amount of time the PTSD persists, which depends on a variety of factors, including the severity of symptoms and the amount of mental health care received. Although symptoms of PTSD can abate without treatment in a minority of cases, PTSD is known to be a persistent condition if left untreated, in which symptoms come and go over long periods of time (Kessler et al. 1995). Because we are interested in estimating the number of servicemembers who may require mental health-care treatment at some point in their lives, once someone develops symptomatic PTSD in our model, he or she does not recover on his or her own.

2.3. PTSD Parameter Estimates

The predicted troop levels track the official DOD troop numbers reasonably well: The average relative monthly deviation is <10%, with larger deviations occurring during the first 7 months of OIF (§1.3 in the online appendix). The parameter estimates for the PTSD model are presented in §2 of the online appendix. The four parameters of the lognormal time lags are derived from sparse longitudinal data (Wolfe et al. 1999, Milliken et al. 2007), and their estimates are $\mu_1 = 2.47$, $s_1 = 2.73$, $\mu_2 = 1.40$, and $s_2 = 0.57$. The military time lag until symptoms (median 11.78 months, mean 40.87 years) is longer and much more heavily tailed than the civilian time lag (median 4.05 months, mean 4.77 months), implying that some career military servicemembers in our model will never exhibit PTSD symptoms.

The four PTSD parameters (mean initial stress, batch size, mean threshold value, recuperation rate) are estimated using a least squares approach based on 17 PTSD rates from MHAT surveys for various groups of servicemembers at various points in time (OSMF 2006a, b). See §2 in the online appendix for more details on the estimation procedure. The mean initial stress ($\alpha^{-1} = 0.0068$) is comparable to the average monthly stress from combat (0.0051 for Army, 0.0090 for Marines) in our model, suggesting that the stress endured during a month of exposure to combat could be greater than the stress previously accumulated in a servicemember's lifetime, including the anticipation of deployment. The mean threshold value ($\gamma^{-1} = 0.130$) is approximately equal to the average stress accumulated during two deployments. The batch size in the compound Poisson process ($b = 0.0631$) is approximately equal to the average stress in one deployment. That is, the stress process is extremely bursty, with rare (e.g., approximately one-third of servicemembers are not exposed to any combat-related stress during a deployment) large jumps that represent particularly stressful events. In addition, servicemembers who do screen for PTSD in our model are going to vary by the ratio of their maximum stress divided by their threshold. Although we are focused on the fraction of servicemembers who get PTSD, to the extent that the severity of PTSD (and hence the type and intensity of treatment required) is related to the maximum stress-to-threshold ratio, our model predicts that there will be a range of severity of symptoms. Finally, there is full recuperation ($\theta = 0$), which implies that there is no accumulation of stress across deployments in our base-case model. However, deploying multiple times does increase the probability of developing PTSD because of the greater exposure to trauma. Such a small value of θ also implies that after he or she returns from a deployment, the stress level of the servicemember will drop below the precombat level. This is not implausible because of the stress (caused by uncertainty and inexperience) leading up to the first deployment.

The model's predicted PTSD rates are compared with the 17 reported MHAT PTSD rates in Table 1. The values in Table 1 are of the form P_j(MHAT-k), which is the probability that a type j servicemember has symptomatic PTSD during the kth MHAT study. For example, "P_{1+2}(MHAT-IV), 1st deployment" is the probability that active and reserve Army servicemembers on their first deployment had symptomatic PTSD during MHAT-IV (which was administered in October 2006). The 18th value in Table 1 is the fraction of troops exposed to no combat. This quantity is not an MHAT PTSD rate but is the fraction of Army soldiers finishing a deployment between 2004 and 2006 who were not exposed to any traumatic combat experiences. See §2 in the online appendix for more details on the values in Table 1 and how they were estimated.

The optimal parameter values achieve an average relative deviation of 15% for the 18 values in Table 1. Our model significantly underestimates the data point from the first MHAT study, P_{1+2}(MHAT-I). Indeed, the fact that this rate is greater than the PTSD rates in MHAT-II (P_{1+2}(MHAT-II)) and MHAT-III (P_{1+2}(MHAT-III)) is somewhat puzzling and is not commented on by the authors of MHAT-II or MHAT-III (OSMF 2005, 2006a). One possible explanation is that early in OIF the servicemembers may have been

Table 1 **Reported PTSD Prevalence Rates and Estimated PTSD Prevalence Rates for the Base-Case Model ($\alpha = 146.7$, $\gamma = 7.72$, $b = 0.0626$, and $\theta = 0$)**

PTSD prevalence rate	Reported	Estimated	References
P_{1+2}(MHAT-I)	0.158	0.072	OSMF (2006a, p. 20)
P_{1+2}(MHAT-II)	0.113	0.109	OSMF (2006a, p. 20)
P_{1+2}(MHAT-III)	0.136	0.133	OSMF (2006a, p. 20)
P_1(MHAT-III), 1st deployment	0.125	0.127	OSMF (2006a, p. 21)
P_1(MHAT-III), >1st deployment	0.184	0.178	OSMF (2006a, p. 21)
P_{1+2}(MHAT-IV)	0.170	0.148	OSMF (2006b, Figure 6)
P_3(MHAT-IV)	0.140	0.166	OSMF (2006b, Figure 6)
P_{1+2}(MHAT-IV), low exposure to trauma	0.080	0.026	OSMF (2006b, Figure 7a)
P_{1+2}(MHAT-IV), medium exposure to trauma	0.140	0.139	OSMF (2006b, Figure 7a)
P_{1+2}(MHAT-IV), high exposure to trauma	0.280	0.281	OSMF (2006b, Figure 7a)
P_3(MHAT-IV), low exposure to trauma	0.060	0.050	OSMF (2006b, Figure 7b)
P_3(MHAT-IV), medium exposure to trauma	0.110	0.159	OSMF (2006b, Figure 7b)
P_3(MHAT-IV), high exposure to trauma	0.280	0.291	OSMF (2006b, Figure 7b)
P_{1+2}(MHAT-IV), 1st deployment	0.150	0.110	OSMF (2006b, Figure 8)
P_{1+2}(MHAT-IV), >1st deployment	0.240	0.236	OSMF (2006b, Figure 8)
P_{1+2}(MHAT-IV), ≤6 months on cur. depl.	0.120	0.118	OSMF (2006b, Figure 9)
P_{1+2}(MHAT-IV), >6 months on cur. depl.	0.190	0.174	OSMF (2006b, Figure 9)
Fraction of troops exposed to no combat	0.320	0.304	Milliken et al. (2007)
Root mean square error		0.030	

Note. P_j(MHAT-k) represents the probability of a type j servicemember having symptomatic PTSD during the administration of the kth MHAT survey, where $j = 1+2$ represents active and Army Reserve soldiers.

overly optimistic that OIF would progress similarly to the Gulf War with little exposure to combat and trauma. When that did not happen, these early deployers were more susceptible to PTSD, whereas servicemembers deploying later had more realistic expectations and perhaps more training to prepare for the nature of the war. This hypothesis suggests that the stress threshold $\bar{D}_{kj}$ may have a time-varying distribution.

Our model also underestimates the PTSD rate for Army servicemembers exposed to the least amount of trauma (P_{1+2}(MHAT-IV), low exposure to trauma). These servicemembers may not have had as much training to prepare them for—nor the proper expectations regarding whether they will be involved in—traumatic incidents. When these servicemembers are exposed to combat or improvised explosive devices (IEDs), these events may have a more significant effect on them. This suggests that stress thresholds may be correlated with stress levels. If data existed at a more refined level (e.g., by occupation), we could incorporate this aspect into our model.

The model overestimates the PTSD rate for Marines exposed to an average amount of trauma (P_3(MHAT-IV), medium exposure to trauma). Of the four Marine PTSD rates, our model overestimates three of them. It is possible that Marines are better able to handle the stress (via self-selection and more intense screening and training) or perhaps less inclined to admit PTSD symptoms, than their Army counterparts. This implies that the threshold distribution may depend on the branch of the military or that the Marines may have a different time lag than the Army. The Marine data point that the model underestimates is the PTSD rate for Marines exposed to the least amount of trauma (P_3(MHAT-IV), low exposure to trauma). If there is both an inadequate training and expectations factor that causes the model to underestimate the PTSD rate for servicemembers exposed to low amounts of trauma (as described in the previous paragraph) and a Marine factor that causes our model to overestimate the PTSD rate, then both of these competing factors will contribute to the estimate of the value "P_3(MHAT-IV), low exposure to trauma."

The only other data point that does not fit well in the model is the PTSD rate for Army servicemembers on their first deployment during MHAT-IV (P_{1+2}(MHAT-IV), 1st deployment). This is more than three years into OIF, so naive expectations should not be a factor. Because this data point only includes first deployers, inexperience may play a role, but the model fits the data point "P_1(MHAT-III), 1st deployment" well (although the MHAT-III value does not include reserve servicemembers).

3. Results

We provide the base-case results in §3.1, followed by three model modifications in §3.2 and four sensitivity analyses in §3.3.

3.1. Base-Case Results

We compute PTSD rates under three possible future withdrawal scenarios. In all three scenarios, the total troop level drops to 140,000 in July 2008 (Burns 2008) (as previously announced by President Bush in

Figure 1 **Cumulative Number of Symptomatic Servicemembers for the Three Withdrawal Scenarios, as Predicted by the Model**

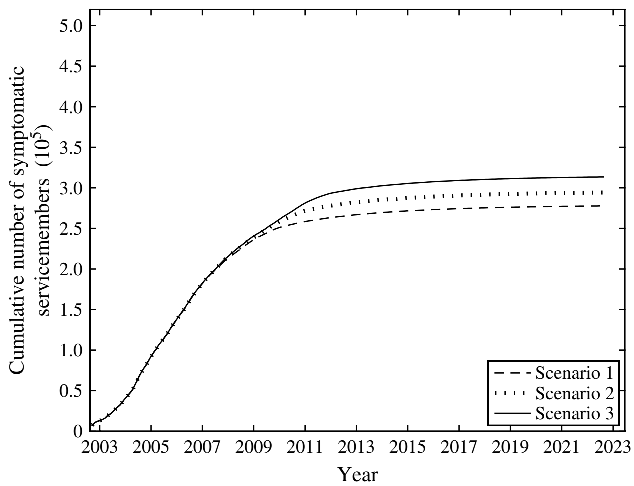
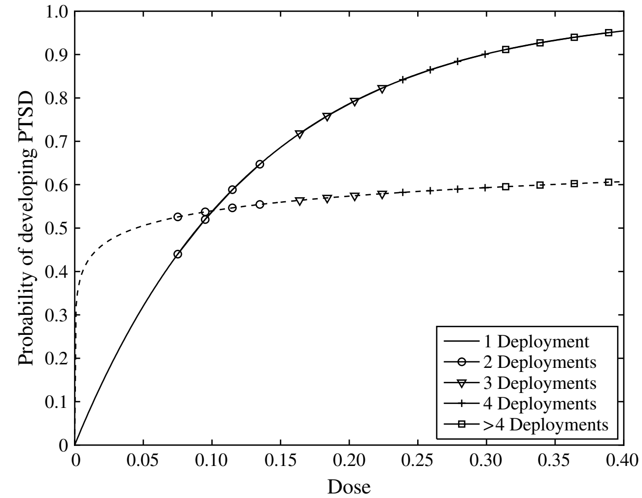


Figure 2 **Poisson (—) vs. Probit (- - -) Dose-Response Curves**



Note. The dose (which represents the maximum cumulative stress experienced by a servicemember) is translated into number of deployments, for the no recuperation scenario, using the average monthly stress (λ) and deployment lengths.

2007), the troop level stays at 140,000 until withdrawal begins, and it takes 13 months to withdraw (modeled as a linear drop from 140,000 to 0) (CBO 2007). In the three scenarios, the withdrawal starts in February 2009, February 2010, and February 2011, respectively. We assume the stress process $\lambda_j(t)$ (calculated separately for Army and Marine servicemembers) in each month starting with October 2008 is equal to the average value of $\lambda_j(t)$ over October 2007–September 2008; because stress is measured as casualties per deployed servicemember, this assumption implies that the casualties are also dropping linearly to 0 during the 13-month withdrawal process. To perform this analysis, we need to specify the troop deployment schedules after September 2008, which we do through a combination of predicting combat units' future deployments using published data and estimating unattached support units' future deployments by assuming that they follow a cyclic deployment and rest schedule (§1.4 of the online appendix).

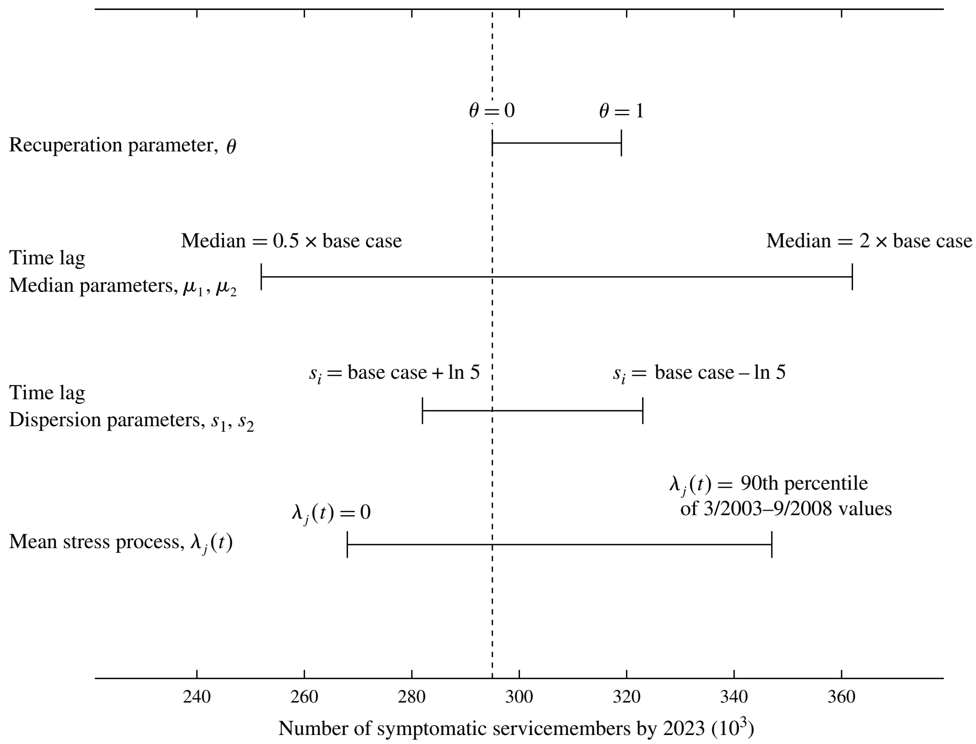
Figure 1 shows the predicted cumulative number of symptomatic PTSD cases as a function of time, starting from the beginning of OIF in March 2003, for the three different withdrawal strategies. Current and past cases of PTSD are when the three curves are together, and future cases are when the three curves split apart. By February 2023, our model predicts that 278,000, 294,000, and 313,000 servicemembers will have exhibited symptoms of PTSD under withdrawal scenarios 1, 2, and 3, respectively. This constitutes ≈40% of the active Army and Marines and ≈32% of the Army Reserve who deploy to OIF (Figure 8 in the online appendix). Because of the difference in the military and civilian time lags, symptomatic cases among the active Army soldiers and Marines lag behind the symptomatic cases in the reserve Army by ≈1–2 years (§3.1 in the online appendix). As expected, there is considerable heterogeneity among servicemembers in both the number of stressful events experienced (in scenario 2, 34.7% of servicemembers experience no stressful events, 0.2% experience ≥9 events; Table 19 in the online appendix) and the maximum cumulative stress/strength threshold ratio (in scenario 2, 71% of servicemembers are less than one order of magnitude from the critical ratio of 1, and 6% of servicemembers are more than two orders of magnitude away from 1; Figure 9 in the online appendix).

3.2. Model Modifications

We analyze several variations of our model to test how robust the model is (§3.2 in the online appendix). Replacing the Poisson dose-response function by the probit leads to a model with no recuperation ($\theta = 1$) and a stronger cumulative effect from multiple deployments. The probit model has full recuperation, and the Poisson model has no recuperation because the probit dose-response curve is much flatter than the Poisson dose-response curve (Figure 2). The joint estimation of the PTSD parameter values leads to a one-dimensional subspace of solutions that yields nearly identical sum-of-squared deviations, which is only slightly lower than the base-case sum of squares; solutions in this subspace have no recuperation ($\theta = 1$) and a similar batch size, and the mean initial stress level and the ID_{50} vary in a systematic way. The probit model predicts ≈5% fewer PTSD cases than the Poisson model, for the entire subspace of solutions.

To isolate the effect of multiple deployments, we consider the hypothetical case in which there is an

Figure 3 Range of the Predicted Number of Servicemembers with Symptomatic PTSD for Each of the Scenarios Described in §3.3



Notes. The labels on the left side refer to the parameter being varied in each scenario (all other parameters remain at their base-case value). The base-case prediction is the dashed vertical line. These results are for withdrawal scenario 2, which begins in February 2010.

involuntary military draft, and each servicemember only deploys once. That is, we maintain the same PTSD parameter values and the same brigade/battalion rotation as in the base-case, but each time a unit is deployed, each servicemember is new. We make this modification for past, current, and future deployments. In this extreme scenario, the fraction of servicemembers that are symptomatic with PTSD is reduced to ≈0.30, but the total number of symptomatic PTSD cases increases by >30% because many more servicemembers are exposed to combat.

The decision to leave the military and return to civilian life may be related to a servicemember's mental health (Hoge et al. 2006). To investigate an extreme version of this phenomenon, we modify the model so that at the end of each deployment the servicemembers who return to civilian life are those that currently have the highest stress-to-threshold ratio (see §3.2 in the online appendix for details). Using the base-case parameters, the number of symptomatic PTSD cases increases by ≈5% relative to the base case; although servicemembers who redeploy may be more resilient, servicemembers who have developed PTSD are likely to separate from the military and be replaced by new servicemembers who are susceptible to PTSD, thereby causing the increase. However, under this variant of the model, the fit of the MHAT probabilities with the base-case parameters is poor (Table 18 in the online appendix). After a recalculation of the optimal parameters for this version of the model, the number of symptomatic PTSD cases increases by ≈60%; the fit is still worse than the base case (see §3.2 in the online appendix), suggesting that the decision to leave the military may depend on a variety of other factors (such as family, finances, camaraderie, morale) and not primarily on a servicemember's ability to cope with the stress that he has been exposed to.

3.3. Sensitivity Analyses

We perform four sensitivity analyses of the PTSD parameter value estimates (§3.3 in the online appendix). Figure 3 presents the range of the predicted number of symptomatic PTSD cases for each of these scenarios. First we disallow recuperation in the base-case model by setting $\theta = 1$. Keeping the other PTSD parameters at their base-case level leads to <10% more symptomatic PTSD cases than in the base case, and reoptimizing the values of the remaining PTSD parameters leads to an increase in the number of symptomatic PTSD cases of <5%. In both cases, the fit of the PTSD model is not much worse than in the base case (Table 18 in the online appendix).

The next two analyses involve the two time lag parameters. Because we have limited data with which to estimate these parameters, we vary the median time lags (for both the military and civilian time lags) by a factor of two while maintaining the same dispersion factors. When the median time lag is increased by a factor of two, the mean threshold level (γ^{-1}) decreases and there is no recuperation ($\theta = 1$), so as to achieve the symptomatic PTSD rates in the MHAT studies (OSMF 2006a, b). In this case, ≈20% more servicemembers develop symptomatic PTSD. Similarly, when the median time lag is decreased by a factor of two from the base-case level, the mean threshold level increases, there is full recuperation ($\theta = 0$), and the number of symptomatic PTSD cases decreases by ≈15%. Because the mean of the military time lag is much greater than the median in the base case, we also varied the dispersion factors. We increased and decreased the dispersion factors (for both the military and civilian time lags) by a factor of five (which is achieved by changing the parameters s_i by $\pm \ln 5$) while maintaining the same medians. This has a significant impact on the variability of the military time lag distribution. The mean of the military time lag decreases from 40.87 years to 1.84 years when we reduce the dispersion and increases to more than 12,000 years when the dispersion increases. Even with these extreme variations from the base-case distribution, the results for these two scenarios were less than a 10% deviation from the base case (see §3.3 in the online appendix; see also Figure 3).

Finally we analyze three different values for the future mean stress process $\lambda_j(t)$. We consider the stress level to be 0, the median value between March 2003 and September 2008 and the 90th percentile value between March 2003 and September 2008. When we decrease the future stress level to 0, ≈10% fewer servicemembers develop symptomatic PTSD, and when we increase the future stress level to the median and 90th percentile value, the number of symptomatic PTSD cases increases by ≈8% and ≈18%, respectively.

4. Discussion

4.1. Limitations of Study

There are many organizational (e.g., training, leadership; see Figure 1 in OSMF 2006b), demographic (e.g., age, gender, marital status), and environmental (weather, uncertain future deployment) factors (OSMF 2003) that affect the behavioral health status of troops. Our model focuses on the impact of two interrelated factors: combat exposure and deployment schedule. The deployment cycle impacts PTSD prevalence in our model in two ways, by allowing for combat exposure during deployment and partial recuperation in between deployments. However, our analysis does not provide a reliable estimate for the recuperation rate because values at the two extremes of 0 and 1 are obtained, depending on the choice of the dose-response function and the value of the median time lag until symptom onset. Moreover, our analysis is unable to shed any light on the nature of the dose-response relationship: when switching from a one-parameter Poisson model to a two-parameter probit model, we appear to be overfitting our model to the available data, as revealed by the one-dimensional subspace of solutions under the probit model.

Our analysis highlights the need for additional data (beyond the need to better estimate the time lag, as noted below), which would allow us to analyze more complex variants of the model. For example, the stress threshold distribution may vary over time and by troop type (see §2.3), and different types of servicemembers engage in different kinds of activities during their time between deployments, implying that the recuperation rate θ may vary by troop type. Furthermore, the aggregation of casualty and PTSD data for soldiers prevented us from attempting to understand the differences in combat exposure and PTSD for different segments of the Army (e.g., combat versus transportation versus medical), even though our model explicitly captures the heterogeneity in combat exposure via the batch size in the compound Poisson process. On a related point, it is possible that IEDs, which became more common during the summer of 2005 (O'Hanlon and Campbell 2007, p. 31), caused additional stress in support troops (e.g., troops involved in transport logistics). However, IEDs are responsible for 80% of Army casualties in OIF (O'Hanlon and Campbell 2007, p. 31), so our measurement of monthly stress should indirectly capture this. Finally, we have assumed that the time lag is independent of the amount of combat exposure, although data suggest that individuals with more combat exposure were more likely to experience longer time lags (Gray et al. 2004).

4.2. Robustness of Results

Nonetheless, from the viewpoint of estimating the cumulative number of servicemembers (more specifically, Army soldiers and Marines) who will develop PTSD from OIF, our results appear to be quite robust: The difference in PTSD rates between allowing full recuperation between deployments and no recuperation is <5% (this small effect is due partially to the fact that our model predicts that more than half of the deployed servicemembers in OIF deploy only once), and the Poisson and probit models generate PTSD rates that differ by <10%. The casualty rate in OIF has been declining since the first half of 2007 (SIAD 2008a; see also Figure 6 in the online appendix), and if this rate continues to decline, then the PTSD rates could drop by as much as 10%–15%. However, if the stress levels increase to those occuring earlier in OIF, then the PTSD rates could increase by 10%–20%.

Among the parameters in our model, the median time lag has the biggest impact on our results (Figure 3). The data available to estimate the time lag parameters are sparse, and it may be that two studies (Wolfe et al. 1999, Milliken et al. 2007) are too few to generate reliable estimates for the time lag parameters. Nonetheless, our base-case estimate gives a PTSD rate that is approximately twice the values in recent OIF surveys (Hoge et al. 2004; OSMF 2003, 2005, 2006a, b, 2008; Tanielian et al. 2008; although this last study includes Air Force and Navy personnel), and Figure 3 shows that the PTSD rate drops by only 20% from the base case when we cut the median time lags in half. Moreover, if we drastically reduce the military time lag distribution so that it equals the civilian distribution (analysis not shown), the PTSD rate is reduced to 27%, which is still much higher than the PTSD rates reported in the recent surveys; note that some of the discrepancy between the surveys and our results is because many surveyed servicemembers will be exposed to additional combat stress after they are surveyed (i.e., in the current or a subsequent deployment). Hence, the twofold punchline of this study is that ignoring the time lag (i.e., assuming it is zero, as is implicitly done in the recent surveys) and the future stress exposure of those surveyed leads to a significant underestimation of the PTSD rate, and further data are required to improve the precision of the time lag. More specifically, there is a need for a large-scale longitudinal study that involves at least three or four time points, which would provide a better understanding of the time lag distributions and hence a more refined forecast of future PTSD cases.

As an aside, our PTSD estimates are also higher than those obtained from the Vietnam War, which is not surprising, given the higher deployment tempo in OIF. A study of Vietnam veterans 15 years after they left the military (Schlenger et al. 1992) estimates a PTSD rate of ≈15%. The National Vietnam Veterans Readjustment Study estimated that ≈30% of Vietnam veterans would develop PTSD during their lifetimes (Kulka et al. 1988), although a recent reevaluation of that study by Dohrenwend et al. (2006) estimated the value was closer to 20%.

When considering whether our results can be extrapolated to all OIF servicemembers, it is important to note that the MHAT studies focus on combat units (OSMF 2003, 2005, 2006a, b), and it would seem that servicemembers in these units may screen for PTSD at higher rates than the general population deployed to OIF. However, MHAT studies II, III, and IV (OSMF 2005, 2006a, b), which contain the bulk of the PTSD data used to calibrate our model, state that their samples should be representative of the larger theater population. Furthermore, combat servicemembers screen for mental health concerns at lower rates than several other occupations (see Figure 3 in annex A of OSMF 2005). Despite this potential bias, our results are likely to be conservative—that is, they are likely to underestimate the true number of servicemembers that will experience PTSD—for several reasons. Our model assumes that someone who develops PTSD stays in that condition. There is some selection bias in that only working, nondisabled servicemembers were surveyed in the MHAT studies (Hoge 2005). Furthermore, the MHAT reports define a servicemember as screening for PTSD through a self-reported survey (OSMF 2006b), which has been validated in military settings (Bliese et al. 2008) and has been used in several other studies analyzing PTSD in OIF servicemembers (Hoge et al. 2004, Hotopf et al. 2006, Smith et al. 2008) and is likely a conservative definition (Hoge et al. 2004, OSMF 2006b, Tanielian et al. 2008). In addition, the stigma associated with mental health problems and the shifting incentives as servicemembers return to civilian life can lead to underreporting and delayed reporting (although we attempt to capture the latter factor with the time lag); see our discussion about time lag dynamics in §2.2. Because of the paucity of longitudinal data, it is difficult to estimate the time lag until symptoms develop, and we may be underestimating the right tail of the time lag distribution by ignoring the right censoring in the longitudinal Gulf War study (Wolfe et al. 1999 and §2 in the online appendix). If the continuation rate depends on a servicemember's exposure to stress or the ability to cope with stress, then servicemembers with PTSD will have higher attrition rates, causing their replacements to receive more combat exposure (§3.2 in the online appendix). Because of the increase in the number of waivers of enlistment standards and less precombat training for recent Army recruits (Thompson 2007), it seems plausible that these soldiers will be more vulnerable to PTSD than the soldiers surveyed in the MHAT studies. Our PTSD estimates do not include servicemembers who never deploy or servicemembers from the Air Force or Navy (Smith et al. 2008), all of whom experience some PTSD, albeit at reduced rates (Smith et al. 2008). Our estimates also do not include the $>10^6$ government contractors participating in OIF, who may have a more difficult time accessing mental health services (Risen 2007).

Our model also does not include servicemembers deployed to Afghanistan in Operation Enduring Freedom (OEF). Forces have been deployed to Afghanistan since 2001 and deployment data (SIAD 2008b) do exist, as well as casualty data (SIAD 2008a) and limited PTSD data (OSMF 2008). For most of the duration of OIF, servicemembers deployed to Afghanistan in OEF were exposed to less combat and screened for PTSD at lower rates than troops deployed to OIF (Hoge et al. 2004, 2006; Tanielian et al. 2008). However, in the months at the end of our study there has been a trend of fewer casualties in OIF and more casualties in OEF (SIAD 2008a). Including OEF would have required a simultaneous deployment model to both theaters and possible estimation of more PTSD parameters with limited additional data. To get a sense of the relative magnitude of OIF and OEF, we note that there have been 13.0 times as many troops wounded and 8.5 times as many troops killed in OIF as there have been in OEF since the start of OIF in March 2003 (SIAD 2008a), and on average there have been 8.1 times more troops deployed per month to OIF than to OEF (SIAD 2008b). These figures suggest that the number of PTSD cases from OEF is roughly an order of magnitude less than the number of OIF cases (i.e., a total of ≈30,000 cases among soldiers and Marines).

Given that the earliest withdrawal date would appear to be no earlier than February 2009, given our sensitivity analyses (§3.2 and §3.3 in the online appendix), and given that the withdrawal itself may take longer than 13 months and may be incomplete, we predict that there will be at least 300,000 soldiers and Marines who develop PTSD and that on the margin, there are ≈20,000 new cases for every year that the war is prolonged. Although it is not possible to put a confidence interval on these estimates, our sensitivity analyses suggest that these figures are likely to be within ±30%.

4.3. Effects of Time Lag and Multiple Deployments

Although our model was unable to tease out the form of the dose-response function or the value of the recuperation rate, its dynamic aspects allow us to understand the impact of the time lag until symptoms and of multiple deployments; in contrast, a traditional logit model—with independent variables such as the total exposure to combat, number of deployments, and deployment lengths—would require the PTSD status of each individual servicemember (this is not publicly available) and could not account for the impact of the time lag, which is crucial for developing an accurate estimate of the eventual PTSD rates. The time lag is shorter after a servicemember separates from the military (Milliken et al. 2007), which may be caused by a variety of factors, including the reluctance of active servicemembers to self-report PTSD symptoms, the two-year time window for VA health benefits after separation from the military, and the difficulties of transitioning back to civilian life. Consequently, our model predicts that, in aggregate, reserves develop symptomatic PTSD ≈1–2 years before active servicemembers, which is not inconsistent with recent VA data that members of the National Guard and Army Reserve have accounted for more than half of the suicides among OIF veterans (Associated Press 2008). Our model also predicts that among servicemembers who screened positive on their second deployment in the MHAT studies, >75% were exposed to PTSD-generating stress during their first deployment.

There are many issues and concerns regarding health, experience, morale, family life, etc. that need to be balanced when determining deployment schedules. From the narrow confines of our model, multiple deployments can be viewed from the individual servicemember's viewpoint or from the perspective of the military as a whole. When viewed from the servicemember's viewpoint, the likelihood of a random servicemember developing PTSD after m months of service in the absence of recuperation between deployments is $\approx 1 - e^{-0.028m}$ for Army soldiers and $\approx 1 - e^{-0.040m}$ for Marines (see Equation (8)). Similarly, superimposing the average dose rate (i.e., stress from combat) and the deployment length on to the horizontal axis of the dose-response curve (Figure 2) shows the increased risk for PTSD incurred by multiple deployers with no recuperation (although the effect is larger for the Poisson curve than the flatter probit curve) and highlights one of the hazards of carrying out a prolonged war with a volunteer military. Even if there is full recuperation, the PTSD rate increases from 0.24 to 0.39 to 0.64 when comparing Marines who deployed for 1, 2, or ≥4 deployments, respectively. However, multiple deployments reduce the total number of PTSD cases, because of the concave nature of the dose-response curve: In the extreme hypothetical example where OIF had utilized an involuntary draft to increase the number of troops to the point where there were no multiple deployments, the number of PTSD cases increases by >30%, that is, by >100,000 cases. Moreover, this increase may be significantly underestimated because the thresholds for draftees would likely be lower than for volunteers (e.g., due to self-selection and motivation).

4.4. Supply vs. Demand

Our primary motivation for forecasting future PTSD incidence is to enable the VA system to plan for adequate supply of PTSD care. Unfortunately, when mapping from future PTSD incidence to future demand for VA mental health services, there are several factors that are considerably more uncertain than our estimates in Figure 1 for the number of OIF servicemembers and veterans who will develop PTSD each month. First, only some servicemembers will be referred to the VA for a mental health evaluation on separation from the military. The Government Accountability Office reports that only 22% of OIF/OEF veterans at risk for PTSD were referred by DOD's health-care providers for a mental health evaluation; this reveals the tension between the VA's attempts at early intervention for rehabilitation and the DOD's retention goals (Bascetta 2006). Even if a veteran is screened for—and diagnosed with—PTSD, there is no guarantee that he will seek treatment. Indeed, the majority of the general U.S. population with mental health problems does not receive treatment (because of stigma or to lack of access and benefits; Kessler et al. 2005, Wang et al. 2005), and veterans may be no different. Even if veterans receive treatment, some of them may receive it in the private sector. Finally, as mentioned earlier, PTSD symptoms can come and go over long periods of time, and full remission is occasionally achieved after initial symptoms in the absence of treatment (Kessler et al. 1995). There are currently not ample data to estimate these other factors and hence to reliably convert future PTSD incidence into demand for VA mental health services. However, if all these other factors occur in a time-homogeneous manner, then Figure 1, combined with recent demand for VA mental health treatment by OIF veterans, can lead to crude estimates for future demand.

As discussed in Atkinson et al. (2008) and Wein (2009), the available supply and demand data suggest cause for concern; Atkinson et al. (2008) also provide a very crude demand-versus-supply analysis. Despite the uncertainties in our PTSD rates and in the factors raised in the previous paragraph, we believe that our analysis justifies making two policy recommendations: 100% of servicemembers should be evaluated by the VA for PTSD on separation from the military, and rapid evidence-based care should be provided to those servicemembers requiring treatment. Early identification and treatment of PTSD may lessen the severity of the condition, and if left untreated, PTSD can lead to comorbidities such as substance abuse and severe depression (Prigerson et al. 2002). A recent study concludes that the evidence is sufficient to conclude the efficacy of exposure therapies in the treatment of PTSD, but inadequate to determine the efficacy of a variety of pharmacotherapies and other psychotherapies (Institute of Medicine 2007). A recent cost analysis estimates that evidence-based PTSD care, which provides complete remission in an estimated 30%–50% of cases (Friedman 2006), would pay for itself within two years, largely by reducing the loss of productivity (Tanielian et al. 2008). The VA's response to workload increases and capacity shortages for mental health care has been to reduce the intensity of service per patient (i.e., fewer patient visits per year, despite no improvements in treatment technology that would warrant this) (Rosenheck and Fontana 2007), which does not bode well for returning veterans with PTSD, who typically require 3–6 months of intensive treatment if there are no comorbidities (National Center for PTSD 2007). To create surge capacity during this crucial time window of troop withdrawal, the government may need to train and compensate mental health professionals in the private sector.

4.5. Conclusion

We provide an integrative modeling approach that links rates of PTSD to troop deployment patterns and combat exposure during deployments. The incorporation of a time delay into the model reveals that raw survey data of active servicemembers during OIF is likely to significantly underestimate the number of PTSD cases ultimately generated. The model and analysis provide a starting point for further refinement of both the model and the parameter values as new data become available. Although it is tempting to employ the model to predict PTSD rates for various types of deployment schedules (e.g., frequent 6-month deployments versus infrequent 12-month deployments), we believe this is premature. Such a comparison would require an accurate estimate of the recuperation rate θ, which could not be obtained from our analysis with the existing data.

5. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

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References

Associated Press. 2008. Most vet suicides among Guard, Reserve troops. (February 12), http://www.msnbc.msn.com/id/23132421/.

Atkinson, M. P., A. Guetz, L. M. Wein. 2008. A dynamic model for post-traumatic stress disorder among U.S. troops in Operation Iraqi Freedom. Working paper, Graduate School of Business, Stanford University, Stanford, CA.

Bascetta, C. A. 2006. Post-traumatic stress disorder: DOD needs to identify the factors its providers use to make mental health evaluation referrals for servicemembers (GAO-06-397). Government Accountability Office, Washington, DC.

Bliese, P. D., K. M. Wright, A. B. Adler, O. Cabrera, C. A. Castro, C. W. Hoge. 2008. Validating the primary care posttraumatic stress disorder screen and the posttraumatic stress disorder checklist with soldiers returning from combat. *J. Consulting Clin. Psych.* **76** 272–281.

Burns, R. 2008. Defense department expects "post-surge" U.S. troop levels in Iraq to be 140,000. (February 25). Associated Press, New York.

Bush, G. W. 2007. In Bush's words: Assessing the war today, and the risks to avoid tomorrow. *New York Times* (September 14).

Centers for Disease Control Vietnam Experience Study. 1988. Health status of Vietnam veterans, I: Psychosocial characteristics. *J. Amer. Medical Assoc.* **259** 2701–2707.

Congressional Budget Office, U.S. Congress. 2005. An analysis of the U.S. military's ability to sustain an occupation of Iraq: An update. Congressional Budget Office, Washington, DC.

Congressional Budget Office, U.S. Congress. 2006. Recruiting, retention, and future levels of military personnel. Congressional Budget Office, Washington, DC.

Congressional Budget Office, U.S. Congress. 2007. Some implications of increasing U.S. forces in Iraq. Congressional Budget Office, Washington, DC.

Dohrenwend, B. P., J. B. Turner, N. A. Turse, B. G. Adams, K. C. Koenen, R. Marshall. 2006. The psychological risks of Vietnam for U.S. veterans: A revisit with new data and methods. *Science* **313** 979–982.

Finney, D. J. 1971. *Probit Analysis*, 3rd ed. Cambridge University Press, Cambridge, UK.

Fontana, A., R. Rosenheck. 1998. Psychological benefits and liabilities of traumatic exposure in the war zone. *J. Traumatic Stress* **11** 485–503.

Friedman, M. J. 2006. Posttraumatic stress disorder among military returnees from Afghanistan and Iraq. *Am. J. Psychiatry* **163** 586–593.

GlobalSecurity.org. 2008. U.S. ground forces end strength. Accessed October 17, http://www.globalsecurity.org/military/ops/iraq_orbat_es.

Gray, M. J., E. E. Bolton, B. T. Litz. 2004. A longitudinal analysis of PTSD symtom course: Delayed-onset PTSD in Somalia peacekeepers. *J. Consulting Clinical Psych.* **72** 909–913.

Hoge, C. W. 2005. Statement before the Committee on Veterans' Affairs, Subcommittee on Health, U.S. House of Representatives, July 27.

Hoge, C. W., J. L. Auchterlonie, C. S. Milliken. 2006. Mental health problems, use of mental health services, and attrition from military service after returning from deployment to Iraq or Afghanistan. *J. Amer. Medical Assoc.* **295** 1023–1032.

Hoge, C. W., C. A. Castro, S. C. Messer, D. McGurk, D. I. Cotting, R. L. Koffman. 2004. Combat duty in Iraq and Afghanistan, mental health problems, and barrier to care. *New Engl. J. Med.* **351** 13–22.

Hotopf, M., L. Hull, N. T. Fear, T. Browne, O. Horn, A. Iverson, M. Jones, et al. 2006. The health of UK military personnel who deployed to the 2003 Iraq War: A cohort study. *Lancet* **367**(9524) 1731–1741.

Institute of Medicine. 2007. *Treatment of PTSD: An Assessment of the Evidence*. The National Academies Press, Washington, DC.

Johnson, R. A. 1988. Stress-strength models for reliability. P. R. Krishnaiah, C. R. Rao, eds. *Handbook of Statistics*, Vol. 7. *Quality Control and Reliability*, Chap. 3. North-Holland Press, Amsterdam.

Kessler, R. C., A. Sonnega, E. Bromet, M. Hughes, C. B. Nelson. 1995. Posttraumatic stress disorder in the National Comorbidity Survey. *Arch. Gen. Psychiatry* **52** 1048–1060.

Kessler, R. C., O. Demler, R. G. Frank, M. Olfson, H. A. Pincus, E. E. Walters, P. Wang, K. B. Wells, A. M. Zaslavsky. 2005. Prevalence and treatment of mental disorders, 1990 to 2003. *New Engl. J. Med.* **352** 2515–2523.

Kulka, R. A., W. E. Schlenger, J. A. Fairbank, R. L. Hough, B. K. Jordan, C. R. Marmar, D. S. Weiss. 1988. *Contractual Report of Findings from the National Vietnam Veterans Readjustment Study*, Vol. 1. U.S. Department of Veterans Affairs, Washington, DC.

Limpert, E., W. A. Stahel, M. Abbt. 2001. Log-normal distributions across the sciences: Keys and clues. *BioScience* **51** 341–352.

Milliken, C. S., J. L. Auchterlonie, C. W. Hoge. 2007. Longitudinal assessment of mental health problems among active and reserve component soldiers returning from the Iraq War. *J. Amer. Medical Assoc.* **298** 2141–2148.

National Center for PTSD. 2007. Treatment of PTSD: Fact sheet. Accessed November 20, http://www.ncptsd.va.gov/ncmain/ncdocs/fact_ shts/fs_treatmentforptsd.html?.

Office of the Surgeon Multinational Force–Iraq and Office of the Surgeon General U.S. Army Medical Command. 2003. Operation Iraqi Freedom Mental Health Advisory Team Report, http://www.armymedicine.army.mil/reports/mhat/mhat/mhat.cfm.

Office of the Surgeon Multinational Force–Iraq and Office of the Surgeon General U.S. Army Medical Command. 2005. Operation Iraqi Freedom Mental Health Advisory Team II Report, http://www.armymedicine.army.mil/reports/mhat/mhat_ii/mhat.cfm.

Office of the Surgeon Multinational Force–Iraq and Office of the Surgeon General U.S. Army Medical Command. 2006a. Mental Health Advisory Team III Operation Iraqi Freedom 04-06 Report, http://www.armymedicine.army.mil/reports/mhat/mhat_iii/mhat-iii.cfm.

Office of the Surgeon Multinational Force–Iraq and Office of the Surgeon General U.S. Army Medical Command. 2006b. Mental Health Advisory Team IV Operation Iraqi Freedom 05-07 Final Report, http://www.armymedicine.army.mil/reports/mhat/mhat_iv/mhat-iv.cfm.

Office of the Surgeon Multinational Force–Iraq and Office of the Surgeon General U.S. Army Medical Command. 2008. Mental Health Advisory Team V Operation Iraqi Freedom 06-08 Final Report, http://www.armymedicine.army.mil/reports/mhat/mhat_v/mhat-v.cfm.

O'Hanlon, M. E., J. H. Campbell. 2007. *Iraq Index: Tracking Variables of Reconstruction & Security in Post-Saddam Iraq*. Brookings Institution, Washington, DC.

Prigerson, H. G., P. K. Maciejewski, R. A. Rosenheck. 2002. Population attributable fractions of psychiatric disorders and behavioral outcomes associated with combat exposure among U.S. men. *Am. J. Public Health* **92** 59–63.

Risen, J. 2007. Back from Iraq, contractors face combat-related stress. *New York Times* (July 5), http://www.nytimes.com/2007/07/05/us/05contractors.html.

Rosenheck, R. A., A. F. Fontana. 2007. Recent trends in VA treatment of post-traumatic stress disorder and other mental disorders. *Health Affairs* **26** 1720–1727.

Schlenger, W. E., R. A. Kulka, J. A. Fairbank, R. L. Hough, B. K. Jordan, C. R. Marmar, D. S. Weiss. 1992. The prevalence of post-traumatic stress disorder in the Vietnam generation: A multi-method, multisource assessment of psychiatric disorder. *J. Traumatic Stress* **5** 333–363.

Smith, T. C., M. A. K. Ryan, D. L. Wingard, D. J. Slymen, J. F. Sallies, D. Kritz-Silverstein. 2008. New onset and persistent symptoms of post-traumatic stress disorder self reported after deployment and combat exposures: prospective population based U.S. military cohort study. *British Medical J.* **336** 366–371.

Statistical Information Analysis Division. 2008a. Military casualty information. Defense Manpower Data Center, Office of the Secretary of Defense. Accessed October 18, http://siadapp.dmdc.osd.mil/personnel/CASUALTY/castop.htm.

Statistical Information Analysis Division. 2008b. Military personnel statistics. Defense Manpower Data Center, Office of the Secretary of Defense. Accessed October 17, http://siadapp.dmdc.osd.mil/personnel/MILITARY/Miltop.htm.

Tanielian, T., L. H. Jaycox and the Invisible Wounds Study Team. 2008. Invisible wounds: summary and recommendations for addressing psychological and cognitive injuries (MG-720/1-CCF). RAND Corp, Santa Monica, CA.

Thompson, M. 2007. Broken down. *Time* (April 16) 28–36.

Tyson, A. S., J. White. 2007. Strained Army extends tours to 15 months. *Washington Post* (April 12) A01.

Wang, P. S., M. Lane, M. Olfson, H. A. Pincus, K. B. Wells, R. C. Kessler. 2005. Twelve-month use of mental health services in the United States. *Arch. Gen. Psychiatry* **62** 629–640.

Wein, L. M. 2009. Counting the walking wounded. *New York Times* (January 26) A23.

Wells, W. F. 1955. *Airborne Contagion and Hygiene*. Harvard University Press, Cambridge, MA.

Wolfe, J., D. J. Erikson, E. J. Sharansky, D. W. King, L. A. King. 1999. Course and predictors of posttraumatic stress disorder among Gulf War veterans: A prospective analysis. *J. Consulting Clin. Psych.* **67** 520–528.

Zatzick, D. F., C. R. Marmar, H. Muller, J. E. Russo, F. P. Rivara, W. Katon et al. 1997. Posttraumatic stress disorder and functioning and quality of life outcomes in a nationally representative sample of male veterans. *Am. J. Psychiatry* **154** 1690–1695.

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APPENDIX

This appendix formulates and calibrates the mathematical model described in the main text. The deployment model is described in §1, the PTSD model parameter estimates are given in §2, and the detailed results are presented in §3.

1 The Deployment Model

In §1.1, we construct deployment schedules for major combat troops and their attached support troops in the active Army, National Guard and Marines from published data at the brigade level (for active Army and National Guard) or the more detailed battalion level (for Marines). In §1.2, we estimate deployment schedules for unattached support troops (e.g., Army Reserve) by using a mathematical model that incorporates cycles of deployment and rest. We present the resulting deployment schedules (which are through September 2008) in §1.3, and construct future deployment schedules (from October 2008 until complete withdrawal) under three possible withdrawal strategies in §1.4.

1.1 Brigade/Battalion Construction

The basic hierarchy of the Army is that several battalions make up a brigade, several brigades make up a division, several divisions make up a corps, and one or more corps form a theater [1]. There are two types of units, nondeployable and deployable [1], and we model only the deployable troops. Of these deployable troops, there are combat units and support units [1]. Combat units also include support troops, and we distinguish between support troops associated with a combat unit (attached support troops) and those that are not associated with a combat unit (unattached support troops). The unattached support troops are members of support units (e.g., engineering, medical, or logistics) that are assigned to a corps or theater and may assist many other combat units when necessary [1]. For more details on the structure of the Army and the different types of troops refer to [1]. In this section we estimate the deployment schedules of Army combat brigades and Marine combat battalions (with attached support troops) directly, and in §1.2 we compute the deployment schedules of unattached support troops indirectly.

It is possible to approximately track the deployment of major combat units in the Army and Marine corps deploying to and returning from OIF by examining multiple publicly available sources, including government reports [1, 2, 3, 4, 5], media reports [6], and various independent compilations and webpages [7, 8]. We built a dataset that approximates the deployment start and end dates for each OIF deployment of each brigade/battalion to the nearest month. Our dataset includes 42 active Army brigades, 21 National Guard brigades, 27 active Marine battalions, and 9 reserve Marine battalions. To construct the dataset, we first consulted compiled documents [7, 8] and military press releases to get a list of major combat units that may have served in OIF. We then performed a Lexis-Nexis [6] search of media reports to independently verify the deployment times of each unit. Because there is no official DOD database that contains the deployment schedules of each unit, local media accounts that provide information about deployment dates are valuable sources. These accounts report when units from local bases will either return from or deploy to OIF. Therefore, many of our references are media accounts, such as newspaper articles. The datasets along with primary references can be found in Table 1 for the active Army brigades, Table 2 for the National Guard brigades, and Table 3 for the Marine battalions. These tables includes estimates of deployment dates prior to September 2008 as well as predictions of deployment dates after September 2008, which are used in §1.4.

Each of these combat units is assumed to have a constant number of troops at full strength depending on unit type. There are between 2.5k to 5k troops for Army combat brigades (see Table 4 for a list of Army brigade sizes by type; these numbers do not include all of the attached support troops). The Army is currently undergoing a transformation to a modular system (i.e., a reorganization aimed at improving agility [1]), and there are usually 3 major combat brigades for pre-modular divisions and 4 major combat brigades for modular divisions. There are approximately 15k troops per division [1] (this includes combat and attached support troops), and thus we assume there are 5k combat and attached support troops for a pre-modular brigade and 4k combat and attached support troops for a modular brigade. Because there are usually 3 major combat battalions per brigade [1], we assume there are 1.5k combat and attached support troops in each Marine battalion.

We assume that all troops starting a deployment stay until the end of the deployment. The fraction of troops from the previous deployment serving on the current deployment depends on the annual continuation rate of their service branch, which is derived in §1.2.2 from data in [218], and the length of time between the start of the two deployments. Discontinuing servicemembers are replaced by new servicemembers to maintain constant troop strength in each unit.

We denote the total number of major combat plus attached support troops of type j at time t by $\hat{N}_j(t)$, where $j = 1, 2, 3$ refer to active Army, reserve Army (which includes Army Reserve and National Guard, although all Army Reserve soldiers are unattached in our model), and (active plus reserve) Marines, respectively, and where t is numbered consecutively with $t = 1$ corresponding to March 2003. This is calculated by a MATLAB program that cycles through each unit and computes the above steps. We define the vector $\hat{n}^{(j)} = (\hat{N}_j(1), \ldots, \hat{N}_j(M))^T$, where M is the number of months.

1.2 Rotation Model for Unattached Troops

Because the unattached support units fall under the umbrella of larger units, it is difficult to find detailed information about when specific units deploy. Therefore, we assume unattached support units are deployed in repetitive cycles. Each unit deploys for a constant number of months, rests for a constant number of months, and continues in this manner throughout OIF. We need to estimate the deployment lengths, rest periods, and continuation rates from available data. Finally, the most difficult part of this estimation process is computing when the unattached units first deploy. We formulate a quadratic program to compute these quantities with constraints given by deployment characteristics at various time points. The mathematical model describing the cyclic deployments is formulated in §1.2.1, the parameter values are estimated in §1.2.2, and the construction of the quadratic program and its constraints is described in §1.2.3.

1.2.1 The Rotation Model

Let $A_j(t)$ be the number of type j unattached support soldiers that first deploy during month t. A deployment cycle for type j soldiers consists of deploying for d_j months and returning home for δ_j months. After each deployment cycle, a fixed fraction r_j of soldiers redeploy for another cycle. Hence, of the soldiers that first deploy in month t, $r_j^l A_j(t)$ remain after l deployments. Discontinuing servicemembers are not replaced by new servicemembers, and thus the troop strength of each unit decreases with each deployment. While $t = 1$ corresponds to March 2003 we assume the unattached troops can deploy as early as October 2002 ($t = -4$) to allow for an initial buildup of troops.

Let $\tilde{N}_j(t)$ be the total number of type j unattached support troops deployed during month t. If $I_{\{x\}}$ is the indicator function for the event x (i.e., equals 1 if x is true and equals 0 otherwise), $x \bmod y$ is the modulo operator (i.e., the remainder after dividing x by y), and

$\lfloor x \rfloor$ is the largest integer less than or equal to x, then

$$\sum_{s=-4}^{t} I_{\{(t-s)\mathrm{mod}(d_j+\delta_j)<d_j\}} r_j^{\lfloor \frac{t-s}{d_j+\delta_j} \rfloor} A_j(s) = \tilde{N}_j(t). \tag{1}$$

It is convenient to write equation (1) in matrix notation, where M is the total number of months under consideration. If we define the vectors $\tilde{n}^{(j)} = (\tilde{N}_j(-4), \tilde{N}_j(-3), \ldots, \tilde{N}_j(M))^T$ and $a^{(j)} = (A_j(-4), A_j(-3), \ldots, A_j(M))^T$, and the $(M+5) \times (M+5)$ deployment matrix B as

$$B_{ts}^{(j)} = \begin{cases} r_j^{\left\lfloor \frac{t-s}{d_j+\delta_j} \right\rfloor} & \text{if} \quad s \leq t \quad \text{and} \quad (t-s)\mathrm{mod}(d_j+\delta_j) < d_j; \\ 0 & \qquad\qquad\qquad\quad \text{otherwise,} \end{cases} \tag{2}$$

then equation (1) can be expressed as

$$B^{(j)} a^{(j)} = \tilde{n}^{(j)} \quad \text{for} \quad j = 1, 2, 3. \tag{3}$$

1.2.2 Parameter Estimation for Unattached Troops

For each type j, we need to estimate the deployment length d_j, the interdeployment length δ_j, the redeployment probability r_j, and the arrival process $A_j(t)$ of unattached troops. In this section we estimate d_j, δ_j, and r_j, and in 1.2.3 we estimate $A_j(t)$.

We assume $d_2 = 12$ and $d_3 = 7$ [219]. We assume $d_1 = 12$ for $t < 39$ and $t \geq 66$, and $d_1 = 15$ for $39 \leq t < 66$. The change in policy at month 39, which corresponds to May 2006, was announced on April 11, 2007, and applied to all currently deployed active Army soldiers [220]. The change in policy at month 66, which corresponds to August 2008, was announced on April 10, 2008, and applied to all troops deploying after August 1, 2008 [221].

Since 2003, the active Army has had 1.2 units out of deployment for every unit in Iraq, and the National Guard has had 4.3 units out of deployment for every unit in Iraq [222]. Hence, $\delta_1 = 1.2(12)$, which we round up to 15 months, and $\delta_2 = 4.3(12)$, which we round up to 52 months. The Marines have $\delta_3 = 9$ months [219, 223].

To estimate the redeployment probabilities, we use the annual continuation rates in [218], which measure the fraction of troops that started the fiscal year in the military who are still in the military at the end of the fiscal year. Figure 1-1 of [218] has continuation rates for 2003-2005 for active Army, National Guard and Army Reserve, and Table 1-1 of [218] gives the fraction of soldiers in each of these 3 parts of the military. Averaging over the 3 years yields an annual continuation rate of 0.838 for active soldiers. Taking a weighted (the weights can be derived from Table 1-1 of [218]) average of the annual continuation rates for the National Guard and Army Reserve gives a 3-year average of 0.823. Using Table 2-1 and Figure 2-1 in [218], we get a weighted (over active and reserve) average annual continuation rate for Marines of 0.822. To convert annual continuation rates into redeployment probabilities r_j, we raise the annual probabilities to the power of $d_j + \delta_j$ (measured here in years). This gives $r_1 = (0.838)^{2.25} = 0.672$, $r_2 = (0.823)^{5.33} = 0.354$, and $r_3 = (0.822)^{1.33} = 0.770$.

1.2.3 Estimation of Unattached Troops First Deployment Date

The most difficult manpower parameters to estimate are the $A_j(t)$'s. We have data on deployment characteristics throughout the first several years of OIF, and our approach is to convert the existing data into linear constraints for $A_j(t)$, and then find the $A_j(t)$'s that violate these constraints as little as possible (in the least squares sense). We derive five types of constraints, as described below.

Total Deployment. Table 5 gives the number of (active plus reserve) Army soldiers and the number of Marines deployed at 3-month time intervals, from March 2003 ($t = 1$) to June 2008 ($t = 64$) [224]. From these data, we subtract the estimated attached troop numbers ($\hat{n}^{(j)}$ from §1.1) to get the number of unattached support troops, then insert these numbers into the right side of (3) (for $\tilde{n}^{(1)} + \tilde{n}^{(2)}$ and $\tilde{n}^{(3)}$) to create 42 constraints (there is no data for March 2007). In addition, Table 6 [225] gives monthly totals over all Army soldiers and Marines (i.e., $\hat{N}_1(t) + \hat{N}_2(t) + \hat{N}_3(t) + \tilde{N}_1(t) + \tilde{N}_2(t) + \tilde{N}_3(t)$) from July 2008 through September 2008 [225], which creates 3 more constraints from (3).

Fraction of Army Soldiers that are Active. Let $g(t)$ be the fraction of Army soldiers that are active during month t. Table 7 gives the values of $g(t)$ for 5 months ([224] and Tables 2 through 6 of [226]). These 5 constraints can be expressed as $\hat{N}_1(t) + \tilde{N}_1(t) = g(t)[\hat{N}_1(t) + \hat{N}_2(t) + \tilde{N}_1(t) + \tilde{N}_2(t)]$, or equivalently $[1-g(t)]\tilde{N}_1(t) - g(t)\tilde{N}_2(t) = g(t)\hat{N}_2(t) - [1-g(t)]\hat{N}_1(t)$, which can be rewritten in matrix notation as the linear constraints

$$[1 - g(t)] \sum_{s=-4}^{t} B_{t,s}^{(1)} A_1(s) - g(t) \sum_{s=-4}^{t} B_{t,s}^{(2)} A_2(s) = g(t)\hat{N}_2(t) - [1 - g(t)]\hat{N}_1(t). \tag{4}$$

Fraction on First Deployment. Table 8 states the fraction of Army soldiers on their first deployment (denoted by $m_{12}(t)$) during October 2005 (active only [227]) and September 2006 (active plus reserve [226]), and the fraction of Marines on their first deployment during September 2006 ($m_3(t)$) [226]. Let $\hat{N}_{jk}(t)$ be the total number of combat plus attached servicemembers of type j who are on their k^{th} deployment. The Army constraint for October 2005 is given by

$$\hat{N}_{11}(t) + \sum_{s=0}^{d_1-1} A_1(t-s) = m_{12}(t) \left(\hat{N}_1(t) + \tilde{N}_1(t) \right), \tag{5}$$

the Army constraint for September 2006 is

$$\sum_{j=1}^{2} \hat{N}_{j1}(t) + \sum_{j=1}^{2} \sum_{s=0}^{d_j-1} A_j(t-s) = m_{12}(t) \sum_{j=1}^{2} \left(\hat{N}_j(t) + \tilde{N}_j(t) \right), \tag{6}$$

and the Marines constraint is

$$\hat{N}_{31}(t) + \sum_{s=0}^{d_3-1} A_3(t-s) = m_3(t) \left(\hat{N}_3(t) + \tilde{N}_3(t) \right). \tag{7}$$

Median Number of Months Deployed on Current Deployment. Table 9 gives the median number of months deployed on their current deployment for (active plus reserve) Army soldiers and for Marines during September 2006 ($t = 43$), which we denote by $\bar{d}_{12}(t)$

and $\bar{d}_3(t)$, respectively [226]. Let $\tilde{N}_{j,dep.\leq x}(t)$ ($\tilde{N}_{j,dep.>x}(t)$, respectively) be the number of unattached troops of type j that have deployed for less than or equal to (greater than, respectively) x months on their current deployment at time t, and define $\hat{N}_{j,dep.\leq x}(t)$ and $\hat{N}_{j,dep.>x}(t)$ accordingly for combat plus attached troops. Because the median can be represented by two linear inequality constraints, these data lead to the four constraints:

$$\sum_{j=1}^{2}\left(\hat{N}_{j,dep.\leq\bar{d}_{12}(t)}(t)+\tilde{N}_{j,dep.\leq\bar{d}_{12}(t)}(t)\right)\geq\sum_{j=1}^{2}\left(\hat{N}_{j,dep.>\bar{d}_{12}(t)}(t)+\tilde{N}_{j,dep.>\bar{d}_{12}(t)}(t)\right), \tag{8}$$

$$\sum_{j=1}^{2}\left(\hat{N}_{j,dep.\geq\bar{d}_{12}(t)}(t)+\tilde{N}_{j,dep.\geq\bar{d}_{12}(t)}(t)\right)\geq\sum_{j=1}^{2}\left(\hat{N}_{j,dep.<\bar{d}_{12}(t)}(t)+\tilde{N}_{j,dep.<\bar{d}_{12}(t)}(t)\right), \tag{9}$$

$$\hat{N}_{3,dep.\leq\bar{d}_3(t)}(t)+\tilde{N}_{3,dep.\leq\bar{d}_3(t)}(t)\geq\hat{N}_{3,dep.>\bar{d}_3(t)}(t)+\tilde{N}_{3,dep.>\bar{d}_3(t)}(t), \tag{10}$$

$$\hat{N}_{3,dep.\geq\bar{d}_3(t)}(t)+\tilde{N}_{3,dep.\geq\bar{d}_3(t)}(t)\geq\hat{N}_{3,dep.<\bar{d}_3(t)}(t)+\tilde{N}_{3,dep.<\bar{d}_3(t)}(t). \tag{11}$$

These constraints can be rewritten as (at $t = 43$, for unattached support Army soldiers it is possible to have troops on their second deployment, and for unattached support Marines it is possible to be on the third deployment)

$$\sum_{j=1}^{2}\sum_{k=1}^{2}\left(\sum_{s=0}^{\bar{d}_{12}(t)-1} B^{(j)}_{t,t-s-(k-1)(d_j+\delta_j)} A_j(t-s-(k-1)(d_j+\delta_j))\right.$$
$$\left.-\sum_{s=\bar{d}_{12}(t)}^{d_j-1} B^{(j)}_{t,t-s-(k-1)(d_j+\delta_j)} A_j(t-s-(k-1)(d_j+\delta_j))\right)$$
$$\geq \sum_{j=1}^{2}\left(\hat{N}_{j,dep.>\bar{d}_{12}(t)}(t)-\hat{N}_{j,dep.\leq\bar{d}_{12}(t)}(t)\right), \tag{12}$$

$$\sum_{j=1}^{2}\sum_{k=1}^{2}\left(\sum_{s=\bar{d}_{12}(t)-1}^{d_j-1} B^{(j)}_{t,t-s-(k-1)(d_j+\delta_j)} A_j(t-s-(k-1)(d_j+\delta_j))\right.$$
$$\left.-\sum_{s=0}^{\bar{d}_{12}(t)-2} B^{(j)}_{t,t-s-(k-1)(d_j+\delta_j)} A_j(t-s-(k-1)(d_j+\delta_j))\right)$$
$$\geq \sum_{j=1}^{2}\left(\hat{N}_{j,dep.<\bar{d}_{12}(t)}(t)-\hat{N}_{j,dep.\geq\bar{d}_{12}(t)}(t)\right), \tag{13}$$

$$\sum_{k=1}^{3}\left(\sum_{s=0}^{\bar{d}_{3}(t)-1} B^{(3)}_{t,t-s-(k-1)(d_3+\delta_3)} A_3(t-s-(k-1)(d_3+\delta_3))\right.$$
$$\left.-\sum_{s=\bar{d}_{3}(t)}^{d_3-1} B^{(3)}_{t,t-s-(k-1)(d_3+\delta_3)} A_3(t-s-(k-1)(d_3+\delta_3))\right)$$
$$\geq \hat{N}_{3,dep.>\bar{d}_{3}(t)}(t)-\hat{N}_{3,dep.\leq\bar{d}_{3}(t)}(t), \tag{14}$$

$$\sum_{k=1}^{3}\left(\sum_{s=\bar{d}_{3}(t)-1}^{d_3-1} B^{(3)}_{t,t-s-(k-1)(d_3+\delta_3)} A_3(t-s-(k-1)(d_3+\delta_3))\right.$$
$$\left.-\sum_{s=0}^{\bar{d}_{3}(t)-2} B^{(3)}_{t,t-s-(k-1)(d_3+\delta_3)} A_3(t-s-(k-1)(d_3+\delta_3))\right)$$
$$\geq \hat{N}_{3,dep.<\bar{d}_{3}(t)}(t)-\hat{N}_{3,dep.\geq\bar{d}_{3}(t)}(t). \tag{15}$$

Mean Total Number of Months Deployed. Finally, Table 10 gives the mean number of months deployed in total (on the current plus past deployments) for currently deployed (active only) Army soldiers in October 2005 ($t = 32$), both for those on their first deployment (denote by $\mu_{1d}(t)$) and for those who have been on multiple deployments ($\mu_{2d}(t)$) [227]. For the active Army troops that we analyze, at time $t = 32$ all units have been on either one or two deployments, so we will denote those on multiple deployments as being on their second deployment. For $k = 1, 2$, let $\tilde{N}_{1,deps=k}(t)$ be the number of unattached active Army troops that are on their k^{th} deployment at time t, and define $\hat{N}_{1,deps=k}(t)$ similarly for combat plus attached active Army troops. Finally, let $\tilde{N}_{1,dep.=k,deps=l}(t)$ be the number of unattached active Army troops who are on their l^{th} deployment and in their k^{th} total (i.e., over all l deployments) month of deployment at time t, and define $\hat{N}_{j,dep.=k,deps=l}(t)$ analogously for combat plus attached troops.

For the unattached active Army troops to be in their second deployment at time $t = 32$, they must have started their first deployment between time -4 and time 5. For second-time deployers, the constraint is given by

$$\begin{aligned}
&\sum_{i=1}^{t} i\hat{N}_{1,dep.=i,deps=2}(t) + \sum_{s=-4}^{t-(d_1+\delta_1)} (d_1 + t - (s + d_1 + \delta_1) + 1)B_{ts}^{(1)} A_1(s) \\
&= \mu_{2d}(t) \left(\hat{N}_{1,deps=2}(t) + \tilde{N}_{1,deps=2}(t) \right). \qquad (16)
\end{aligned}$$

Moving variables to the left and constant terms to the right gives a linear constraint:

$$\begin{aligned}
&\sum_{s=-4}^{t-(d_1+\delta_1)} (d_1 + t - (s + d_1 + \delta_1) + 1)B_{ts}^{(1)} A_1(s) - \mu_{2d}(t)\tilde{N}_{j,deps=2}(t) \\
&= \mu_{2d}(t)\hat{N}_{1,deps=2}(t) - \sum_{i=1}^{t} i\hat{N}_{1,dep.=i,deps=2}(t). \qquad (17)
\end{aligned}$$

Similarly for first deployers, we have

$$\sum_{s=0}^{d_1-1} (s+1)A_1(t-s) - \mu_{1d}(t)\tilde{N}_{1,deps=1}(t) = \mu_{1d}(t)\hat{N}_{1,deps=1}(t) - \sum_{i=1}^{t} i\hat{N}_{1,dep.=i,deps=1}(t). \quad (18)$$

Taken together, we can write the constraints (3)-(18) as $Ea = b$ and $Ca \geq c$ for matrices E and C and vectors b and c. Because there is no nonnegative solution that satisfies this set of constraints, we resort to minimizing the weighted squared deviation of the equality constraints, subject to the inequality constraints and nonnegativity constraints. As explained in Table 11, we weight these constraints to account for the scaling inherent in each constraint as well as the relative importance of the constraints. We impose these weights by scaling the rows of E and b to create the weighted matrix $\tilde{E}$ and vector $\tilde{b}$. That is, we solve the quadratic program

$$\min_a \; a^T \tilde{E}^T \tilde{E} a - 2\tilde{b}^T \tilde{E} a + \tilde{b}^T \tilde{b}, \tag{19}$$

$$\text{subject to} \;\; Ca \geq c, \tag{20}$$

$$a \geq 0. \tag{21}$$

Substituting the solution $A_j(t)$ to (19)-(21) into equation (1) yields $\tilde{N}_j(t)$, which is the number of unattached type j troops during month t.

1.3 Results

The solution $A_j(t)$ to (19)-(21) appears in Table 12. Fig. 1 displays the monthly troop levels for Army and Marines, broken down by combat plus attached troops vs. unattached troops, and Fig. 2 gives the monthly troop levels for active Army ($j = 1$), reserve Army ($j = 2$) and Marines ($j = 3$). Fig. 3 shows that our computed troop levels track the official DOD and project post-surge troop numbers reasonably well. The average monthly relative deviation (i.e., $\frac{|\text{reported}-\text{computed}|}{\text{reported}} \times 100\%$) in Figs. 3a, 3b and 3c are 5.9%, 9.8% and 0.0%, respectively, with larger deviations occurring during the first 7 months of OIF. Tables 13-16 compare the computed vs. reported values of the fraction of Army soldiers that are active, the median number of months deployed during the current deployment, the mean number of months deployed over all OIF deployments, and the fraction of troops on first deployment. The average relative deviation over the 12 comparisons in Tables 13-16 is 6.1%, and the largest relative deviation is 15.2%, which is for the mean time deployed for first deployers during October 2005.

1.4 Future Deployments

To determine the incidence of symptomatic PTSD over the coming years, we need to estimate troop deployments from October 2008 through the end of OIF.

For each of the three withdrawal strategies defined in the main text, we define corresponding target trajectories, which are troop levels that remain constant until the beginning of withdrawal and then decrease linearly to 0 over the 13-month withdrawal process. We compute separate target trajectories for the Army (both active and reserves) and for the Marines by assuming that the fraction of servicemembers in the Army (Marines, respectively) from October 2008 until complete withdrawal is the same as the observed fraction between October 2007 and September 2008, which is 0.819 (0.181, respectively). In the remainder of this subsection, we describe how to construct withdrawal schedules to achieve these target trajectories.

To fit the withdrawal trajectories, we need to determine when brigades/battalions should deploy in the future. First, for any unit that is deployed during September 2008 and that does not have an estimated return date, we assume it deploys for the standard deployment length for its troop type (15 months for active Army before August 2008 and 12 months after August 2008, 12 months for reserve Army, and 7 months for Marines). Any unit deployed after September 2008 also deploys for the standard deployment length. Next, there are three types of units that could be deployed: combat and attached units, unattached units that deployed before October 2008 (these were estimated in section §1.2), and unattached units that first deploy after October 2008. We will refer to unattached units that deployed before October 2008 as unattached 1 units and the unattached units that first deploy after

October 2008 as unattached 2 units. Unattached units will refer to both unattached 1 and unattached 2 units.

Combat and Attached Units. Because we are considering a withdrawal strategy, we allow combat and attached units to deploy only when replacing other combat and attached units that are finishing a tour of duty. We only replace an outgoing unit with a new unit if adding the unit will not make the total troop level greater than the target withdrawal trajectory at any point during the deployment.

To determine what order to deploy combat and attached units in the future, we form a queue based upon possible future deployment dates. Tables 1, 2, and 3 have estimates of past deployment dates as well as estimates of future deployment dates based upon published information. The queue is ordered with the units estimated to deploy closest to October 2008 at the front and those estimated to deploy farthest from October 2008 at the end. If units have the same estimated future deployment date, then the unit that is the most rested is the closest to the front of the queue. If units have the same estimated future deployment date and are equally rested, then they are ordered according to Tables 1, 2, and 3. Thus, in the Army queue, the 56th Brigade of the 36th Infantry Division of the National Guard is at the front of the queue and the 155th Brigade of the National Guard is at the end of the queue.

With this queue in place, we use the following procedure. In every month we see if any units finish a deployment. If they do then we deploy replacements according to the order of the queue, if adding them does not put the troop level greater than the target trajectory level. Because of modularity, it is possible that the unit at the front of the queue cannot be deployed but another in the queue can. In this case, we deploy the first unit in the queue that can be deployed.

It is possible that units in the queue never deploy, deploy at dates different than the estimates in Tables 1, 2, and 3, or that all units in the queue deploy. If all the units in the queue have deployed then we consider candidates for future deployment in a manner related to using the most rested unit. Tables 1, 2, and 3 only list deployments for OIF. Units also deploy to Afghanistan or other expeditions, including humanitarian relief efforts. Thus, after the last recorded deployment of each combat and attached unit, we assume that the unit continues on a standard deployment cycle (e.g., for Marines there are 9 months of rest, then 7 months of deployment, then 9 months of rest, etc.). Based upon actual as well as these theoretical deployments, in each month we can define units that are eligible to deploy that month, and if a unit is needed to replace an outgoing unit we deploy the eligible unit that is most rested. A unit is eligible if the number of months since it last completed a deployment (real or theoretical) is within some range. The minimum of this range is the rest period for the standard deployment cycle (15 months for active Army, 52 months for reserve Army, and 9 months for Marines), and the maximum of the range is 24 months for active Army and 15 months for Marines, but we set no upper bound for reserve Army brigades. If it has been longer than this maximum value, we assume the unit would have gone on another theoretical deployment and thus would not be available to deploy. The eligible unit that is the most rested (from a real or theoretical deployment) is the next unit to deploy. If there are several most-rested eligible units, then they are ordered according to Tables 1, 2, and 3.

Unattached 1 Units. For each month from October 2008 until the end of the withdrawal period, we first determine if any combat and attached units have finished a deployment, and if so we replace them according to the method described above. Next we look at all the unattached 1 units and determine which are scheduled to deploy that month, assuming they stay on the standard deployment cycle. If an unattached 1 unit is scheduled to redeploy and deploying that unit does not put troop levels above the withdrawal trajectory, then we deploy this unattached 1 unit. Otherwise that unit is not deployed and we assume that this unattached 1 unit is finished deploying and it will not redeploy at any future time. If there are several unattached 1 units that are scheduled for deployment during the same month, then we attempt to deploy them in a largest-unit-first manner.

Unattached 2 Units. After determining the deployment schedules for the combat and attached units and unattached 1 units for each month from October 2008 until the end of the withdrawal period, we have future troop levels that are by construction less than the target withdrawal trajectory. We add unattached 2 units to fit the withdrawal trajectory as well as possible. We determine their deployment schedule in an analogous fashion as described in section §1.2, i.e., embedding a system of equations, $Ba = n$, into a quadratic program. The vector n is the target withdrawal trajectory minus the combat plus attached and unattached 1 troop levels, a gives the number of unattached 2 servicemembers initially deploying in a given month, and the elements of the matrix B are the fraction of servicemembers who initially deploy in a given month who are deployed during another month. The only difference between this method and the one in §1.2 is that unattached 2 units can only deploy if they can finish their entire deployment before the end of the withdrawal period, and thus there are no unattached 2 units that only deploy for a few months at the end of the withdrawal period.

Figure 4 displays the target withdrawal trajectories and the monthly troop levels for Army and Marines, broken down by combat plus attached troops vs. unattached 1 troops vs. unattached 2 troops, and Figure 5 gives the target withdrawal trajectories and the monthly troop levels for active Army ($j = 1$), reserve Army ($j = 2$), and Marines ($j = 3$).

2 PTSD Parameter Estimation

In this section, we estimate the PTSD model parameters, which are the mean initial stress α^{-1}, the mean monthly stress exposures $\lambda_j(t)$, the batch size b, the recuperation parameter

θ, the mean stress threshold γ^{-1}, and the time lag parameters μ_i and s_i, $i = 1, 2$. We begin by estimating $\lambda_j(t)$ from monthly casualty data. We then estimate μ_i and s_i from some limited longitudinal data, and finally jointly estimate α, b, θ, and γ from PTSD data.

Average Monthly Stress. For lack of disaggregated casualty data on active vs. reserve Army soldiers, we assume that $\lambda_1(t) = \lambda_2(t)$ and estimate $\lambda_1(t)$ and $\lambda_3(t)$ using monthly data on fatalities and wounded [228]. We computed the correlation between the number of fatalities and the number wounded in each month to be 0.75 for Army soldiers ($j = 1$ and 2 combined), 0.85 for Marines ($j = 3$), and 0.79 for the aggregated Army and Marines. The total number (from March 2003 to September 2008) wounded is 6.99 times as many as the total number of fatalities for Army and 8.51 for Marines. It is difficult to estimate the relative amount of stress caused by exposure to a fatality vs. exposure to a wounded servicemember, and for concreteness, we define the mean amount of stress to be (6.99×fatalities + wounded) divided by the number of troops deployed for Army and (8.51×fatalities + wounded) divided by the number of troops deployed for Marines, for each month t. That is, we divide the monthly (6.99×fatalities + wounded) quantity by $\hat{N}_1(t) + \tilde{N}_1(t) + \hat{N}_2(t) + \tilde{N}_2(t)$ for Army and we divide the monthly (8.51×fatalities + wounded) quantity by $\hat{N}_3(t)+\tilde{N}_3(t)$ for Marines to obtain our values for $\lambda_1(t)$ and $\lambda_3(t)$, where $\hat{N}_1(t)+\tilde{N}_1(t)+\hat{N}_2(t)+\tilde{N}_2(t)$ and $\hat{N}_3(t)+\tilde{N}_3(t)$ are linearly interpolated from the quarterly data in Table 5 from March 2003 to June 2008, and by assuming that the post-June 2008 monthly total (Army plus Marines) deployments in Table 6 are in the same proportion (0.822 Army and 0.178 Marines) as they were over the September 2007-June 2008 time period. The resulting values for $\lambda_1(t)$ and $\lambda_3(t)$ appear in Table 17 and Figure 6.

Time Lag Parameters. We estimate the four time lag parameters μ_i and s_i, $i = 1, 2$, from sparse longitudinal data from two separate studies. Our approach is to solve five equations for six unknowns, which are the four time lag parameters plus the unknown number of servicemembers who had PTSD in each of the studies. The two studies, one from OIF and one from the Gulf War, screen servicemembers for PTSD at two time points, at the end of a deployment and then some months later. In each study, we assume that the PTSD rates are the same for active and reserve servicemembers (indeed, the rates are similar at the first time point in each study), but that the time lag depends upon whether the servicemember is physically with the military or has returned to civilian life.

The data for the OIF study appears in Table 3 of [229] and generates 3 equations. Let n_1 be the unknown number of 88,235 (56,350 active and 31,885 reserve or National Guard) surveyed (at the end of a deployment between September 2004 and October 2006) servicemembers who eventually experienced PTSD due to combat exposure during OIF (i.e., some may not have experienced symptoms until after the second time point). We first need an estimate of the time in the deployment when the PTSD-generating event occurs. Using the deployment history vectors $C_{kj}(t)$ (which are defined in the Model Overview section of the main text) for both unattached support troops and combat and attached support troops, we determine the average total OIF deployment for an Army soldier that finished a deployment between September 2004 and October 2006. We analyze soldiers who have completed only one deployment separately from those who have completed two deployments. For soldiers completing their first deployment, the median length of the deployment is 12 months. Let V_1 be a [0,12] uniform random variable representing the time (during the 12 months of combat) of the PTSD-generating event. Even though we have a measure of the frequency of traumatic events over time, $\lambda_1(t)$, for simplicity we assume that the PTSD-generating event occurs uniformly during the period of deployment. For soldiers who have completed their second deployment we have that the median length of the first deployment is 12 months, the median length between deployments is 15 months, and the median length of the second deployment is 12 months. Let V_2 be a uniform random variable over the broken interval $[0, 12] \cup [27, 39]$, which represents the time (during the two deployments) of the PTSD-generating event. A fraction 0.79 of the soldiers have completed one deployment and we define Z, the time between the PTSD-generating event and the time of the first screening, to be a mixture of $12 - V_1$ with probability 0.79 and $39 - V_2$ with probability 0.21. Finally, let U_1 be a [3,9] month uniform random variable representing the time interval between the two screening time points in this study. Combining the data for active and reserve personnel for the first time points gives

$$P(T_1 < Z) = \frac{3474 + 2119}{n_1}. \tag{22}$$

Considering only the active soldiers at the second time point yields

$$P(T_1 < Z + U_1) = \frac{3474 + 3697}{\left(\frac{56{,}350}{88{,}235}\right) n_1}. \tag{23}$$

Considering the reserve soldiers in between the first and second time points leads to

$$P(T_2 < U_1) = \frac{3457}{\left(\frac{31{,}885}{88{,}235}\right) n_1 - 2119}. \tag{24}$$

The final two equations come from a study of Gulf War veterans [230], in which 2949 servicemembers were assessed for PTSD at two time points: 5 days post-return after an average of 4 months in combat, and 18-24 months later. The mix was 72% (i.e., 2123 servicemembers) reserve and 28% (i.e., 826 servicemembers) active. At the first time point, 3% (i.e., 88 servicemembers) screened positive for PTSD. Because the active vs. reserve did not play a significant role in the PTSD rate at the first time point, we assume that 72% of the 88, or 63, were reserve servicemembers and 25 were active. At the second time point, 8% (i.e., 236 servicemembers) screened positive for PTSD. Furthermore, 79% of those who screened positive for PTSD at the second time point (i.e., 186 servicemembers) did not screen positive at the first time point, and hence 274 (i.e., 88+186) servicemembers had screened positive for PTSD by the second time point. Let n_2 be the unknown number of the 2949 servicemembers that eventually experienced PTSD due to the combat (i.e., some may not have experienced symptoms until after the second time point), let U_2 be a [0,4] uniform random variable that represents the time (during the 4 months of combat) of the PTSD-generating event, and let U_3 be a [18,24] uniform random variable representing the time interval between the two measurement points in this study. The equation emanating from the first time point is

$$P(T_1 < 4 - U_2) = \frac{88}{n_2}. \tag{25}$$

We also know that the odds ratio for screening positive for PTSD at the second time point was 2.0 (unconditioned on what happened at the first time point). If we let p_r and p_a be the PTSD rates for reserve and active servicemembers at the second time point, then it follows that p_r and p_a satisfy the two equations

$$2123p_r + 826p_a = 236, \tag{26}$$

$$\frac{p_r(1-p_a)}{(1-p_r)p_a} = 2. \tag{27}$$

Solving (26)-(27) yields $p_r = 0.092$ and $p_a = 0.048$ and so 196 reserve servicemembers and 40 active servicemembers screened positive for PTSD at the second time point. Of the servicemembers who screened positive for PTSD at the first time point, 62% also screened positive at the second time point, and thus 157 reserve servicemembers experienced symptoms between the two time points. Therefore our final equation arising from focusing on the reserve servicemembers in between the two time points is

$$P(T_2 < U_3) = \frac{157}{0.72n_2 - 63}. \tag{28}$$

Because we have five equations and six variables, we set $n_2 = 306$, which gives the right side of equation (28) to be unity, and thus we conservatively assume that all reserve servicemembers show symptoms within 2 years (to test the impact of this assumption, we analyze a scenario with stochastically larger time lag distributions in §3.3). To solve for μ_1, s_1, μ_2, s_2, and n_1, we minimize the sum of the squared deviations between the left side and right side of equations (22)-(25) and (28). Finally we have bounds on our least squares problem. We require $s_1, s_2, n_1 > 0$, the right sides of equations (22)-(25) and (28) to be between 0 and 1, and because the left side of (25) is less than the left side of (22), which is less than the left side of (23), and the left side of (24) is less than the left side of (28), we require that the right sides of these equations follow the same relationships.

The solution to this problem is $\mu_1 = 2.47$, $s_1 = 2.73$, $\mu_2 = 1.40$, $s_2 = 0.57$, $n_1 = 19448$, and $n_2 = 306$. The median military time lag is $e^{\mu_1} = 11.78$ months, the mean time lag is $e^{\mu_1 + \frac{s_1^2}{2}} = 40.87$ years, and the dispersion factor is $e^{s_1} = 15.35$ (implying that, e.g., 95% of the time lags are between $\frac{11.78}{15.35^2} = 0.05$ months and $\frac{15.35^2(11.78)}{12} = 231.16$ years). The median civilian time lag is $e^{\mu_2} = 4.05$ months, the mean time lag is $e^{\mu_2 + \frac{s_2^2}{2}} = 4.77$ months, and the dispersion factor is $e^{s_2} = 1.77$ (and hence 95% of the time lags are between 1.29 and 12.72 months).

PTSD Parameters. Finally, we jointly estimate α, b, θ, and γ by using a least squares approach, where the objective function is the sum of 18 squares. Seventeen of the 18 squares correspond to 17 reported (from Mental Health Advisory Team (MHAT) studies) vs. predicted (by our model) PTSD rates (Table 18). Table 18 presents the same information as Table 1 in the main text, but Table 18 contains several additional scenarios. The seventeen MHAT values in Table 18 are of the form $P_j(t)$, which is the probability a type j servicemember has symptomatic PTSD during month t. These months all correspond to one of the first four MHAT studies. MHAT-I occurred in September 2003 ($t = 7$) [227], MHAT-II occurred in October 2004 ($t = 20$) [227], MHAT-III occurred in October 2005 ($t = 32$) [227], and MHAT-IV occurred in September 2006 ($t = 43$). For example, "$P_1(32)$, > 1st deployment" is the probability that active Army servicemembers who were on at least their second deployment during MHAT-III had symptomatic PTSD in month $t = 32$ (MHAT-III). Another example, "$P_{1+2}(43)$, $\leq$6 mo on cur. dep." is the probability that active and reserve Army servicemembers who had been on their current deployment for less than 6 months during MHAT-IV had symptomatic PTSD in month $t = 43$ (MHAT-IV). The MHAT rates in Table 18 all appear in either the MHAT-III [227] or the MHAT-IV study [226]. The first three values in Table 18 appear in the figure on page 20 of the MHAT-III report [227] . The category on the left side of that figure (Acute Stress Symptoms) is what we use for the MHAT PTSD rates in Table 18. This rate is reported for OIF-I (MHAT-I, $t = 7$), OIF-II (MHAT-II, $t = 20$), and OIF-04-06 (MHAT-III, $t = 32$). The remaining MHAT values in Table 18 are reported in a similar manner on page 21 of [227] and on pages 20 through 24 of [226]. The category of interest is always "Acute Stress" and OIF-05-07 refers to MHAT-IV and month $t = 43$.

The final value in Table 18 (corresponding to the 18^{th} term in our objective function) is "Fraction of troops exposed to no combat." This quantity does not come from an MHAT study and, unlike the first 17 values in Table 18, it is not a PTSD rate. This value pertains to the fact that the fraction of Army soldiers finishing a deployment between September 2004 and October 2006 who were exposed to a potentially traumatic combat experience was 0.68 [229]. As noted earlier, 0.79 of servicemembers that finished a deployment during September 2004 and October 2006 completed their first deployment during that time and were deployed for a median of 12 months. From Table 17, we find that the average value of $\lambda_1(t)$ from October 2003 (which is 12 months prior to the start of the survey) to October 2006 was 0.0054. Because the batch size is b, the number of Poisson events during the 12 months of deployment was approximately $\frac{0.065}{b}$. The remaining 0.21 of servicemembers completed their second deployment during September 2004 and October 2006, and their first deployment lasted a median of 12 months, their second deployment a median of 12 months, and the break between deployments 15 months. Since the total cycle of the two deployments took 39 months to complete, the servicemembers could not have finished their deployment earlier than May 2006. From Table 17, we find that the average value of $\lambda_1(t)$ from the first deployment, which could occur between March 2003 and July 2004 (which is 27 months prior to the end of the survey), and the second deployment, which could occur between June 2005 (12 months prior to May 2006) and October 2006 was 0.0047. The number of Poisson events during the 24 noncontiguous months of deployment was approximately $\frac{0.113}{b}$. Thus the average number of Poisson events during deployments from both servicemembers completing one and two deployment is $\frac{0.075}{b}$. Hence, the 18$^{\text{th}}$ term in our least squares objective is $(e^{-0.075/b} - 0.32)^2$. Finally, 5% of servicemembers screen positive for PTSD pre-deployment [231, 232]. This value is similar to the PTSD rate in the general population [232], and therefore we take 5% to be the baseline PTSD symptomatic rate pre-deployment. We choose the parameter values to ensure this initial rate, and thus this is an equality constraint for our least squares problem. We assume everyone who develops PTSD before their initial deployment (i.e., their initial stress is greater than their threshold) is also symptomatic pre-deployment. For the Poisson model this equality constraint is equivalent to $\gamma = \frac{\alpha}{19}$.

We denote $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$, for $i = 1, \ldots, 17$, to be the model's estimate of the i^{th} MHAT probability in Table 18, given $\alpha = \tilde{\alpha}$, $b = \tilde{b}$, $\theta = \tilde{\theta}$, and $\gamma = \tilde{\gamma}$. For example, $f_1(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$ is the model's estimate (for specific parameter values) of the PTSD rate of active and reserve Army servicemembers in month $t = 7$ ($P_{1+2}(7)$). To determine the optimal values of α, b, θ, and γ, we need to be able to evaluate the function $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$. Once we are able to do this, we can compute for the optimal parameter values by solving the following least squares problem:

$$\left(\alpha, b, \theta, \gamma\right) = \arg\min_{\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma}} \sum_{i=1}^{17} \Big(f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma}) - \text{MHAT}_i \Big)^2 + (e^{-0.075/\tilde{b}} - 0.32)^2, \tag{29}$$

$$\text{subject to } \tilde{\gamma} = \frac{\tilde{\alpha}}{19}, \tag{30}$$

where MHAT_i is the i^{th} reported MHAT PTSD rate in Table 18 and the constraint and the last term in the objective function are described in the previous paragraph.

There does not appear to be any way to evaluate $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$ either analytically or numerically. The calculation involves manipulating and comparing many different types of random variables for servicemembers with different deployment schedules. Consequently, we use Monte Carlo methods to estimate $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$ as follows. From the deployment submodel in §1, we have the deployment history for each servicemember, given by $C_{kj}(t)$. For each servicemember we generate an initial stress level (an exponential random variable with mean $\tilde{\alpha}^{-1}$) and a stress threshold $\bar{D}_{kj}$ (an exponential random variable with mean $\tilde{\gamma}^{-1}$). We then accumulate and decrease the servicemember's monthly stress level, $D_{kj}(t)$, according to his deployment history and equations (1) and (2) in the main text. This is a random process because we must generate the monthly stress, $E_{kj}(t)$, which is a compound Poisson random variable with mean $\lambda_j(t)$ and batch size b. After computing $D_{kj}(t)$ for each servicemember throughout OIF, we can determine if and when the servicemember develops PTSD ($\bar{t}_{kj}$ in equation (3) in the main text) and his maximum stress level up until $t = 43$ (this quantity is used to compute $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$ for the MHAT-IV rates in Table 18 with low, medium, or high exposure to trauma). Finally, we generate a lognormal time lag (using the parameters estimated in the previous subsection) and compute when the servicemember reports symptoms. We only need the military time lag because the MHAT surveys occur during a deployment and no reserve servicemembers in our model deploy multiple times before MHAT-IV ($t = 43$). We then determine $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$ by computing the fraction of servicemembers corresponding to the population in the i^{th} row of Table 18 who report symptoms. For example, to compute $f_{13}(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$ (an estimate of $P_3(43)$, high exposure to trauma) we run the procedure described in this paragraph and then rank the Marines who were deployed in month $t = 43$ (i.e., those Marines such that $C_{k3}(43) = 1$) according to their maximum stress level up until $t = 43$ (i.e., $\max_{t \le 43} D_{k3}(t)$). The Marines ranked in the upper third (i.e., the one third of the deployed Marines with the highest maximum stress level) are the population used to compute $f_{13}(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$. We determine what fraction of this population has symptomatic PTSD by $t = 43$, and this is $f_{13}(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$. We have only described one iteration of this process, but in practice, to compute $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$ we run this procedure several times to get a more precise point estimate of $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$. Once we have this algorithm to evaluate $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$ for all $i = 1, \ldots, 17$, we can solve for α, b, θ, and γ in (29)–(30). Substituting these optimal parameters into $f_i(\alpha, b, \theta, \gamma)$ yields the model estimates for the MHAT probabilities in Table 18. Because the objective function in (29) is complicated and has uncertainty, it is difficult to compute the global minimum. However, through various search and optimization methods we are confident that our calculated minimum is a reasonable estimate of the global minimum.

For the Poisson model, the minimum sum of squares of the 18 terms is 0.016, which gives a root mean square error of $\sqrt{\frac{0.016}{18}} = 0.030$. The average relative deviation for these 18 PTSD rates is 15.1%. The optimal solution is $\alpha = 146.7$, $\gamma = 7.72$, $b = 0.0626$, and $\theta = 0$. Because the amount of rest between deployments is usually at least 10 months (it is shorter for some Marine battalions), it is difficult to determine the optimal θ when there is near full recuperation between deployments (e.g., $0.3^{10} = 6 \times 10^{-6}$). Therefore to be conservative we set $\theta = 0$, although we perform a sensitivity analysis where we increase the value of θ to 1 in §3.3. The mean stress pre-deployment, α^{-1}, is comparable to the average monthly stress from combat (0.0051 for Army, 0.0090 for Marines), and the mean threshold value, γ^{-1}, is comparable though larger than average stress accumulated during a yearlong deployment.

Table 18 compares the actual MHAT PTSD rates with the PTSD rates estimated using these parameter values ($f_i(\alpha, b, \theta, \gamma)$). We discuss the model fit to the MHAT rates in §2.3 of the main text. We also computed the optimal solution using a weighted least squares approach with weights $(P_i(1 - P_i))^{-1}$, where P_i are the actual MHAT PTSD rates in Table 18. The solution using the weighted least squares objective function was very similar to the one given above using the standard least squares approach.

3 Results

The estimates for the future troop deployments from §1, the PTSD model from the main text, and the parameter estimates from §2 allow us to simulate our system and compute the cumulative number of symptomatic servicemembers over time. Because the difference between the civilian and military time lags is significant we need to determine when active Army soldiers and Marines separate from the military and return to civilian life. We assume that if a servicemember stops deploying before its unit's final deployment then that servicemember enters civilian life as soon as the servicemember completes his last deployment. For the cohorts of servicemembers who complete their brigrade's final deployment, we assume that every year following that final deployment a fraction of those servicemembers enter civilian life according to the retention rate, which is given in §1.2.2. The base-case results appear in §3.1, several model modifications are considered in §3.2, and sensitivity analyses are performed in §3.3.

3.1 Base-case Results

Figure 1 in the main text shows the cumulative number of troops who are symptomatic for the three withdrawal scenarios described in the main text. Figures 7 and 8 also illustrate the difference between the civilian and military time lags. Because the reserve Army servicemembers switch to the civilian time lag as soon as they return from a deployment, the gap between the number who devlop PTSD and the number of symptomatic cases closes quickly for the reserve Army at the end of the withdrawal. In contrast, the active Army and Marine servicemembers switch only once they separate from the military, and so the gap between the number who develop PTSD and the number of symptomatics closes more slowly for active Army and Marines. The average time lag for active servicemembers (active Army and Marines) is 24.6 months and the average time lag for reserve servicemembers is 7.5 months. The 90$^{\text{th}}$ percentile for active service members is 68 months and the 90$^{\text{th}}$ percentile for reserve servicemembers is 15 months, implying that it could be years before all of the active servicemembers with PTSD exhibit symptoms.

Table 19 gives the probability mass function for the number of stressful events (i.e., the number of Poisson events in our model) for servicemembers in withdrawal scenario 2. For these same servicemembers, Figure 9 displays a histogram of the $\frac{\text{maximum cumulative stress}}{\text{strength threshold}}$ ratio. These data reveal the heterogeneity of stressful experiences and PTSD severity among servicemembers.

3.2 Model Modifications

In this section we modify our model under three different scenarios. In the first scenario we replace the Poisson dose-response model with the probit model, in the second we assume each servicemember only deploys once, and finally we analyze the situation where the servicemembers leave the military and return to civilian status based on how resilient they are to the stress they have faced. The first and third scenarios require the re-estimation of the PTSD parameters. Table 18 gives the estimated parameter values and the predicted PTSD probabilities for these two cases. Figure 10 shows the cumulative number of symptomatics over time for all three scenarios.

Probit Model. For the probit model, in which $\bar{D}_{jk} = \mathrm{ID}_{50} \exp\left(\frac{\Phi^{-1}(u_{jk})}{\beta}\right)$ (see the §2.2 of the main text), we need to solve for α, ID_{50}, β, b, and θ by minimizing the sum of squared deviations between the actual PTSD rates in Table 18 and the predicted rates from the model. To ensure that 5% of servicemembers have PTSD pre-deployment, for a given ID_{50} and α we compute the β that yields this rate. The resulting root mean square error is 0.028; as expected, this is smaller than in the base-case, which has one less parameter to optimize over. Our analysis yields what appears to be a one-dimensional subspace of parameters that achieve close to the minimum sum of squares, thereby making it difficult to pinpoint the exact optimal parameter values. For all near-optimal solutions, $\theta = 1$ and $b \approx 0.065$; i.e., in contrast to the Poisson base-case model, there is no recuperation in the probit case. By increasing α and decreasing ID_{50} at the proper relative rates (β is just a function of α and ID_{50} to ensure the initial PTSD rate is 0.05), the sum of squares can be maintained near its minimum value. We have found solutions that have α ranging from 500 to 10^7, β ranging from 0.1 to 0.4, and ID_{50} ranging from 0.03 to 0.10. We compared several solutions with widely varying α's and they all yield a similar fit to the MHAT probabilities in Table 18 and predict a similar number of symptomatics. The parameter values $\alpha = 8.495 \times 10^6$, $\mathrm{ID}_{50} = 0.0447$, $\beta = 0.124$, $b = 0.0682$, and $\theta = 1$ achieved the minimum sum of squares, and so we use these parameters to compute the values in column A of Table 18 and Figure 10(a). The probit model provides a better fit to, although still underestimates, the PTSD rate for Army soldiers receiving the lowest level of stress ($P_{1+2}(43)$, low exposure to trauma), but it underestimates the PTSD rate of Marines receiving the highest level of stress ($P_3(43)$, high exposure to trauma).

In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.39 for active Army, 0.31 for reserve Army, and 0.37 for Marines. Overall there are $\approx 5\%$ fewer servicemembers who develop PTSD in the probit model compared to the Poisson model.

No Multiple Deployments. To isolate the effects of multiple deployments, we look at the extreme hypothetical case in which there are no multiple deployments. That is, we use the base-case PTSD parameters and the base-case brigade/battalion rotation schedule, except that each time a unit is deployed each servicemember is new; presumably, this state of affairs could only be achieved with an involuntary draft. Figure 10(b) gives the predicted number of symptomatic servicemembers for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.33 for active Army, 0.30 for reserve Army, and 0.28 Marines.

The fraction of troops who develop symptomatic PTSD does drop substantially, especially for Marines because of their frequent deployments and short deployment length. However $\approx 30\%$ more servicemembers develop symptomatic PTSD than in the base-case. To maintain troop levels while simultaneously disallowing multiple deployments would certainly require a draft, and it is possible that drafted servicemembers would be less prepared to handle the stress than volunteer servicemembers and would be more susceptible to PTSD; this aspect is not taken into account by our model. Disallowing multiple deployments in the probit model leads to a somewhat lower PTSD rate for individual servicemembers (0.32 for active Army, 0.30 for reserve Army, and 0.26 for Marines), and an overall increase in symptomatic PTSD cases of $\approx 30\%$ over the probit model with multiple deployments allowed.

Stress-based Attrition. It is possible that servicemembers choose when to leave the military and return to civilian status based partially upon the amount of stress they have been exposed to and their ability to handle that stress. Therefore, in this analysis we use the same cohort and deployment schedule; however, after each deployment, the servicemembers who return to civilian life are the servicemembers with the highest $\frac{\text{stress}}{\text{threshold}}$ value at the end of that deployment. Thus the servicemembers who serve on multiple tours of duty should be better equipped to handle the stress than in the base-case model. While servicemembers who deploy several times are more resilient to stress, there are more vulnerable servicemembers exposed to stress in this scenario because the servicemembers who develop PTSD leave the military, and hence must be replaced sooner than in the base-case model. Figure 10(c) gives the predicted number of symptomatic servicemembers for this situation using the base-case parameter values. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.44 for active Army, 0.32 for reserve Army, and 0.44 Marines. The total number of servicemembers who develop symptomatic PTSD is $\approx 5\%$ more than the base-case. Using the probit dose-response for this scenario yields a similar increase over the base-case probit model.

Column B of Table 18 shows that the estimated MHAT probabilities under this modeling assumption are much worse than the other scenarios; the resulting root mean square error is 0.078. Because of the time lag before the onset of symptoms, many of the servicemembers in our model who screen for PTSD during the MHAT surveys developed PTSD on a prior deployment. However, for this modeling scenario most servicemembers who develop PTSD return to civilian life rather than another deployment, and therefore almost all of the estimated MHAT probabilities are smaller in this scenario than in the base-case. The difference is most evident for $P_1(32)$, >1st deployment and $P_{1+2}(43)$, >1st deployment because in the base-case over 75% of these troops who screened positive for PTSD developed PTSD during an earlier deployment. Because the base-case parameters yield such a poor fit, we recompute the optimal parameters for this scenario. The optimal parameter values are $\alpha = 306.6$, $\gamma = 16.14$, $b = 0.0588$, and $\theta = 1$ and the estimated MHAT probabilities are in column C of Table 18. The root mean square error is 0.049, which is much better than using the base-case parameters, but it is still worse than most of the other scenarios in Table 18. To achieve the MHAT PTSD rates in Table 18, more servicemembers need to develop PTSD to compensate for both the larger number of servicemembers with PTSD who leave the military before the later MHAT surveys occur and the increased resiliency of those servicemembers who stay in the military and take part in the later MHAT surveys. This decreases the mean threshold level γ^{-1}, which is why γ and α increase (α is a function of γ to ensure the initial PTSD rate is 0.05), and causes there to be no recuperation. Figure 10(d) gives the predicted number of symptomatic servicemembers for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.69 for active Army, 0.47 for reserve Army, and 0.65 Marines. The total number of servicemembers who develop symptomatic PTSD is $\approx 60\%$ more than the base-case. Using the probit dose-response for this scenario yields a similar increase over the base-case probit model. The decision to stay in the military or return to civilian life is based on many factors related to family, health, finances, morale, and camraderie. The poor fit of this model to the MHAT probabilities (column C of Table 18) suggests that a servicemember's ability to handle the stress he is exposed to may be a second-order consideration in this decision.

3.3 Sensitivity Analyses

This subsection reports on the results of three sensitivity analyses, each of which test how the results change when we adjust the parameter values. We first set $\theta = 1$ and assume there is no recuperation, we next analyze different time lag parameters, and finally we adjust the future monthly stress level. The first two analyses require the re-estimation of the PTSD parameters. Table 18 gives the estimated parameter values and the predicted PTSD probabilities for these two cases, and Figure 11 shows the cumulative number of symptomatics over time. The results for the final analysis are shown in Figure 12.

No Recuperation. The base-case model has $\theta = 0$ and thus predicts no cumulative effects from multiple deployments. We investigate how our model changes if we set $\theta = 1$, thereby disallowing recuperation between deployments. First, we keep the same parameter values as in the base-case ($\alpha = 146.7$, $\gamma = 7.72$, and $b = 0.0626$), but set $\theta = 1$. The root mean square error is 0.031. Column D of Table 18 and Figure 11(a) give the estimated MHAT probabilities and the predicted number of symptomatic servicemembers, respectively, for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.45 for active Army, 0.32 for reserve Army, and 0.45 for Marines. Increasing θ from 0 to 1 does not alter the fit of the MHAT probabilities significantly, and $< 10\%$ more servicemembers develop PTSD in this scenario compared to the base-case.

Next we assume $\theta = 1$ and recompute the optimal values for the remaining parameters, which yield $\alpha = 132.9$, $\gamma = 6.99$, $b = 0.0625$. Because there is no recuperation, servicemembers on average will have higher levels of stress. Consequently, to achieve the same PTSD rates in the MHAT studies, there needs to be a larger mean threshold (γ^{-1}) and a smaller α (α is a function of γ to ensure the initial PTSD rate is 0.05). The root mean square error is 0.031. Column E of Table 18 and Figure 11(b) give the estimated MHAT probabilities and the predicted number of symptomatic servicemembers, respectively, for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.43 for active Army, 0.30 for reserve Army, and 0.44 Marines. The total number of servicemembers who develop symptomatic PTSD is $< 5\%$ more than the base-case.

Time Lag Parameters. We investigate the impact of varying the time lag parameters μ_1 and μ_2, while maintaining the same dispersion factors. First, we increase μ_1 and μ_2 so that the median for both the civilian and military time lags increases by a factor of 2, which is equivalent to increasing μ_1 and μ_2 by $\ln(2)$. The optimal parameter values are $\alpha = 197.0$, $\gamma = 10.37$, $b = 0.0624$, and $\theta = 1$, with a root mean square error of 0.030. To achieve the MHAT PTSD rates in Table 18 when the time lags are longer, more servicemembers need to develop PTSD during the time period of those studies because a smaller fraction of those who develop PTSD will be symptomatic. This decreases the mean threshold level γ^{-1}, which is why γ and α increase (α is a function of γ to ensure the initial PTSD rate is 0.05). Column F of Table 18 and Figure 11(c) give the estimated MHAT probabilities and the predicted number of symptomatic servicemembers, respectively, for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.51 for active Army, 0.37 for reserve Army, and 0.50 for Marines, which is $\approx 20\%$ higher than in the base-case. Because the threshold is lower, more servicemembers will develop PTSD although it will take longer for the symptoms to appear.

Next we decrease μ_1 and μ_2 so that the median for both the civilian and military time lags decrease by a factor of 2, which is equivalent to decreasing μ_1 and μ_2 by $\ln(2)$. The optimal parameter values change to $\alpha = 110.9$, $\gamma = 5.83$, $b = 0.0626$, and $\theta = 0$, with a root mean square error of 0.030. Because the median time lags have decreased, to achieve the same PTSD rates in the MHAT studies, fewer servicemembers need to develop PTSD during the time period of those studies because a larger fraction of those who develop PTSD will be symptomatic, which leads to a larger mean threshold level and full recuperation ($\theta = 0$). Column G of Table 18 and Figure 11(d) give the estimated MHAT probabilities and the predicted number of symptomatic servicemembers, respectively, for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.35 for active Army, 0.27 for reserve Army, and 0.35 for Marines, which is $\approx 15\%$ fewer PTSD cases than in the base-case.

We also analyze the impact of varying the time lag parameters s_1 and s_2, while maintaining the same medians. This change has less of an impact than varying the medians, so we just state the results here. When we increase the dispersion parameters by ln(5) (which is a more significant modification than we analyzed for the median parameters) the optimal parameter values are $\alpha = 137.3$, $\gamma = 7.22$, $b = 0.0626$, and $\theta = 0$, with a root mean square error of 0.030. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.39 for active Army, 0.30 for reserve Army, and 0.39 for Marines, which is $\approx 5\%$ less than in the base-case. When we decrease the dispersion parameters by ln(5) (we set $s_2 = 0.01$) the optimal parameter values are $\alpha = 150.3$, $\gamma = 7.90$, $b = 0.0633$, and $\theta = 1$, with a root mean square error of 0.035. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.45 for active Army, 0.32 for reserve Army, and 0.46 for Marines, which is $\approx 10\%$ higher than in the base case.

Future Stress Level. In the base-case we define the average monthly stress process from October 2008 until the end of the withdrawal, $\lambda_j(t)$ for $t \geq 68$, to equal the average of $\lambda_j(t)$ between October 2007 – September 2008 (calculated separately for Army and Marines). This value is 0.0030 for Army and 0.0013 for Marines. We consider several other possible values for the future mean monthly stress because this value is used for up to three and half years of deployments.

Because the stress process is close to its lowest values between October 2007 – September 2008 (see Figure 6), we set $\lambda_1(t) = \lambda_3(t) = 0$ for $t \geq 68$ to model the best-case scenario where violence, and hence stress, continues to decrease. Figure 12(a) gives the predicted number of symptomatic servicemembers for this situation. These curves are not exactly the same because even though no future servicemembers develop PTSD via exposure to stress during OIF, servicemembers can still develop PTSD from pre-deployment stress. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.37 for active Army, 0.27 for reserve Army, and

0.39 for Marines, which is $\approx 10\%$ fewer PTSD cases than in the base-case.

We next assume the future average monthly stress is equal to the median value between March 2003 and September 2008. This value is 0.0049 for Army and 0.0084 for Marines, which is a significant increase over the base-case estimates, especially for the Marines. Figure 12(b) gives the predicted number of symptomatic servicemembers for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.43 for active Army, 0.34 for reserve Army, and 0.46 for Marines, which is $\approx 8\%$ more PTSD cases than in the base-case.

Finally, to model the worst-case scenario, we set the future average monthly stress equal to the 90^{th} percentile of stress between March 2003 and September 2008. This value is 0.0081 for Army and 0.00164 for Marines. Figure 12(c) gives the predicted number of symptomatic servicemembers for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.46 for active Army, 0.38 for reserve Army, and 0.51 for Marines, which is $\approx 18\%$ more PTSD cases than in the base-case.

In all three scenarios, the deviation from the base case becomes more significant the farther in the future the withdrawal occurs.

References

[1] Congressional Budget Office, U.S. Congress. Options for restructuring the Army. Washington, D. C., May 2005.

[2] Congressional Budget Office, U.S. Congress. An analysis of the U.S. Military's ability to sustain an occupation of Iraq: an update. Washington, D. C., October 5, 2005.

[3] Congressional Budget Office, U.S. Congress. Some implications of increasing U.S. forces in Iraq. Washington, D. C., April 2007.

[4] L. B. Carter. Iraq: summary of U.S. forces. Congressional Research Service Report RL 31763, Washington, D. C., November 28, 2005.

[5] S. Bowman. Iraq: U.S. military operations. Congressional Research Service Report RL 31701, Washington, D. C., July 15, 2007.

[6] Lexis-nexis academic. Website. `http://www.lexisnexis.com/us/lnacademic/`.

[7] L. Korb, P. Rundlet, M. Bergmann, S. Duggan, P. Juul. Beyond the Call of Duty. Center for American Progress, Washington, D. C., 2007.

[8] GlobalSecurity.org. U.S. forces order of battle. Accessed at `http://www.globalsecurity.org/military/ops/swa-ops.htm` on on October 17, 2008.

[9] A. Rizzo. Airborne Brigade celebrates return to their home base after a year in Iraq. *The Associated Press*, March 12, 2004.

[10] D. D'Ambriosio. Fort Carson soldiers return to one of biggest homecomings yet. *The Associated Press State & Local Wire*, February 26, 2004.

[11] U.S. Fed News. They've all come home. *U.S. Fed News*, March 16, 2006.

[12] A. K. Stairrett. 2007 was big year at Fort Hood. *Killeen Daily Herald*, December 31, 2007.

[13] 11th ACR Homepage. 2007. `http://www.irwin.army.mil/Units/11TH+Armored+Cavalry+Regiment/11th+CR/default.htm`.

[14] CNN. 'Great pride' as troops leave Iraq. *CNN.com*, July 4, 2004.

[15] M. Mitchell. Ft Lewis Strykers will have their Iraq tours extended by 3 months. *The Associated Press State & Local Wire*, April 12, 2007.

[16] M. Mitchell. Fort Lewis brigade to go to Iraq in April, 2 months early. *The Associated Press State & Local Wire*, January 12, 2007.

[17] J. Mannion. Rumsfeld approves deployment of additional troops to Iraq: officials. *Agence France Presse – English*, December 19, 2003.

[18] 82nd Airborne cancels annual celebration at Fort Bragg, says too many troops deployed. *The Associated Press*, March 6, 2007.

[19] K. Maurer. 82nd Airborne coming back home early. *The Associated Press State & Local Wire*, May 16, 2008.

[20] DoD Announces Iraq Deployments. *U.S. Department of Defense*, September 30, 2008.

[21] E. Thompson. Fort Bragg troops return from Iraq after 11 months. *The Associated Press State & Local Wire*, January 19, 2004.

[22] Newspaper says 82nd Airborne troops may go to Iraq. *The Associated Press State & Local Wire*, November 24, 2004.

[23] R. Burns. First drop in U.S. troop levels in Iraq may come within weeks. *The Associated Press*, February 1, 2005.

[24] E. Thompson. U.S. Army's 82nd Airborne quick-reaction force faces possible problem: Full deployment. *The Associated Press Worldstream*, January 19, 2007.

[25] Troop drawdown due to success. *UPI*, March 10, 2008.

[26] E. Thompson. About 2,500 Fort Bragg paratroopers headed to Iraq. *The Associated Press State & Local Wire*, July 24, 2003.

[27] Rumsfeld Approves Brigade Deployment, Extension. *American Forces Press Service*, December 19, 2003 .

[28] 82nd Airborne's 3rd brigade preparing to return home from Iraq. *The Associated Press State & Local Wire*, October 4, 2007.

[29] Army Prepares for Next Rotations to Iraq. *Army Public Affairs*, May 19, 2008.

[30] K. Hefling. 101st Airborne Division receives deployment orders. *The Associated Press State & Local Wire*, February 6, 2003.

[31] States News Service. 101st Airborne Division Transfers Authority to Task Force Olympia. *States News Service*, February 5, 2004.

[32] R. French. 101st Airborne Division soldiers return to Fort Campbell from Iraq, more big groups coming. *The Associated Press*, August 11, 2006.

[33] K. M. Hall. 101st Airborne Prepares for Deployment. *Associated Press Online*, September 7, 2007.

[34] Last of 101st Airborne soldiers to arrive home for Thanksgiving. *The Associated Press State & Local Wire*, November 20, 2006.

[35] J. Krane. General: Army's 1st Armored Division to leave Iraq in July. *Associated Press Worldstream*, June 11, 2004.

[36] Editorials from the Kentucky New Era, The Kentucky Post, The News-Enterprise. *The Associated Press State & Local Wire*, February 7, 2007.

[37] L. C. Baldour. U.S. sending more troops into Iraq from neighboring Kuwait. *Associated Press Worldstream*, May 30, 2006.

[38] U.S. Fed News. Reintegration classes help soldiers in iraq get into 'Going home mindset'. *U.S. Fed News*, September 26, 2006.

[39] U.S. Fed News. USAREUR units identified for deployment in 2008. *U.S. Fed News*, November 28, 2007.

[40] 3rd Brigade, 1st Armored webpage. 2007. `http://www.riley.army.mil/view/article.aspx?articleId=240-2002-04-23-36228-42`.

[41] Fort Riley has ceremony for brigade going to Iraq. *The Associated Press State & Local Wire*, September 30, 2008.

[42] J. Mannion. Rumsfeld signs rotation orders, Marines to go back to Iraq : defense officials. *Agence France Presse – English*, November 6, 2003.

[43] DoD Announces Units for Next Operation Iraqi Freedom Rotation. *Defense Department Documents and Publications*, July 27, 2006.

[44] States News Service. Soldier touched by recognition from Vice President. *States News Service*, February 27, 2008.

[45] Four brigades tapped for Iraq rotation. *Army Public Affairs*, July, 2008.

[46] U.S. Fed News. Dod Announces Deployment Adjustments for Units in Iraq. *U.S. Fed News*, October 30, 2004.

[47] U.S. Fed News. Black Jack soldiers begin movement home. *U.S. Fed News*, January 4, 2008.

[48] N. A. Youssef & R. Schoof. Troop levels in Iraq likely to remain above 130,000. *Knight Ridder Washington Bureau*, September 12, 2007, 2006.

[49] Sgt. S. Wood, U.S American Forces Press Service. DoD Announces Tour Extension of Army Unit, Early Deployment of Another. *Defense Department Documents and Publications*, September 25, 2006.

[50] Last of 1st Cavalry troops return to Fort Bliss. *The Associated Press State & Local Wire*, December 25, 2007.

[51] 1st Brigade, 1st Infantry webpage. 2007. `http://www.riley.army.mil/view/article.aspx?articleId=499-2002-05-07-34854-2`.

[52] 2nd Brigade, 1st Infantry webpage. 2007. `http://www.2bct.1id.army.mil/Primary%20Sites/history/history_oif2.htm`.

[53] U.S. Fed News. 'Vanguard' battalion wrapping up its 15-month deployment in Iraq. *U.S. Fed News*, November 8, 2007.

[54] T. Shanker. G.I.'s on the Road From Kuwait to Baghdad Still Run a Gantlet of Danger. *The New York Times*, March 9, 2004.

[55] J. Milburn. Another Fort Riley brigade begins buildup for deployment to war. *The Associated Press State & Local Wire*, April 18, 2007.

[56] 2ID brigade deploys from Korea for OIF. *U.S. Department of Defense Information*, August 10, 2004.

[57] Sgt. S. Wood. Army Announces Repositioning Plans. *U.S. Department of Defense Information*, July 27, 2005.

[58] A. Santiago. Final Group Returns Home To Fort Carson. *CBS4Denver*, Jan 14, 2008.

[59] 3rd Brigade, 2nd Infantry webpage. 2007. `http://www.lewis.army.mil/3bde/History.htm`.

[60] More Stryker brigade members returning to Fort Lewis. *The Associated Press State & Local Wire*, September 10, 2007.

[61] 1st Brigade, 3rd Infantry webpage. 2007. `http://www.stewart.army.mil/3DIDWeb/1st%20BCT/1stBrigadehom.htm`.

[62] R. Bynum. Fort Stewart troops return from 3rd tour in Iraq. *The Associated Press State & Local Wire*, March 27, 2008.

[63] B. MacQuarrie. Rebuilding Iraq/Homecoming; Third Infantry gets its marching orders. *The Boston Globe*, July 27, 2003.

[64] Fort Stewart soldier killed in Iraq. *The Associated Press State & Local Wire*, December 27, 2005.

[65] J. Garamone. Gates Says No D-Day for Troop Surge Into Iraq. *American Forces Press Service*, January 11, 2007.

[66] 3rd Brigade, 3rd Infantry webpage. 2007. `http://www.stewart.army.mil/3DIDWeb/3rd%20BCT/3rdBrigadehom.htm`.

[67] R. Bynum. 3rd Infantry to Stay Deployed in Iraq. *Associated Press Online*, July 14, 2003.

[68] 4th Brigade, 3rd Infantry webpage. 2007. `http://www.stewart.army.mil/3DIDWeb/4th%20BCT/4thBrigadehom.htm`.

[69] Army: 4K Ga. troops set to return early from Iraq. *The Associated Press State & Local Wire*, October 15, 2008.

[70] P. Jelinek. Rumsfeld oks plan to bring out troops in Iraq, send in new ones. *The Associated Press State & Local Wire*, July 23, 2003.

[71] U.S. Fed News. 4ID Brigade wraps up ntc training, preps for Iraq. *U.S. Fed News*, September 22, 2005.

[72] U.S. Fed News. Texas radio personalities wrap up Camp Liberty 'deployment'. *U.S. Fed News*, March 21, 2008.

[73] First wave of 4th ID troops returns to Fort Hood. *The Associated Press State & Local Wire*, September 5, 2006.

[74] U.S. Fed News. 4th Inf. div. cases colors as soldiers deploy to Iraq. *U.S. Fed News*, November 1, 2007.

[75] Houston officer killed in Iraq was due home in December. *The Associated Press State & Local Wire*, November 29, 2006.

[76] States News Service. Cobras become Long Knives: 4th bct, 4th Inf. Div. re-flags to join 1st Cav. Division. *States News Service*, March 11, 2008.

[77] Less fanfare this time as original Stryker brigade gears up for return to Iraq. *The Associated Press State & Local Wire*, October 15, 2005.

[78] U.S. Fed News. Sryker regiment cases its colors as its readies for second deployment to Iraq. *U.S. Fed News*, August 3, 2007.

[79] U.S. Fed News. Kids Get Taste Of Soldier Life, Outdoor Fun During Spur Ride. *U.S. Fed News*, September 8, 2008.

[80] G. Edmonson. U.S. Expands Force in Iraq as Country's Election Date Nears. *Cox News Service*, December 1, 2004.

[81] Staff Sgt. J. Cupp. 'Ironhorse' Brigade Officer Reflects on Progress in Iraq. *Defense Department Documents and Publications*, December 27, 2007.

[82] Staff Sgt. C. Yazzie. 25th Soldiers return from OIF deployments. *United States Army, Pacific*, September 21, 2007.

[83] Welcome home 4th brigade. *States News Service*, December 19, 2007.

[84] Spc. L.B. Edgar 'Arctic Wolves' head home from Iraq. *Defense Department Documents and Publications*, November 27, 2006.

[85] Chris Freiberg. Training met with fervor as Strykers prep for war. *Fairbanks Daily News-Miner*, March 15, 2008.

[86] G. J. Gilmore. Iraqi Forces 'Getting Better Every Day,' U.S. Officer Says. *Defense Department Documents and Publications*, June 30, 2006.

[87] P. Hess Analysis: Iraq deployment extensions. *UPI*, April 4, 2007.

[88] Analysis: Iraq deployment extensions. *The Associated Press State & Local Wire*, August 10, 2005.

[89] P. Jelinek. Army's 'most-deployed' brigade coming home from Iraq: 52 killed, 270 wounded, 2 missing. *Associated Press Worldstream*, October 5, 2007.

[90] U.S. Fed News. 10th Mountain Division's 4th brigade combat team ready for deployment to Iraq. *U.S. Fed News*, November 8, 2007.

[91] R. Burns. Most of newly alerted National Guard units have served in Iraq. *The Associated Press State & Local Wire*, October 19, 2007.

[92] DoD Announces Force Adjustments. *U.S. Department of Defense*, December 3, 2007.

[93] U.S. Fed News. Wisconsin Guard Announces Largest Deployment Since WWII. *U.S. Fed News*, September 9, 2008.

[94] T. Jones. Ohio National Guard readies for deployment. *The Associated Press State & Local Wire*, March 23, 2008 .

[95] J. Garamone. Four Guard Brigades Alerted for Iraq Duty. *American Forces Press Service*, April 9, 2007.

[96] States News Service. Lincoln on departure of 39th to Middle East. *States News Service*, March 23, 2008.

[97] Oklahoma's 45th begins deployment. *The Associated Press State & Local Wire*, January 23, 2008.

[98] U.S. States News Employers visit army National Guard soldiers during pre-mobilization process at Fort Dix. *U.S. States News*, March 17, 2008.

[99] States News Service. Garrett Honors New Jersey's 50th Infantry Brigade. *States News Service*, June 17, 2008.

[100] States News Service. Governor O'Malley Welcomes Home National Guard Soldiers From Iraq. *States News Service*, July 27, 2008.

[101] M. Grant. Local soldiers could soon be home. *WTHI*, Oct 13, 2008.

[102] Seattle-based National Guard unit going back to Iraq. *The Associated Press State & Local Wire*, March 20, 2008.

[103] U.S. States News. Demobilization Ceremony Set for Members of 3rd Battalion 116th Cavalry After Return from Iraq. *U.S. States News*, November 14, 2005.

[104] Barbour: Miss. 155th Brigade Combat Team coming home. *The Associated Press State & Local Wire*, December 6, 2005.

[105] 155th gets annual training. *The Associated Press State & Local Wire*, June 13, 2008.

[106] For reservists from New Orleans, a bittersweet homecoming. *Agence France Presse – English*, September 16, 2005.

[107] H. Mohr. Training of Vermont troops help fuel economic boom in Mississippi. *The Associated Press State & Local Wire*, January 16, 2005.

[108] Ceremony marks deployment for Pennsylvania-based 28th Infantry. *The Associated Press State & Local Wire*, June 25, 2005.

[109] U.S. Fed News. Rep. Kline Speaks in Honor of Minnesota National Guard. *U.S. Fed News*, April 5, 2006.

[110] DoD announces units for 'troop surge'. *UPI*, January 25, 2007.

[111] M. Basu. 48th will get safer assignment. *Cox News Service*, September 10, 2005.

[112] Lance Cpl. D. J. Redding. Beach city rolls out red carpet for grunt unit. *U.S. Department of Defense Information*, November 4, 2004.

[113] States News Service. Newport Beach Honors Marines. *States News Service*, October 18, 2005.

[114] U.S. Fed News. Marine Paints the Town for Incoming Battalion. *U.S. Fed News*, August 6, 2006.

[115] *City News Service*, January 20, 2006.

[116] U.S. Fed News. Marines dig into digital environment. *U.S. Fed News*, April 5, 2007.

[117] M. Lowry. "First of the First" return from 7 month deployment. *Defense Department Documents and Publications*, February 20, 2008.

[118] 2nd Battalion, 1st Marines webpage. 2007. `http://www.i-mef.usmc.mil/DIV/1MAR/2bn/history.asp`.

[119] SD Marine Homecoming. *City News Service*, January 28, 2005.

[120] U.S. Fed News. Hawaii-Based Marines Arrive in Al Anbar Province. *U.S. Fed News*, March 18, 2006.

[121] Cpl. A.C. Schnell. Marines find it's a small Marine Corps, build lasting friendship. *U.S. Department of Defense Information*, November 7, 2005.

[122] Sgt. A. Hurt. Open-minded attitude, trust key to BLT 3/1 integration with Iraqi Army. *Defense Department Documents and Publications*, September 3, 2007.

[123] MarineParents.com. Marine Units Deployed to Iraq and Afghanistan. Accessed at `http://www.marineparents.com/usmc/back-to-iraq.asp` on October 19, 2008.

[124] Marine welcomed home from Iraq with billboard. *The Associated Press State & Local Wire*, July 11, 2003.

[125] Ground troops from Camp Lejeune deploy this week for Iraq. *The Associated Press State & Local Wire*, July 5, 2004.

[126] 24th Marine Expeditionary Unit set to return home. *U.S. Department of Defense Information*, February 7, 2005.

[127] U.S. Fed News. Eodmu 2 Det. 22 Completes Deployment with Esg 8. *U.S. Fed News*, May 4, 2006.

[128] U.S. Fed News. Marines Take Moment to Reenlist in Iraq. *U.S. Fed News*, July 25, 2007.

[129] Lt. Col. Winand Battalion Commander March Newsletter. *1st Battalion, 2nd Marines webpage*, February 7, 2008. `http://www.iimefpublic.usmc.mil/1bn2mar`.

[130] K. Gray. Marine gets reacquainted with wife, son. *The Associated Press State & Local Wire*, June 10, 2003.

[131] States News Service. I Marine Expeditionary Force Names Rotation Units. *States News Service*, February 26, 2004.

[132] U.S. Fed News. Lejeune marines say goodbye to Mahmudiyah community. *U.S. Fed News*, September 17, 2004.

[133] U.S. Fed News. 2/2 Returns home to family, friends. *U.S. Fed News*, February 13, 2006.

[134] US Fed News. Marines participate in casualty assistance training. *US Fed News*, March 5, 2008.

[135] Some soldiers trickling home while other anxious families wait. *The Associated Press State & Local Wire*, July 22, 2003.

[136] Marine forces prepare for Iraqi deployment beginning January 2005; Release 1101-04-0925. *U.S. Department of Defense Information*, April 20, 2005.

[137] Cpl. R. Lewis. M3/2 Marines return, end seven-month deployment in Iraq. *Defense Department Documents and Publications*, March 5, 2007.

[138] Cpl. B. Hall. 3rd Bn., 2nd Marines hits ground running. *Defense Department Documents and Publications*, November 6, 2007.

[139] 31st MEU ends operations in Iraq, prepares for long journey home. *U.S. Department of Defense Information*, February 11, 2005.

[140] U.S. Fed News. 1/3 returns after seven-month Iraq deployment. *U.S. Fed News*, September 27, 2007.

[141] 1st Battalion, 3rd Marines webpage. News From Iraq, September 30, 2008. `http://www.mcbh.usmc.mil/3mar/1dbn/1-3NewsFromIraq.htm`.

[142] U.S. Fed News. 2nd battalion, 3rd marine regiment returns to Isle. *U.S. Fed News*, April 13, 2007.

[143] US Fed News. Island warriors arrive in Iraq, receive visit from RCT commander. *US Fed News*, February 12, 2008.

[144] Cpl. L. Blom. Hawaii-based 2nd Battalion, 3rd Marines replace sister Marine battalion in Iraq's Haditha region. *Defense Department Documents and Publications*, October 8, 2006.

[145] U.S. Fed News. 'America's Battalion' Departs K-Bay for Iraq Deployment. *U.S. Fed News*, August 3, 2007.

[146] More marines, sailors coming home from Mideast. *The Associated Press State & Local Wire*, February 28, 2008.

[147] T. Labbe & A. Shadid. U.S. to Delay Transfer of Security Powers; Polish-Led Force Was to Take Over In South This Week. *The Washington Post*, September 1, 2003.

[148] 11th Marine Expeditionary Unit begins return home. *U.S. Department of Defense Information*, February 7, 2005.

[149] Cpl. A. Kalwitz. Al Qaim detachment plays role in Iraq transition. *Defense Department Documents and Publications*, May 23, 2007.

[150] 1st Battalion, 4th Marines webpage. June 2008 Newsletter. `http://www.i-mef.usmc.mil/div/1mar/1bn4/_downloads/China_News_June_2008.pdf`.

[151] SD Marine Homecoming. *City News Service*, September 24, 2004.

[152] U.S. Fed News. President Authorizes Unit Tour Extensions in Iraq. *U.S. Fed News*, January 11, 2007.

[153] CNN. Airstrike targets senior Iraqi officials. *CNN.com*, April 7, 2003.

[154] U.S. Fed News. 3rd battalion, 4th marine regiment gets send-off from division commander. *U.S. Fed News*, July 9, 2004 Friday.

[155] U.S. Fed News. 3/4 Returns to Iraq for Third Time. *U.S. Fed News*, January 20, 2005.

[156] U.S. Fed News. Some stay, some go, all proud. *U.S. Fed News*, January 27, 2006.

[157] 4,000 Marines to be extended in Iraq. *Marine Corps Times*, January 12, 2007. `http://www.marinecorpstimes.com/news/2007/01/mc.marinesurge070111/`.

[158] US Fed News 'Darkside' deploys to western al Anbar again. *US Fed News*, February 25, 2008.

[159] R. Tomkins. Feature: Johnny comes marching home. *United Press International*, May 26, 2003.

[160] SD Marine Rotation. *City News Service*, June 22, 2004.

[161] U.S. Fed News. 1st Battalion, 5th Marines, Arrives at Ar Ramadi. *U.S. Fed News*, February 28, 2005.

[162] Cpl. T. Sloan. 1/5 returns from tour in Iraq. *U.S. Department of Defense Information*, October 6, 2005.

[163] 2nd Battalion, 5th Marines webpage. 2007. `http://www.i-mef.usmc.mil/DIV/5MAR/2bn/history.asp`.

[164] Marine unit returns from Iraq with zero casualties. *PrairiePundit*, October 9, 2007.

[165] Winchester Marine dies after being wounded in Iraq. *The Associated Press State & Local Wire*, April 15, 2003.

[166] Cpl. M. Escobar. 1/6 hits the ground in Baharia, prepares for busy deployment. *U.S. Department of Defense Information*, March 22, 2005.

[167] U.S. Fed News. RCT-5 commander welcomes 'Betio bastards' to fight. *U.S. Fed News*, July 19, 2006.

[168] U.S. Fed News. 3/5 returns to Fallujah. *U.S. Fed News*, October 28, 2007.

[169] Cpl. M. Escobar. 1/6 HARD' welcomes new keeper of good order and discipline. *Defense Department Documents and Publications*, December 20, 2005.

[170] Marine battalion comes home, lost 12 members in Iraq. *The Associated Press State & Local Wire*, May 20, 2007.

[171] Lejeune MEU coming home Memorial Day. *The Associated Press State & Local Wire*, May 20, 2003.

[172] Lance Cpl. E. L. Mennenga. 2/6 Marines come home. *U.S. Department of Defense Information*, March 15, 2005.

[173] 900 Marines, sailors return to Camp Lejeune from Iraq. *The Associated Press State & Local Wire*, April 13, 2006.

[174] U.S. Fed News. 2/6 Heads to California Desert, Begins Training for Upcoming Deployment. *U.S. Fed News*, November 5, 2006.

[175] Cpl H. E. Loredo. 3/6 convoy training takes to the streets of 29 Palms. *U.S. Department of Defense Information*, July 1, 2005.

[176] Cpl. A. Rosas. California-based Marine battalion starts second tour in Iraq's Qa'im region. *Defense Department Documents and Publications*, March 26, 2006.

[177] U.S. Fed News. 3/6 Marines Get Acquainted with New Neighbors. *U.S. Fed News*, February 10, 2007.

[178] 3rd Battalion, 6th Marines webpage. October 2008 Newsletter. `http://www.iimefpublic.usmc.mil/Public%2FInfolineMarines.nsf/0/5259F05D5C78487A432574E200404561/$FILE/3-6Newsletter%20October%2008.pdf`.

[179] B. Hemmer and M. Savidge. Marines Cautious with Civilians. *Cnn*, April 6, 2003.

[180] AAV Marine recalls units deployment to Iraq. *U.S. Department of Defense Information*, May 3 2005.

[181] U.S.MC Press Release: I MEF units deploying to Iraq. December 23, 2005. usmc.mil.

[182] U.S. Fed News. 1/7 marines return from third deployment to warm welcome. *U.S. Fed News*, September 23, 2006.

[183] Sgt. J. E. Haskamp. 2/7 says goodbye to brothers in arms. *U.S. Department of Defense Information*, November 15, 2004.

[184] War Dogs' advance party comes home. *Defense Department Documents and Publications*, January 15, 2006.

[185] U.S. Fed News. 2/7 Takes to Field before Iraq Deployment. *U.S. Fed News*, October 31, 2006.

[186] J. Finer. Troops in Kuwait, at the Edge of War; Marines Find Time Is of the Essence Near Front Lines. *The Washington Post*, February 4, 2003.

[187] U.S. Fed News. Marines return home. *U.S. Fed News*, August 6, 2004.

[188] U.S. Fed News. Sun sets on 12 'cutting edge' marines. *U.S. Fed News*, June 22, 2006.

[189] Simulated City Preps Marines for Reality in Iraq. *National Public Radio*, April 13, 2007.

[190] T. Susman. Trash is Marines' new enemy. *Los Angeles Times*, October 26, 2007.

[191] U.S. Fed News. 3/7 Departs For Iraq. *U.S. Fed News*, August 27, 2008.

[192] Cpl. A. C. Schnell. Leaving Fallujah for family. *U.S. Department of Defense Information*, February 3, 2005.

[193] U.S. Fed News. 1st Battalion, 8th Marines departs for Ramadi. *U.S. Fed News*, September 16, 2007.

[194] Marines deploy for Afghanistan, some just months after Iraq. *The Associated Press State & Local Wire*, November 24, 2003.

[195] Lance Cpl. C. E. McKelvey. Brothers transition from football field to battle field. *Defense Department Documents and Publications*, December 25, 2007.

[196] Cpl. Mike Escobar. No rest for the weary: 3/8 commences pre-deployment training, once again. *Defense Department Documents and Publications*, December 14, 2005.

[197] Camp Lejeune Marines, sailors return from Iraq. *The Associated Press State & Local Wire*, October 3, 2006.

[198] U.S. Fed News. 1/9 marines train during desert talon: 'The Walking Dead' strut their stuff. *U.S. Fed News*, June 21, 2007.

[199] A. Doud Marines return home from Iraq. *WNCT TV*, October 16, 2008.

[200] Lance Cpl. B. Woodruff. Marines, Sailors Deploy to Support OIF. *Military.com*, October 2, 2008.

[201] 3rd Battalion, 9th Marines webpage. Battalion Website. `http://www.mnfwest.usmc.mil/Public%2FIIMEFpublic.nsf/unitSites/3bn9mar`.

[202] Marine reservists from Texas prepare to go to Iraq. *The Associated Press State & Local Wire*, June 6, 2004.

[203] D. Murphy. In Iraq with 'reservists that fight'. *Christian Science Monitor*, March 24, 2005.

[204] C. Ivey. Marine Corps' only reserve battalion to fight Iraq war comes home. *The Associated Press State & Local Wire*, May 29, 2003.

[205] Sgt. A. Rigez. Morocco hosts California Marines, Sailors during Exercise African Lion. *Defense Department Documents and Publications*, April 26, 2007.

[206] Some Arkansas Marines to return for second tour of Iraq. *The Associated Press State & Local Wire*, April 25, 2007.

[207] Marines return from Middle East. *The Associated Press State & Local Wire*, September 27, 2003.

[208] Cpl. R. M. Blaich. Haditha booming thanks to Marines of past, present. *Defense Department Documents and Publications*, November 26, 2007.

[209] Gunnery Sgt. M. Oliva 1st Battalion , 24th Marine Regiment newest RCT-5 battalion in Fallujah. *Defense Department Documents and Publications*, October 17, 2006.

[210] Broken Arrow Marines prepare to deploy. *The Associated Press State & Local Wire*, September 11, 2007.

[211] 2nd Battalion, 24th Marines webpage. 2007. `http://www.mfr.usmc.mil/4thmardiv/24thMar/2dBn/History.htm`.

[212] Des Moines Marine reservists to return home. *The Associated Press State & Local Wire*, August 21, 2008.

[213] 3rd Battalion, 24th Marines webpage. 2007. `http://www.marforres.usmc.mil/4thmardiv/24thmar/3dbn/HISTORY_.HTM`.

[214] U.S. Fed News. 1/25 Comes to Combat Center, Trains for Deployment to Iraq. *U.S. Fed News*, January 10, 2006.

[215] Marine reservists come home combat veterans. *The Associated Press State & Local Wire*, July 31, 2003.

[216] U.S. Fed News. Picatinny's Golf Company, 2nd Battalion, 25th Marine Regiment Deploys In Support of War on Terror. *U.S. Fed News*, September 8, 2008.

[217] Ohio-based Marine unit preparing to return home, families say. *The Associated Press State & Local Wire*, September 16, 2005.

[218] Congressional Budget Office, U.S. Congress. Recruiting, retention, and future levels of military personnel. Washington, D. C., October, 2006.

[219] J. C. Huly, U.S. Marine Corps. The Army's and the Marine Corps' troop rotations for Operation Iraqi Freedom (OIF) 3 and Operation Enduring Freedom (OEF) 6 and the Army's involuntary mobilization of the Individual Ready Reserve. Testimony before the House Armed Services Committee, July 7, 2004.

[220] A. S. Tyson, J. White. Strained Army extends tours to 15 months. *Washington Post*, pg. A1, April 12, 2007.

[221] S. L. Myers, T. Shanker. Bush signals no further reduction of troops in Iraq. *New York Times*, pg. A1, April 11, 2008.

[222] J. M. Gilmore, Assistant Director for National Security. Issues that affect the readiness of the Army National Guard and Army Reserve. Congressional Budget Office testimony before the Commission on the National Guard and Reserves, May 16, 2007. Accessed at `http://www.cbo.gov/ftpdoc.cfm?index=8111&type=0` on July 24, 2007.

[223] D. Isenberg, United Press International. How over-extended is the U.S. military. Accessed at

http://www.spacewar.com/reports/How_Over_Extended_Is_The_US_Military_999.html on July 24, 2007.

[224] Statistical Information Analysis Division, Defense Manpower Data Center, Office of the Secretary of Defense. Military personnel statistics. Accessed at http://siadapp.dmdc.osd.mil/personnel/MILITARY/Miltop.htm on October 18, 2008.

[225] DoD News Briefing with Lt. Gen. Ham from the Pentagon. *U.S. Department of Defense*, February 25, 2008.

[226] Office of the Surgeon Multinational Force - Iraq and Office of the Surgeon General United States Army Medical Command. Mental Health Advisory Team (MHAT) IV Operation Iraqi Freedom 05-07, November 17, 2006.

[227] Office of the Surgeon Multinational Force - Iraq and Office of the Surgeon General United States Army Medical Command. Mental Health Advisory Team (MHAT-III) Operation Iraqi Freedom 04-06, May 29, 2006.

[228] Statistical Information Analysis Division, Defense Manpower Data Center, Office of the Secretary of Defense. Military Casualty Information. Accessed at http://siadapp.dmdc.osd.mil/personnel/CASUALTY/castop.htm on October 18, 2008.

[229] C. S. Milliken, J. L. Auchterlonie, C. W. Hoge. Longitudinal assessment of mental health problems among active and reserve component soldiers returning from the Iraq war. *JAMA* **298**, 2141-2148, 2007.

[230] J. Wolfe, D. J. Erikson, E. J. Sharansky, D. W. King, L. A. King. Course and predictors of posttraumatic stress disorder among Gulf War veterans: a prospective analysis. *J. Consulting Clin. Psychology* **67**, 520-528, 1999.

[231] Office of the Surgeon Multinational Force - Iraq and Office of the Surgeon General United States Army Medical Command. Mental Health Advisory Team (MHAT-II) Operation Iraqi Freedom 04-06, January 30, 2005.

[232] C. W. Hoge, C. A. Castro, S. C. Messer, D. McGurk, D. I. Cotting, R. L. Koffman. Combat duty in Iraq and Afghanistan, mental health problems, and barrier to care. *New Engl. J. Med.* **351**, 13-22, 2004.

Figure Legends

Fig. 1. Attached and unattached computed monthly troop levels for OIF from February 2003 until September 2008.

Fig. 2. Computed monthly troop levels for OIF by service branch from February 2003 until September 2008.

Fig. 3. Troop level comparisons from February 2003 until September 2008. **(a)** Comparison of computed Army troop levels vs. official DOD troop numbers [224]; **(b)** comparison of computed Marines troop levels vs. official DOD troop numbers [224]; **(c)** comparison of computed troop levels vs. Post-surge projected troop numbers [225].

Fig. 4. Attached and unattached predicted monthly troop levels for OIF from February 2003 until the end of the withdrawal. **(a)** Withdrawal scenario 1, withdrawal begins February 2009; **(b)** withdrawal scenario 2, withdrawal begins February 2010; **(c)** withdrawal scenario 3, withdrawal begins February 2011.

Fig. 5. Predicted monthly troop levels for OIF by service branch from February 2003 until the end of the withdrawal. **(a)** Withdrawal scenario 1, withdrawal begins February 2009; **(b)** withdrawal scenario 2, withdrawal begins February 2010; **(c)** withdrawal scenario 3, withdrawal begins February 2011.

Fig. 6. Average monthly stress for Army soldiers, $\lambda_1(t)$, (—) and Marines, $\lambda_3(t)$, (- - -) between March 2003 and September 2008.

Fig. 7. Predicted cumulative number of servicemembers who develop PTSD (—) and are symptomatic (- - -) by service branch from March 2003 until February 2023. **(a)** Withdrawal scenario 1, withdrawal begins February 2009; **(b)** withdrawal scenario 2, withdrawal begins February 2010; **(c)** withdrawal scenario 3, withdrawal begins February 2011.

Fig. 8. Predicted cumulative fraction of servicemembers (out of total deployed) who develop PTSD (—) and are symptomatic (- - -) by service branch from March 2003 until February

2023.**(a)** Withdrawal scenario 1, withdrawal begins February 2009; **(b)** withdrawal scenario 2, withdrawal begins February 2010; **(c)** withdrawal scenario 3, withdrawal begins February 2011.

Fig. 9. Histogram of the ratio $\frac{\text{maximum cumulative stress}}{\text{strength threshold}}$ for servicemembers in withdrawal scenario 2. The red dashed line at 1 partitions the servicemembers into whether they develop PTSD or not. The extreme bins of the histogram contain the remaining mass for the corresponding tail of the distribution.

Fig. 10. Predicted cumulative number of servicemembers symptomatic with PTSD for the model modifications described in §3.2. **(a)** Probit model; **(b)** Poisson base-case model, servicemembers only deploy once; **(c)** Poisson base-case model, servicemembers return to civilian status according to their stress-to-threshold ratio; **(d)** Poisson model, parameters reestimated, servicemembers return to civilian status according to their stress-to-threshold ratio.

Fig. 11. Predicted cumulative number of servicemembers symptomatic with PTSD for the sensitivity analyses described in §3.3. **(a)** Poisson base-case model with $\theta = 1$; **(b)** Poisson model, θ constrained to 1, other parameters reestimated; **(c)** Poisson model, μ_1 and μ_2 increase by ln(2), other parameters reestimated; **(d)** Poisson model, μ_1 and μ_2 decrease by ln(2), other parameters reestimated.

Fig. 12. Predicted cumulative number of servicemembers symptomatic with PTSD for various values of the future average monthly stress. The Poisson base-case parameters are used for all scenarios. **(a)** Future monthly stress = 0; **(b)** Future monthly stress = Median stress between March 2003 and September 2008; **(c)** Future monthly stress = 90$^{\text{th}}$ percentile stress between March 2003 and September 2008.

Division	Brigade	Type	Refs	d1 start	d1 end	d2 start	d2 end	d3 start	d3 end	modular date
–	173rd AB	ACR	[9]	04/01/03	04/01/04	–	–	–	–	10/01/06
–	3rd ACR	ACR	d1–[10] d2–[11] d3–[12]	04/01/03	03/01/04	04/01/05	04/01/06	11/01/07	02/01/09	–
–	11th ACR (TF HQ)	ACR	[13]	01/01/05	03/01/06	–	–	–	–	–
–	2nd CR/4th SBCT 2nd ID	Light	d1–[14] d2–[15] [16]	04/01/03	07/01/04	05/01/07	08/01/08	–	–	06/01/06
82nd Airborne	1st BCT	Light	d1–[17] d2–[18][19] d3–[20]	01/01/04	05/01/04	07/01/07	08/01/08	07/01/09	07/01/10	06/01/06
82nd Airborne	2nd BCT	Light	d1–[21] d2–[22][23] d3–[24][25]	01/01/03	02/01/04	12/01/04	04/01/05	01/01/07	04/01/08	06/01/06
82nd Airborne	3rd BCT	Light	d1–[26] [27] d2–[28] d3–[29]	08/01/03	05/01/04	08/01/06	11/01/07	11/01/08	11/01/09	06/01/06
82nd Airborne	4th BCT	Light	d1–[20]	05/01/09	05/01/10	–	–	–	–	06/01/06
101st Airborne	1st BCT	Light	d1–[30] [31] d2–[32] d3–[33]	03/01/03	02/01/04	10/01/05	10/01/06	10/01/07	01/01/09	09/01/05
101st Airborne	2nd BCT	Light	d1–[30] [31] d2–[32] d3–[33]	03/01/03	02/01/04	10/01/05	10/01/06	10/01/07	01/01/09	09/01/05
101st Airborne	3rd BCT	Light	d1–[30] [31] d2–[32] d3–[33]	03/01/03	02/01/04	10/01/05	10/01/06	10/01/07	01/01/09	09/01/05
101st Airborne	4th BCT/ 506th IR/ ABN	Light	[34]	10/01/05	10/01/06	–	–	–	–	09/01/05
1st Armored	1st BCT	Heavy	d1–[35] d2–[36]	05/01/03	08/01/04	01/01/06	03/01/07	–	–	–
1st Armored	2nd BCT	Heavy	d1–[35] d2–[37][38] d3–[39]	05/01/03	08/01/04	05/01/06	11/01/06	05/01/08	08/01/09	–
1st Armored	3rd BCT	Heavy	d1–[40] d2–[40] d3–[41]	03/01/03	04/01/04	01/01/05	02/01/06	10/01/08	10/01/09	04/01/07
1st Cavalry	1st BCT	Heavy	d1–[42] d2–[43][44] d3–[45]	03/01/04	03/01/05	11/01/06	02/01/08	01/01/09	01/01/10	05/01/05
1st Cavalry	2nd BCT	Heavy	d1–[46] d2–[43][47] d3–[45]	02/01/04	02/01/05	10/01/06	01/01/08	02/01/09	02/01/10	05/01/05
1st Cavalry	3rd BCT	Heavy	d1–[42] d2–[43][48] d3–[29]	04/01/04	04/01/05	09/01/06	11/01/07	11/01/08	11/01/09	05/01/05
1st Cavalry/1st Armored	4th BCT	Heavy	d1–[49][50] d2–[20]	11/01/06	12/01/07	04/01/09	04/01/10	–	–	05/01/05
1st Infantry	1st BCT	Heavy	[51]	09/01/03	09/01/04	–	–	–	–	–
1st Infantry	2nd BCT	Heavy	d1–[52] d2–[53] d3–[29]	02/01/04	02/01/05	09/01/06	12/01/07	12/01/08	12/01/09	03/01/08
1st Infantry	3rd BCT	Heavy	[54]	02/01/04	02/01/05	–	–	–	–	–
1st Infantry	4th BCT	Light	d1–[55] d2–[20]	02/01/07	05/01/08	07/01/09	07/01/10	–	–	08/01/06
2nd Infantry	2nd BCT	Stryker	d1–[56][57] d2–[58]	08/01/04	08/01/05	10/01/06	01/01/08	–	–	09/01/05
2nd Infantry	3rd BCT	Stryker	d1–[59] d2–[60] d3–[20]	11/01/03	11/01/04	07/01/06	09/01/07	06/01/09	06/01/10	01/01/03
2nd Infantry	5th BCT	Stryker	[20]	07/01/09	07/01/10	–	–	–	–	05/01/07
3rd Infantry	1st BCT	Heavy	d1–[61] d2–[61] d3–[61][62]	02/01/03	09/01/03	02/01/05	01/01/06	01/01/07	04/01/08	02/01/04
3rd Infantry	2nd BCT	Heavy	d1–[63] d2–[64] d3–[65]	01/01/03	09/01/03	02/01/05	01/01/06	05/01/07	08/01/08	02/01/04
3rd Infantry	3rd BCT	Heavy	d1–[66][67] d2–[66] d3–[65]	02/01/03	08/01/03	02/01/05	01/01/06	03/01/07	06/01/08	02/01/04
3rd Infantry	4th BCT	Heavy	d1–[68] d2–[62][69]	02/01/05	01/01/06	11/01/07	01/01/09	–	–	02/01/04
4th Infantry	1st BCT	Heavy	d1–[70] d2–[71] d3–[72]	04/01/03	04/01/04	12/01/05	12/01/06	03/01/08	06/01/09	12/01/04
4th Infantry	2nd BCT	Heavy	d1–[73] d2–[73] d3–[29]	04/01/03	04/01/04	12/01/05	12/01/06	10/01/08	10/01/09	12/01/04
4th Infantry	3rd HBCT	Heavy	d1–[73] d2–[73] d3–[74]	04/01/03	04/01/04	12/01/05	12/01/06	12/01/07	03/01/09	12/01/04
4th Infantry	4th BCT	Heavy	d1–[75] d2–[76]	12/01/05	12/01/06	07/01/08	10/01/09	–	–	12/01/04
25th Infantry	1st SBCT	Stryker	d1–[77] d2–[78][79]	10/01/04	10/01/05	09/01/07	11/01/08	–	–	01/01/06
25th Infantry	2nd BCT	Stryker	d1–[80] d2–[81]	01/01/04	03/01/05	01/01/08	04/01/09	–	–	01/01/06
25th Infantry	3rd BCT	Light	d1–[82] d2–[29]	08/01/06	10/01/07	11/01/08	11/01/09	–	–	01/01/06
25th Infantry	4th BCT	Light	[83]	10/01/06	12/01/07	–	–	–	–	01/01/06
25th Infantry	172nd IBCT / 1st B	Stryker	d1–[84] d2–[85]	08/01/05	12/01/06	10/01/08	10/01/09	–	–	01/01/07
10th Mountain	1st BCT	Light	d1–[86] d2–[87]	08/01/05	08/01/06	09/01/07	12/01/08	–	–	06/01/04
10th Mountain	2nd BCT	Light	d1–[88] d2–[89]	06/01/04	06/01/05	08/01/06	11/01/07	–	–	06/01/04
10th Mountain	4th BCT	Light	[90]	12/01/07	03/01/09	–	–	–	–	06/01/04

Table 1: Active Army deployment dataset. For each brigade, the start and end dates of each deployment, and the time each brigade was modularized. The notation di stands for the i^{th} deployment for i=1,2,3.

Division	Brigade	Type	Refs	d1 start	d1 end	d2 start	d2 end	modular date
–	29th IBCT	Light	d1–[91] d2–[91]	03/01/05	03/01/06	11/01/08	11/01/09	–
–	30th IBCT	Light	d1–[91] d2–[91]	02/01/04	01/01/05	04/01/09	04/01/10	–
–	32th IBCT	Light	[92][93]	05/01/09	03/01/10	–	–	–
–	37th IBCT	Light	[94]	05/01/08	02/01/09	–	–	–
–	39th BCT	Light	d1–[95] d2–[95][96]	04/01/04	04/01/05	04/01/08	02/01/09	–
–	41th IBCT	Light	[92]	06/01/09	06/01/10	–	–	–
–	45th IBCT	Light	[97]	03/01/08	11/01/08	–	–	–
–	50th IBCT	Light	[98][99]	09/01/08	06/01/09	–	–	–
–	58th IBCT	Light	[100]	08/01/07	07/01/08	–	–	–
–	72th IBCT	Light	[29]	08/01/09	08/01/10	–	–	–
–	76th IBCT	Light	[95][101]	04/01/08	11/01/08	–	–	–
–	81st ABCT	Armored	d1–[91] d2–[91][102]	03/01/04	03/01/05	11/01/08	11/01/09	–
–	116th Cav B	Heavy	[103]	12/01/04	12/01/05	–	–	–
–	155th ABCT	Heavy	d1–[104] d2–[105]	01/01/05	01/01/06	11/01/09	11/01/10	–
–	256th BCT	Heavy	d1–[106] d2–[29]	10/01/04	10/01/05	07/01/09	07/01/10	–
–	278th ACR	ACR	d1–[107] d2–[29]	11/01/04	11/01/05	10/01/09	10/01/10	–
28th Infantry	2nd BCT	Heavy	d1–[108] d2–[29]	06/01/05	06/01/06	08/01/09	08/01/10	–
28th Infantry	56th SBCT	Heavy	[91]	02/01/09	02/01/10	–	–	06/01/06
34th Infantry	1st BCT	Heavy	[109] [110]	04/01/06	08/01/07	–	–	–
35th Infantry	48th IB	Heavy	[111]	05/01/05	05/01/06	–	–	–
36th Infantry	56th BCT	Heavy	d1–[91] d2–[91]	01/01/05	01/01/06	10/01/08	10/01/09	–

Table 2: National Guard deployment dataset. For each brigade, the start and end dates of each deployment, and the time each brigade was modularized. The notation di stands for the i^{th} deployment for i=1,2.

Regiment	Battalion	Reserve/Active	Refs	d1 start	d1 end	d2 start	d2 end	d3 start	d3 end	d4 start	d4 end	d5 start	d5 end
1st RCT	1–1	A	d1–[112][113] d2–[114] [115] d3–[116][117]	01/01/05	08/01/05	01/01/06	09/01/06	07/01/07	02/01/08	–	–	–	–
1st RCT	2–1	A	d1–[118] d2–[118]	02/01/04	09/01/04	10/01/05	01/01/06	–	–	–	–	–	–
1st RCT	3–1	A	d1–[8] d2–[119] d3–[120] [121] d4–[122][123]	03/01/03	07/01/03	07/01/04	02/01/05	09/01/05	04/01/06	04/01/07	11/01/07	–	–
2nd RCT	1–2	A	d1–[8] [124] d2–[125][126] d3–[127] d4–[123][128] d5–[129]	03/01/03	07/01/03	07/01/04	02/01/05	11/01/05	06/01/06	03/01/07	10/01/07	08/01/08	03/01/09
2nd RCT	2–2	A	d1–[8] [130] d2–[131][132] d3–[133] d4–[134]	03/01/03	06/01/03	03/01/04	10/01/04	07/01/05	02/01/06	04/01/08	11/01/08	–	–
2nd RCT	3–2	A	d1–[8] [135] d2–[136] d3–[137] d4–[123][138]	03/01/03	06/01/03	02/01/05	09/01/05	07/01/06	02/01/07	10/01/07	05/01/08	–	–
3rd RCT	1–3	A	d1–[139] d2–[140] d3–[141]	11/01/04	03/01/05	03/01/07	10/01/07	09/01/08	04/01/09	–	–	–	–
3rd RCT	2–3	A	d1–[142] d2–[143]	09/01/06	04/01/07	02/01/08	09/01/08	–	–	–	–	–	–
3rd RCT	3–3	A	d1–[144] d2–[145][146]	03/01/06	10/01/06	08/01/07	03/01/08	–	–	–	–	–	–
1st RCT	1–4	A	d1–[8] [147] d2–[148] d3–[123][149] d3–[123][150]	03/01/03	10/01/03	07/01/04	03/01/05	05/01/07	12/01/07	11/01/08	06/01/09	–	–
5th RCT	2–4	A	d1–[131][151] d2–[152]	02/01/04	09/01/04	11/01/06	04/01/07	–	–	–	–	–	–
7th RCT	3–4	A	d1–[8]-[153] d2–[131] [154] d3–[155][156] d4–[123][157] d5–[158]	03/01/03	07/01/03	03/01/04	08/01/04	01/01/05	08/01/05	09/01/06	07/01/07	03/01/08	10/01/08
5th RCT	1–5	A	d1–[8][159] d2–[131][160] d3–[161] [162]	03/01/03	07/01/03	03/01/04	10/01/04	03/01/05	10/01/05	–	–	–	–
5th RCT	2–5	A	d1–[8][163] d2–[161][163] d3–[123][164]	03/01/03	08/01/03	09/01/04	03/01/05	03/01/07	10/01/07	–	–	–	–
5th RCT	3–5	A	d1–[8] [165] d2–[166] d3–[167] d4–[168]	03/01/03	07/01/03	08/01/04	04/01/05	01/01/06	08/01/06	10/01/07	05/01/08	–	–
6th RCT	1–6	A	d1–[169] d2–[170]	03/01/05	09/01/05	09/01/06	05/01/07	–	–	–	–	–	–
6th RCT	2–6	A	d1–[8][171] d2–[172] d3–[173] d4–[123][174]	03/01/03	06/01/03	09/01/04	03/01/05	09/01/05	04/01/06	04/01/07	12/01/07	–	–
6th RCT	3–6	A	d1–[175][176] d2–[123][177] d3–[178]	09/01/05	04/01/06	01/01/07	09/01/07	04/01/08	12/01/08	–	–	–	–
7th RCT	1–7	A	d1–[8] [179] d2–[180] d3–[181][182] d4–[158] d5–[123]	03/01/03	10/01/03	09/01/04	04/01/05	03/01/06	09/01/06	08/01/07	03/01/08	05/01/09	12/01/09
7th RCT	2–7	A	d1–[131][183] d2–[184] d3–[185]	02/01/04	09/01/04	07/01/05	02/01/06	01/01/07	08/01/07	–	–	–	–
7th RCT	3–7	A	d1–[8][186] d2–[131][187] d3–[188] d4–[189][190] d5–[191]	03/01/03	09/01/03	02/01/04	09/01/04	09/01/05	03/01/06	05/01/07	01/01/08	09/01/08	04/01/09
8th RCT	1–8	A	d1–[8] d2-[192] d3–[193]	03/01/03	06/01/03	07/01/04	02/01/05	10/01/07	05/01/08	–	–	–	–
8th RCT	2–8	A	d1–[8][194] d2–[114] d3–[123][195]	03/01/03	06/01/03	08/01/06	03/01/07	10/01/07	05/01/08	–	–	–	–
8th RCT	3–8	A	d1–[136][196] d2–[196] [197]	01/01/05	08/01/05	03/01/06	10/01/06	–	–	–	–	–	–
9th RCT	1–9	A	[198][199]	03/01/08	10/01/08	–	–	–	–	–	–	–	–
9th RCT	2–9	A	[200]	10/01/08	05/01/09	–	–	–	–	–	–	–	–
9th RCT	3–9	A	[123][201]	05/01/09	12/01/09	–	–	–	–	–	–	–	–
23rd RCT	1–23	R	[202] [203]	09/01/04	03/01/05	–	–	–	–	–	–	–	–
23rd RCT	2–23	R	d1–[204] d2–[205]	03/01/03	06/01/03	11/01/08	06/01/09	–	–	–	–	–	–
23rd RCT	3–23	R	d1–[206] [207] d2–[206][208]	04/01/03	10/01/03	10/01/07	05/01/08	–	–	–	–	–	–
24th RCT	1–24	R	[209][210]	10/01/06	05/01/07	–	–	–	–	–	–	–	–
24th RCT	2–24	R	d1–[211] d2–[212]	08/01/04	04/01/05	01/01/08	08/01/08	–	–	–	–	–	–
24th RCT	3–24	R	[213]	03/01/04	10/01/04	–	–	–	–	–	–	–	–
25th RCT	1–25	R	[214][209]	03/01/06	10/01/06	–	–	–	–	–	–	–	–
25th RCT	2–25	R	d1–[215] d2–[216]	03/01/03	08/01/03	10/01/08	05/01/09	–	–	–	–	–	–
25th RCT	3–25	R	[217]	03/01/05	09/01/05	–	–	–	–	–	–	–	–

Table 3: Marine deployment dataset. For each battalion, the start and end dates of each deployment. The notation di stands for the i^{th} deployment for i=1,2,3,4,5.

Type of brigade	pre-modular	modular
Heavy	3500	3700
Light	2500	3200
Stryker	3900	3900
ACR	4800	4800

Table 4: Estimated number of personnel per brigade [1].

Date	t	Army	Marines
03/2003	01	99,664	66,166
06/2003	04	179,320	22,885
09/2003	07	152,815	6545
12/2003	10	138,120	2557
03/2004	13	155,291	25,568
06/2004	16	120,703	32,636
09/2004	19	101,932	35,216
12/2004	22	135,700	30,500
03/2005	25	121,400	30,500
06/2005	28	113,600	23,100
09/2005	31	132,400	25,900
12/2005	34	137,600	27,400
03/2006	37	105,100	26,700
06/2006	40	103,300	23,300
09/2006	43	119,500	25,600
12/2006	46	100,200	23,200
06/2007	52	125,300	26,700
09/2007	55	138,500	31,300
12/2007	58	125,800	26,900
03/2008	61	126,000	27,100
06/2008	64	117,100	24,500

Table 5: Monthly OIF troop deployments for Army and Marines [224].

Date	t	Troops
07/2008	65	140,000
08/2008	66	140,000
09/2008	67	140,000

Table 6: Recent monthly OIF troop deployments (Army plus Marines) [225].

Date	t	Fraction Active
03/2003	01	0.82
09/2003	07	0.88
10/2004	20	0.54
10/2005	32	0.69
09/2006	43	0.79

Table 7: Fraction of Army soldiers that are active [224, 226].

Date	t	Army	Marines
10/2005	32	0.55	–
09/2006	43	0.71	0.67

Table 8: Fraction of troops on first deployment [226].

Date	t	Army	Marines
09/2006	43	9	6

Table 9: Median number of months deployed during current deployment [226].

Date	t	First Deployment	Second Deployment
10/2005	32	10	20

Table 10: Mean total number of months deployed (over all OIF deployments) [227].

Constraint Type	Assigned Weight
DOD troop totals	5
DOD post-surge troop level estimates	1
Fraction of Army soldiers active	20
Mean service time	1
Fraction on first deployment	20

Table 11: Constraint weights for unattached support troop estimation. DOD troop totals occur with lower frequency and are presumably more reliable than the post-surge estimates, so they have a higher weight in our model. Other weights were chosen such as to keep violation of any constraint less than 10%.

Month t	$A_1(t)$	$A_2(t)$	$A_3(t)$	Month t	$A_1(t)$	$A_2(t)$	$A_3(t)$	Month t	$A_1(t)$	$A_2(t)$	$A_3(t)$
-4	25089	5123	14448	20	0	0	7336	44	1724	1040	0
-3	0	5123	8921	21	24913	2922	0	45	1724	1040	0
-2	0	0	0	22	0	2922	1195	46	1724	1040	0
-1	3145	0	4131	23	0	0	2317	47	0	0	0
0	0	0	2471	24	0	0	0	48	0	0	0
1	0	10130	3263	25	0	0	0	49	0	0	0
2	11477	0	0	26	0	0	0	50	0	0	2094
3	0	0	0	27	0	0	0	51	0	0	0
4	0	0	0	28	0	0	0	52	0	0	0
5	0	0	0	29	0	1528	0	53	0	0	55
6	245	0	664	30	0	1528	0	54	0	0	55
7	245	0	0	31	0	1528	0	55	0	765	0
8	0	33860	0	32	0	21724	0	56	528	528	0
9	0	0	0	33	0	0	0	57	528	528	0
10	0	0	0	34	0	0	0	58	528	528	0
11	1388	3853	0	35	2422	0	0	59	0	0	0
12	0	3853	0	36	2422	0	0	60	0	0	1224
13	0	3853	0	37	2422	0	0	61	0	0	334
14	0	0	0	38	7746	0	6652	62	0	0	519
15	0	0	0	39	0	0	0	63	0	0	519
16	0	0	0	40	0	0	0	64	0	0	519
17	0	0	0	41	5977	0	781	65	154	154	154
18	0	0	0	42	5977	0	1062	66	3803	3803	3803
19	0	0	1242	43	5977	0	1323	67	0	0	0

Table 12: The solution $A_j(t)$ to (19)-(21), where month $t = 1$ corresponds to March 2003.

Date	Month	Computed Fraction Active	Reported Fraction Active
03/2003	01	0.770	0.820
9/2003	07	0.850	0.880
10/2004	20	0.610	0.540
10/2005	32	0.616	0.690
09/2006	43	0.799	0.790

Table 13: Computed vs. reported fraction of Army soldiers that are active.

Type	Computed	Reported	Computed Combat + Attached	Computed Unattached
Army	9	9	9	7
Marines	6	6	2	6

Table 14: Computed vs. reported median number of months deployed during current deployment in September 2006 ($t = 43$). The computed median number of months is also broken down for combat plus attached troops vs. unattached troops.

Type	Computed	Reported	Computed Combat + Attached	Computed Unattached
First Deployers	8.48	10	5.45	12.00
Multiple Deployers	17.89	20	15.79	20.03

Table 15: Computed vs. reported mean number of months deployed over all OIF deployments in October 2005 ($t = 32$). The computed mean is also broken down for combat plus attached troops vs. unattached troops.

Type	Date	Month	Computed	Reported	Computed Combat + Attached	Computed Unattached
Army	10/2005	32	0.50	0.55	0.52	0.48
Army	09/2006	43	0.69	0.71	0.49	0.98
Marines	09/2006	43	0.66	0.67	0.56	0.76

Table 16: Computed vs. reported fraction of troops on first deployment. The computed fraction is also broken down for combat plus attached troops vs. unattached troops.

Month t	$\lambda_1(t)$	$\lambda_3(t)$	Month t	$\lambda_1(t)$	$\lambda_3(t)$	Month t	$\lambda_1(t)$	$\lambda_3(t)$
1	0.0022	0.0072	24	0.0049	0.0061	47	0.0092	0.0099
2	0.0040	0.0066	25	0.0040	0.0042	48	0.0067	0.0146
3	0.0015	0.0026	26	0.0058	0.0089	49	0.0081	0.0103
4	0.0017	0.0014	27	0.0064	0.0140	50	0.0094	0.0098
5	0.0030	0.0021	28	0.0062	0.0154	51	0.0114	0.0061
6	0.0025	0.0013	29	0.0054	0.0082	52	0.0101	0.0052
7	0.0030	0.0000	30	0.0063	0.0148	53	0.0070	0.0080
8	0.0048	0.0000	31	0.0051	0.0070	54	0.0073	0.0054
9	0.0062	0.0000	32	0.0064	0.0163	55	0.0053	0.0029
10	0.0039	0.0004	33	0.0046	0.0142	56	0.0037	0.0012
11	0.0035	0.0002	34	0.0047	0.0093	57	0.0031	0.0006
12	0.0019	0.0003	35	0.0037	0.0086	58	0.0028	0.0001
13	0.0029	0.0085	36	0.0040	0.0095	59	0.0037	0.0012
14	0.0085	0.0333	37	0.0046	0.0084	60	0.0029	0.0010
15	0.0069	0.0113	38	0.0054	0.0164	61	0.0043	0.0009
16	0.0047	0.0094	39	0.0052	0.0161	62	0.0048	0.0026
17	0.0046	0.0121	40	0.0058	0.0116	63	0.0022	0.0025
18	0.0064	0.0194	41	0.0050	0.0107	64	0.0026	0.0021
19	0.0071	0.0154	42	0.0057	0.0148	65	0.0018	0.0013
20	0.0054	0.0147	43	0.0074	0.0153	66	0.0019	0.0022
21	0.0075	0.0468	44	0.0081	0.0243	67	0.0022	0.0004
22	0.0046	0.0144	45	0.0063	0.0157			
23	0.0054	0.0184	46	0.0102	0.0193			

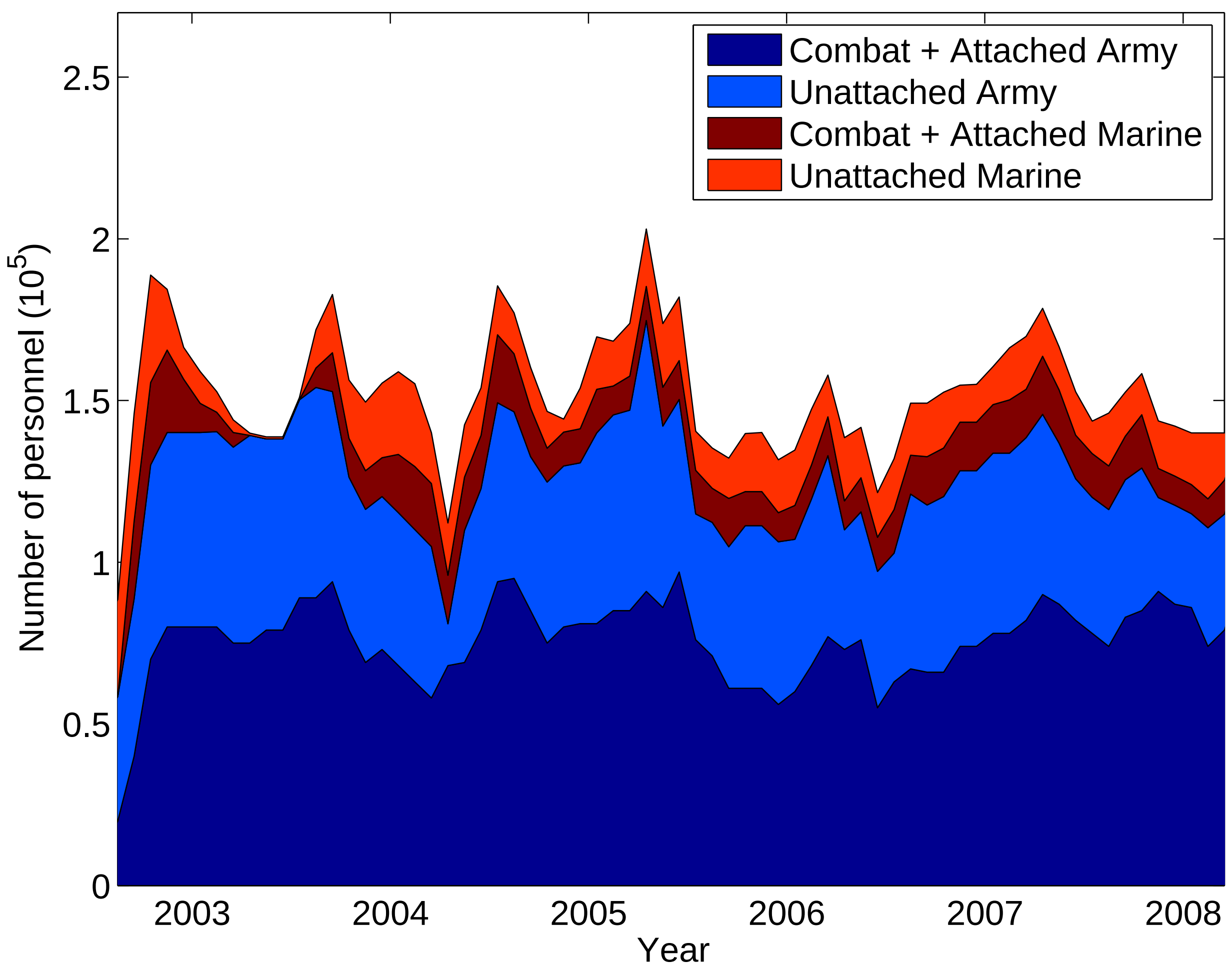
Table 17: Monthly mean stress levels for Army ($\lambda_1(t)$) and Marines ($\lambda_3(t)$), where month $t = 1$ corresponds to March 2003.

PTSD Prevalence Rate	Reported	Base	A	B	C	D	E	F	G	References
$P_{1+2}(7)$	0.158	0.072	0.075	0.072	0.086	0.072	0.070	0.070	0.073	Pg. 20, [227]
$P_{1+2}(20)$	0.113	0.109	0.112	0.110	0.144	0.109	0.106	0.104	0.112	Pg. 20, [227]
$P_{1+2}(32)$	0.136	0.133	0.137	0.092	0.134	0.136	0.130	0.131	0.133	Pg. 20, [227]
$P_1(32)$, 1st deployment	0.125	0.127	0.127	0.126	0.168	0.126	0.122	0.120	0.129	Pg. 21, [227]
$P_1(32)$, >1st deployment	0.184	0.178	0.189	0.046	0.106	0.189	0.180	0.185	0.174	Pg. 21, [227]
$P_{1+2}(43)$	0.170	0.148	0.151	0.090	0.146	0.152	0.146	0.148	0.147	Fig. 6, [226]
$P_3(43)$	0.140	0.166	0.163	0.119	0.195	0.174	0.167	0.164	0.168	Fig. 6, [226]
$P_{1+2}(43)$, low exposure to trauma	0.080	0.026	0.050	0.018	0.020	0.026	0.026	0.026	0.026	Fig. 7a, [226]
$P_{1+2}(43)$, medium exposure to trauma	0.140	0.139	0.136	0.083	0.135	0.135	0.129	0.133	0.136	Fig. 7a, [226]
$P_{1+2}(43)$, high exposure to trauma	0.280	0.281	0.267	0.168	0.283	0.296	0.283	0.284	0.280	Fig. 7a, [226]
$P_3(43)$, low exposure to trauma	0.060	0.050	0.050	0.037	0.045	0.050	0.050	0.050	0.050	Fig. 7b, [226]
$P_3(43)$, medium exposure to trauma	0.110	0.159	0.167	0.105	0.185	0.152	0.144	0.146	0.156	Fig. 7b, [226]
$P_3(43)$, high exposure to trauma	0.280	0.291	0.272	0.214	0.355	0.321	0.308	0.297	0.299	Fig. 7b, [226]
$P_{1+2}(43)$, 1st deployment	0.150	0.110	0.112	0.110	0.143	0.110	0.106	0.105	0.112	Fig. 8, [226]
$P_{1+2}(43)$, >1st deployment	0.240	0.236	0.239	0.045	0.152	0.249	0.237	0.246	0.227	Fig. 8, [226]
$P_{1+2}(43)$, $\leq$6 mo on cur. dep.	0.120	0.118	0.119	0.054	0.099	0.119	0.115	0.118	0.117	Fig. 9, [226]
$P_{1+2}(43)$, >6 mo on cur. dep.	0.190	0.174	0.178	0.121	0.187	0.181	0.173	0.174	0.174	Fig. 9, [226]
Fraction of troops exposed to no combat	0.320	0.304	0.335	0.304	0.282	0.304	0.304	0.303	0.304	[229]
Root mean square error		0.030	0.028	0.078	0.049	0.031	0.031	0.030	0.030	

Table 18: Reported PTSD prevalence rates and estimated PTSD prevalence rates for the base-case Poisson model ($\alpha = 146.7$, $\gamma = 7.72$, $b = 0.0626$, and $\theta = 0$) as well as several variations described in §3.2 and §3.3. $P_j(t)$ represents the probability of a type j servicemember having symptomatic PTSD in month t, where $j = 1 + 2$ represents active and reserve Army soldiers. **(A)** Probit model, $\alpha = 8.495 \times 10^6$, $\text{ID}_{50} = 0.0447$ $\beta = 0.124$, $b = 0.0682$, and $\theta = 1$; **(B)** Poisson model with servicemembers returning to civilian status according to their stress-to-threshold ratio, base-case parameters, $\alpha = 146.7$, $\gamma = 7.72$, $b = 0.0626$, and $\theta = 0$; **(C)** Poisson model with servicemembers returning to civilian status according to their stress-to-threshold ratio, parameters reestimated, $\alpha = 306.6$, $\gamma = 16.14$, $b = 0.0588$, and $\theta = 1$; **(D)** Poisson base-case model with $\theta = 1$, $\alpha = 146.7$, $\gamma = 7.72$, $b = 0.0626$; **(E)** Poisson model, θ constrained to 1, other parameters reestimated, $\alpha = 132.9$, $\gamma = 6.99$, $b = 0.0625$, and $\theta = 1$; **(F)** Poisson model, μ_1 and μ_2 increased by ln(2), $\alpha = 197.0$, $\gamma = 10.37$, $b = 0.0624$, and $\theta = 1$; **(G)** Poisson model, μ_1 and μ_2 decreased by ln(2), $\alpha = 110.9$, $\gamma = 5.83$, $b = 0.0626$, and $\theta = 0$.

Number of stressful events	Fraction of servicemenbers
0	0.347
1	0.281
2	0.172
3	0.097
4	0.052
5	0.027
6	0.013
7	0.006
8	0.003
9	0.001
≥ 10	0.001

Table 19: Probability mass function for the number of stressful events servicemembers are exposed to during withdrawal scenario 2



Combat + Attached Army
Unattached Army
Combat + Attached Marine
Unattached Marine
2.5
2
1.5
1
0.5
0
Number of personnel (10^5)
2003
2004
2005
2006
2007
2008
Year

Figure 1:

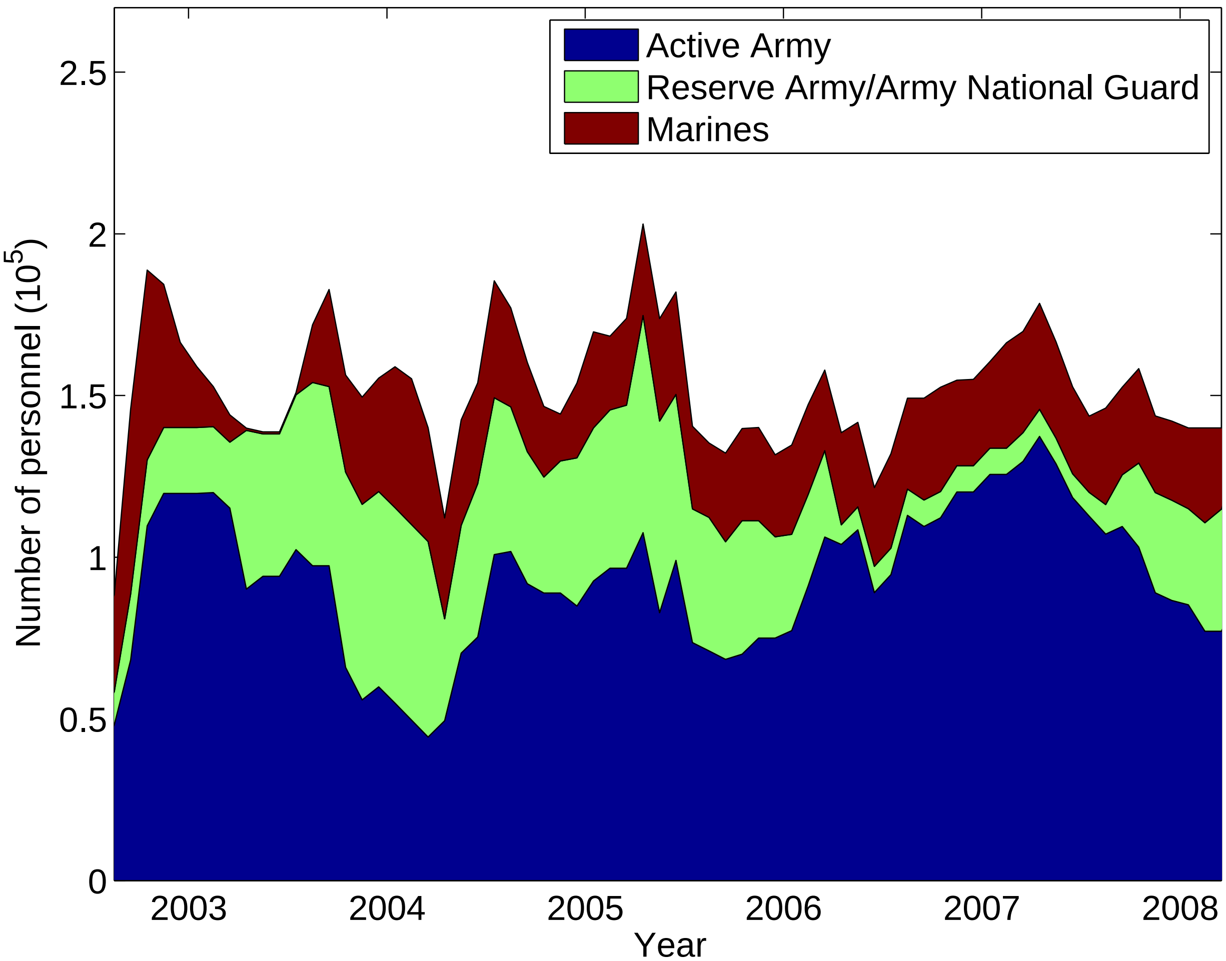
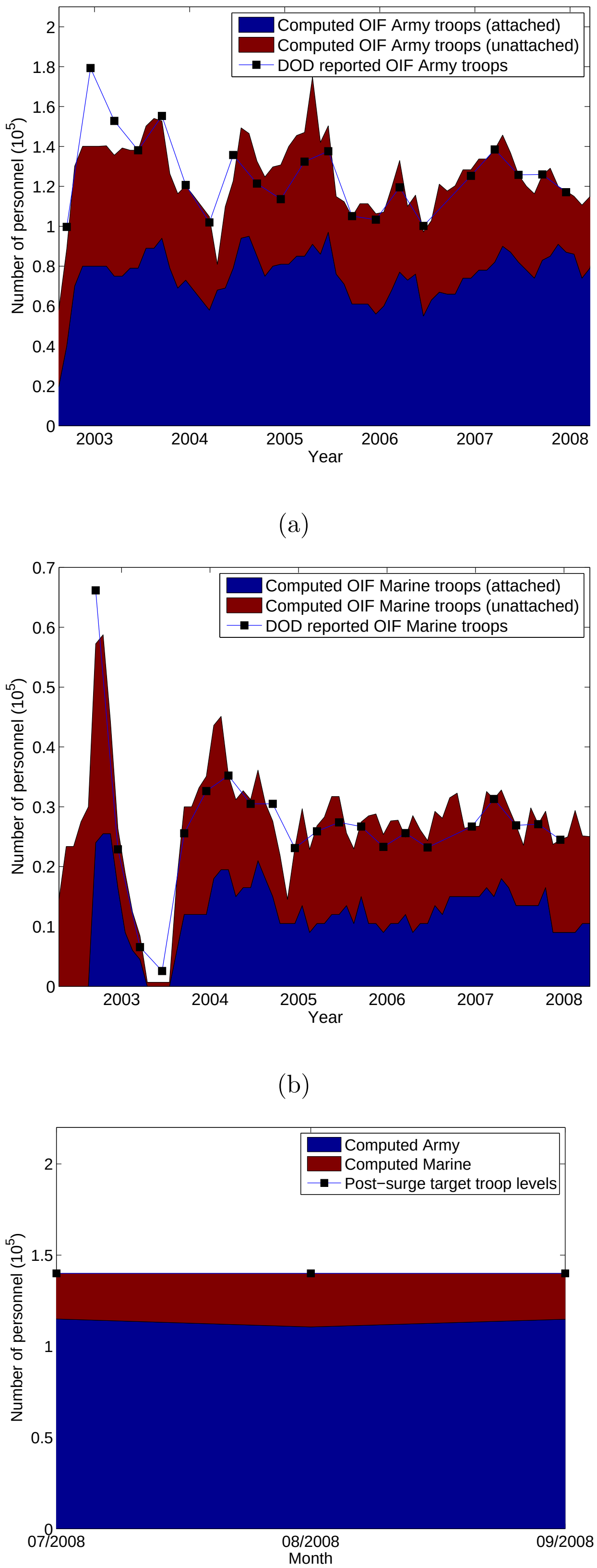

Active Army
Reserve Army/Army National Guard
Marines
Number of personnel (10^5)
0
0.5
1
1.5
2
2.5
2003
2004
2005
2006
2007
2008
Year

Figure 2:


Computed OIF Army troops (attached)
Computed OIF Army troops (unattached)
DOD reported OIF Army troops
Number of personnel (10^5)
Year
2003 2004 2005 2006 2007 2008
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

(a)

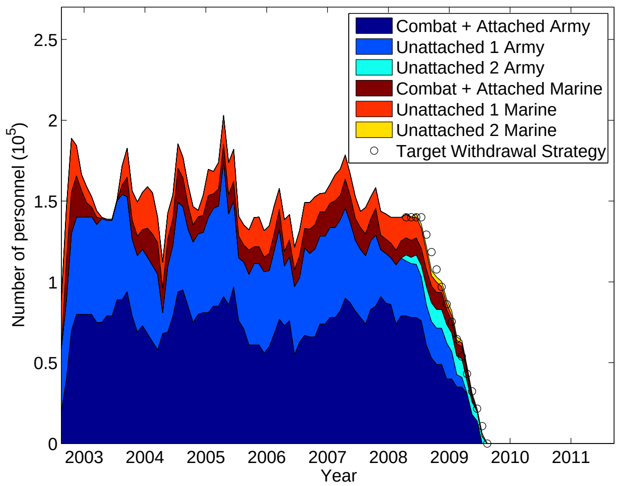
Computed OIF Marine troops (attached)
Computed OIF Marine troops (unattached)
DOD reported OIF Marine troops
Number of personnel (10^5)
Year
2003 2004 2005 2006 2007 2008
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7

(b)

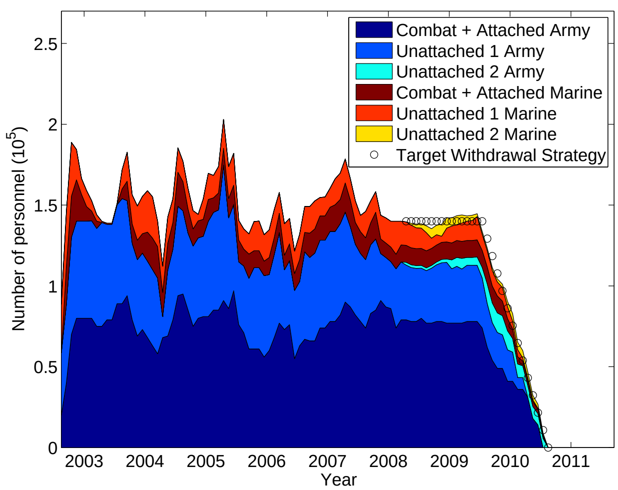
Computed Army
Computed Marine
Post−surge target troop levels
Number of personnel (10^5)
Month
07/2008 08/2008 09/2008
0 0.5 1 1.5 2

(c)

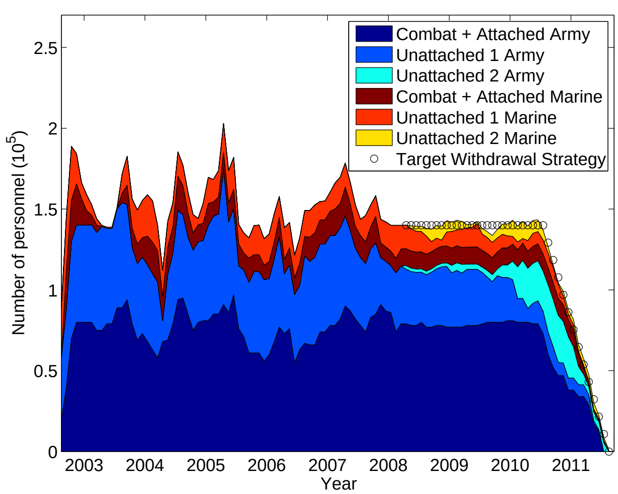
Figure 3:


Combat + Attached Army
Unattached 1 Army
Unattached 2 Army
Combat + Attached Marine
Unattached 1 Marine
Unattached 2 Marine
Target Withdrawal Strategy
Number of personnel (10^5)
0
0.5
1
1.5
2
2.5
2003
2004
2005
2006
2007
2008
2009
2010
2011
Year

(a)

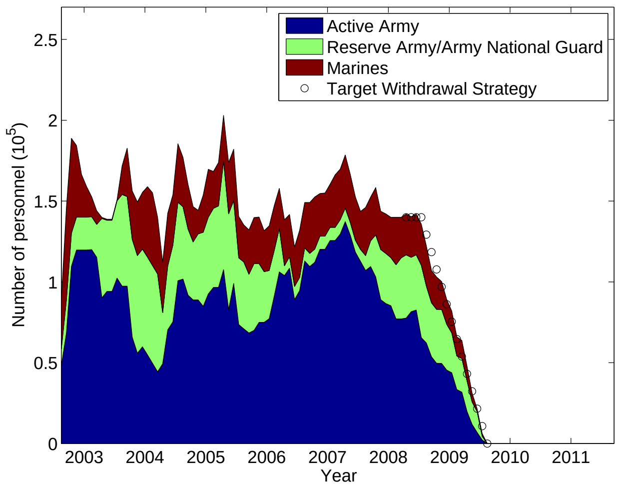

Combat + Attached Army
Unattached 1 Army
Unattached 2 Army
Combat + Attached Marine
Unattached 1 Marine
Unattached 2 Marine
Target Withdrawal Strategy
Number of personnel (10^5)
0
0.5
1
1.5
2
2.5
2003
2004
2005
2006
2007
2008
2009
2010
2011
Year

(b)


Combat + Attached Army
Unattached 1 Army
Unattached 2 Army
Combat + Attached Marine
Unattached 1 Marine
Unattached 2 Marine
Target Withdrawal Strategy
Number of personnel (10^5)
0
0.5
1
1.5
2
2.5
2003
2004
2005
2006
2007
2008
2009
2010
2011
Year

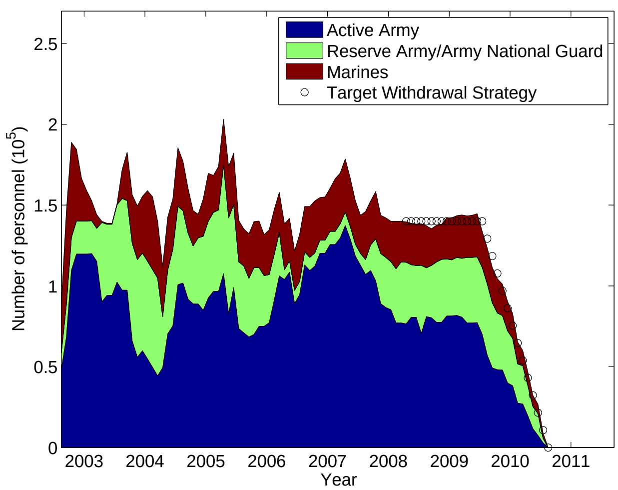
(c)

Figure 4:



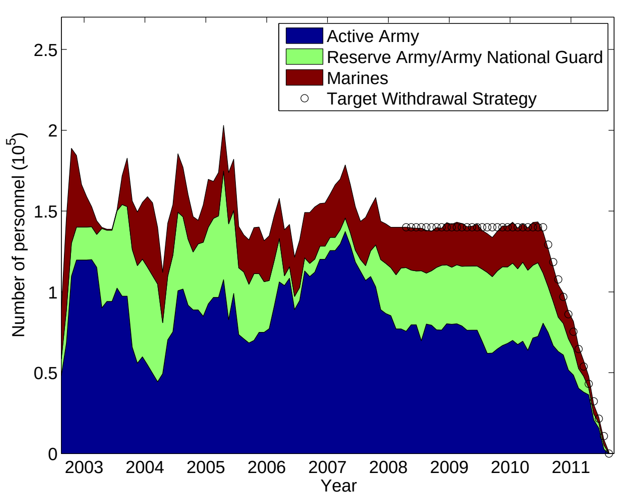
Active Army
Reserve Army/Army National Guard
Marines
Target Withdrawal Strategy
Number of personnel (10^5)
2.5
2
1.5
1
0.5
0
2003 2004 2005 2006 2007 2008 2009 2010 2011
Year

(a)



Active Army
Reserve Army/Army National Guard
Marines
Target Withdrawal Strategy
Number of personnel (10^5)
2.5
2
1.5
1
0.5
0
2003 2004 2005 2006 2007 2008 2009 2010 2011
Year

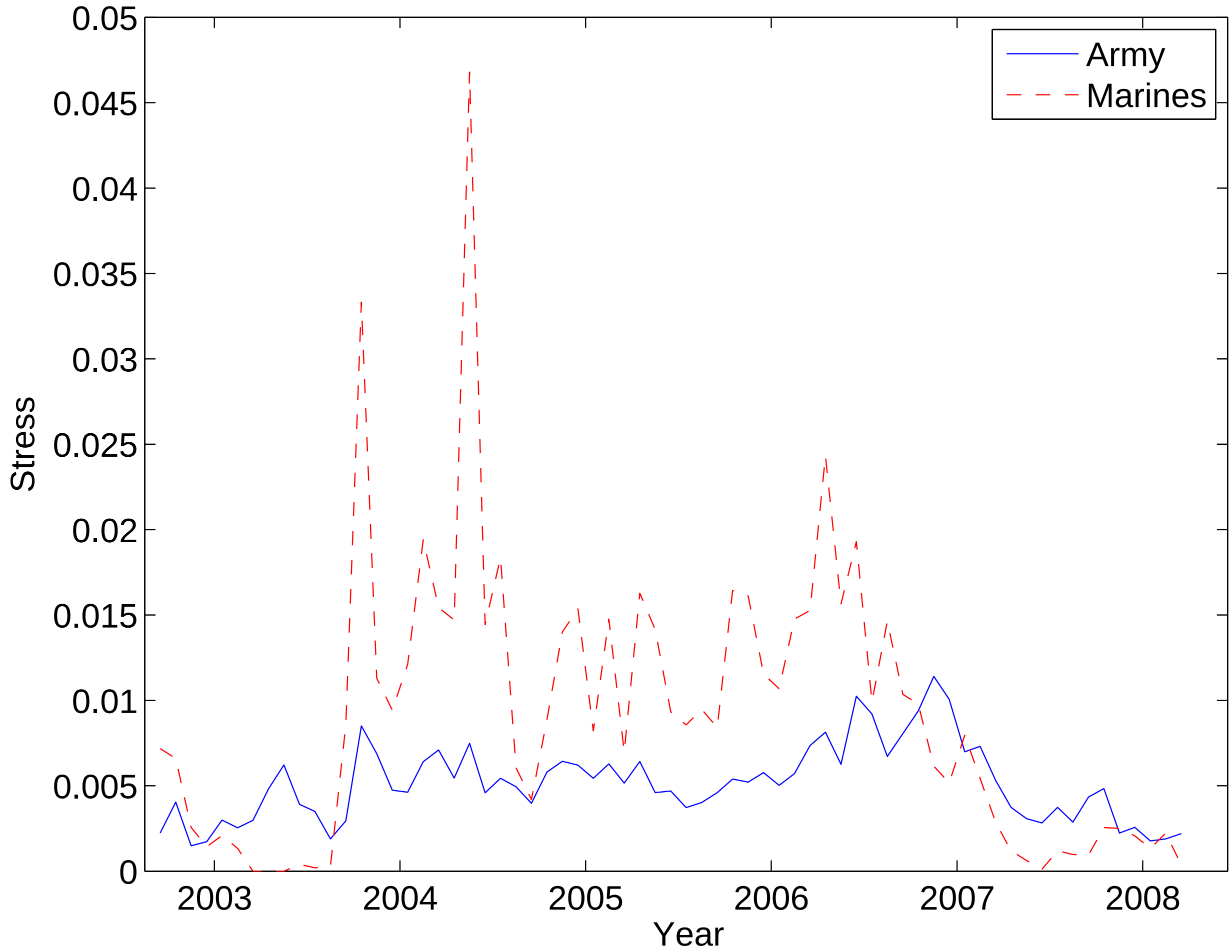
(b)



Active Army
Reserve Army/Army National Guard
Marines
Target Withdrawal Strategy
Number of personnel (10^5)
2.5
2
1.5
1
0.5
0
2003 2004 2005 2006 2007 2008 2009 2010 2011
Year

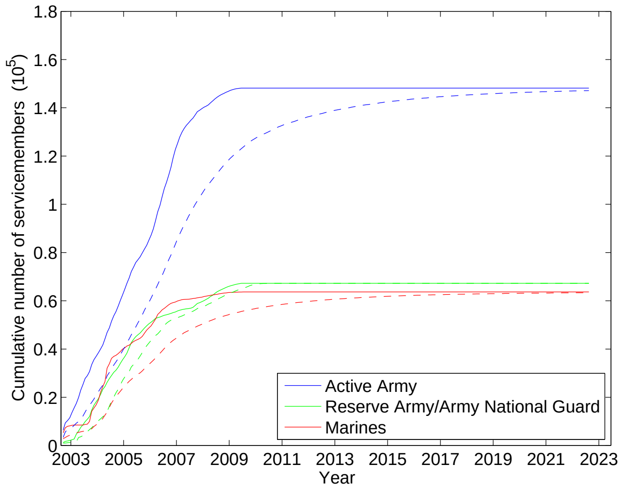
(c)

Figure 5:



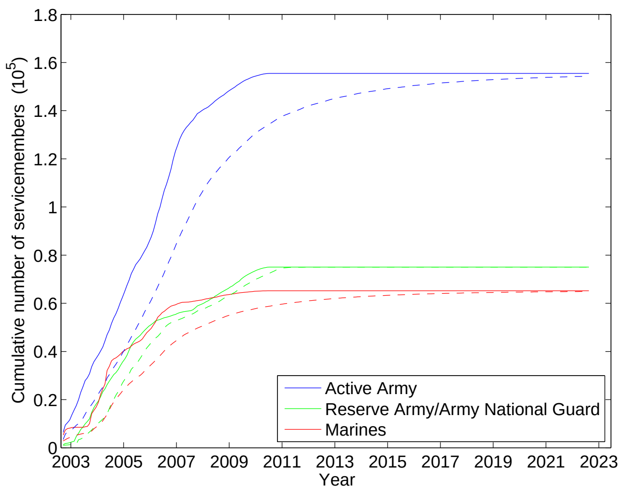
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Stress
2003
2004
2005
2006
2007
2008
Year
Army
Marines

Figure 6:



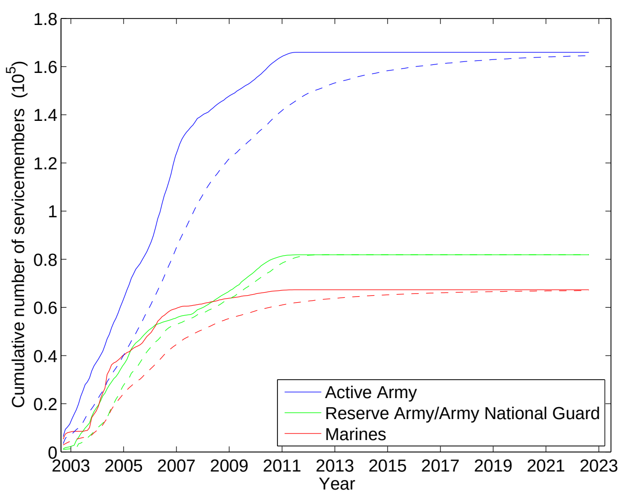
Cumulative number of servicemembers (10^5)
Year
Active Army
Reserve Army/Army National Guard
Marines

(a)



Cumulative number of servicemembers (10^5)
Year
Active Army
Reserve Army/Army National Guard
Marines

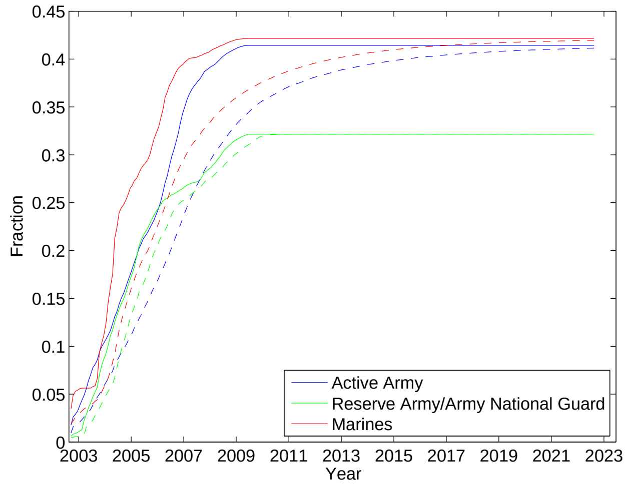
(b)



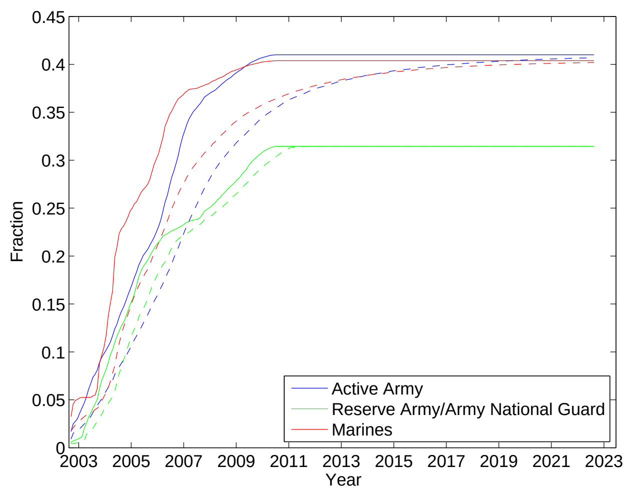
Cumulative number of servicemembers (10^5)
Year
Active Army
Reserve Army/Army National Guard
Marines

(c)

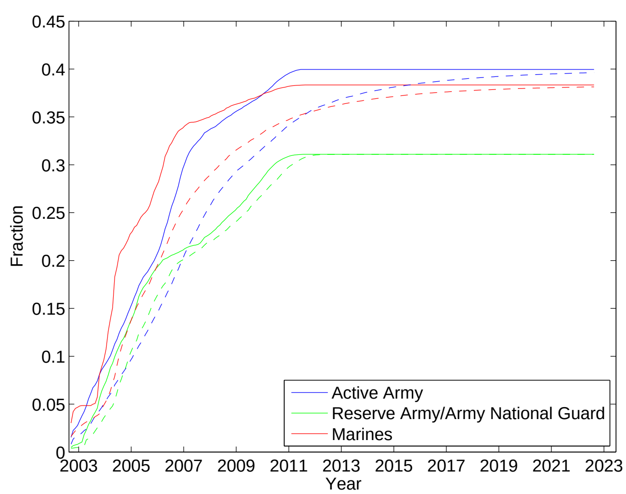
Figure 7:


Fraction
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
2003 2005 2007 2009 2011 2013 2015 2017 2019 2021 2023
Year
Active Army
Reserve Army/Army National Guard
Marines

(a)


Fraction
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
2003 2005 2007 2009 2011 2013 2015 2017 2019 2021 2023
Year
Active Army
Reserve Army/Army National Guard
Marines

(b)


Fraction
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
2003 2005 2007 2009 2011 2013 2015 2017 2019 2021 2023
Year
Active Army
Reserve Army/Army National Guard
Marines

(c)

Figure 8:

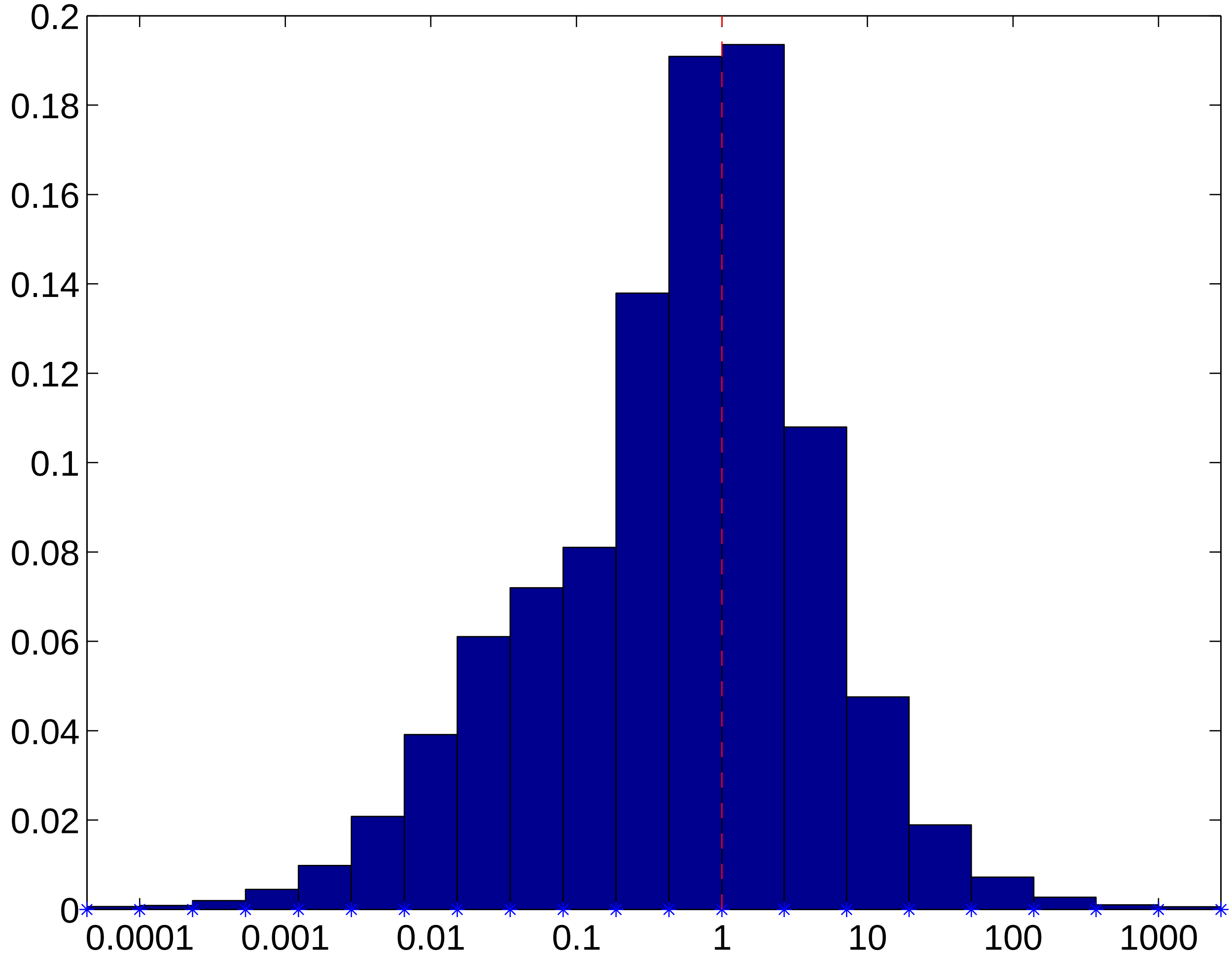
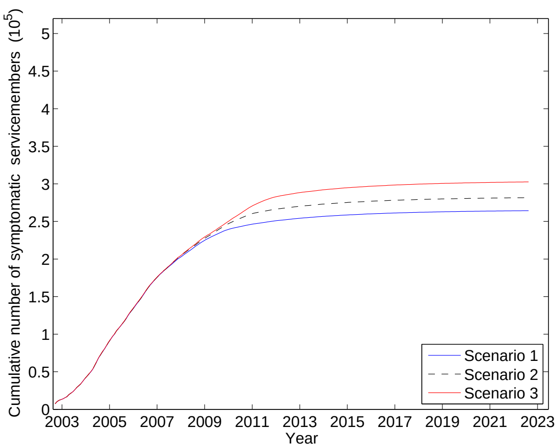
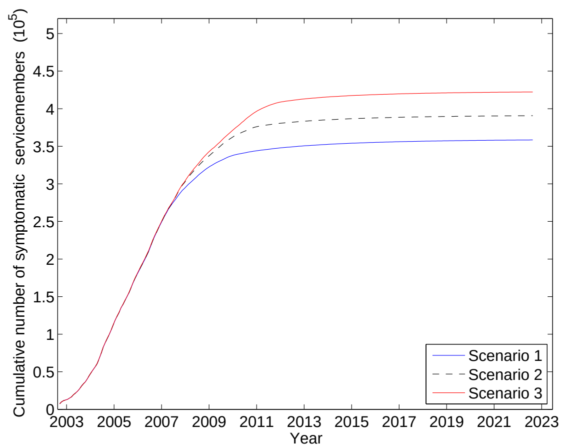

0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.0001
0.001
0.01
0.1
1
10
100
1000

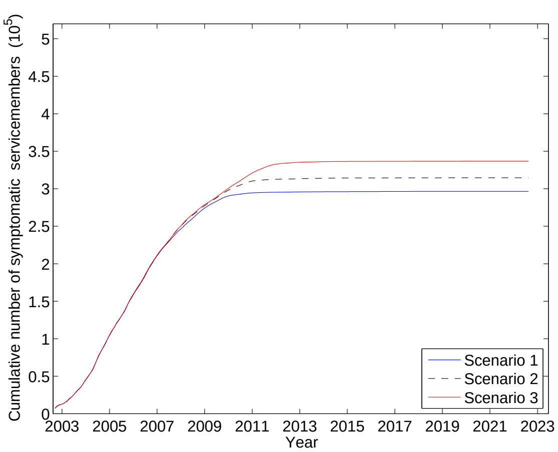
Figure 9:



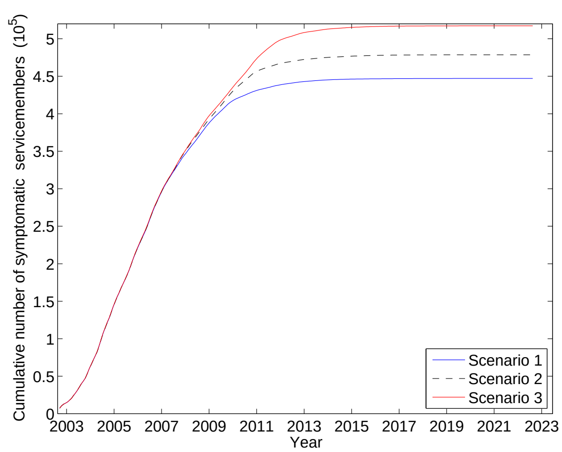
(a)



(b)

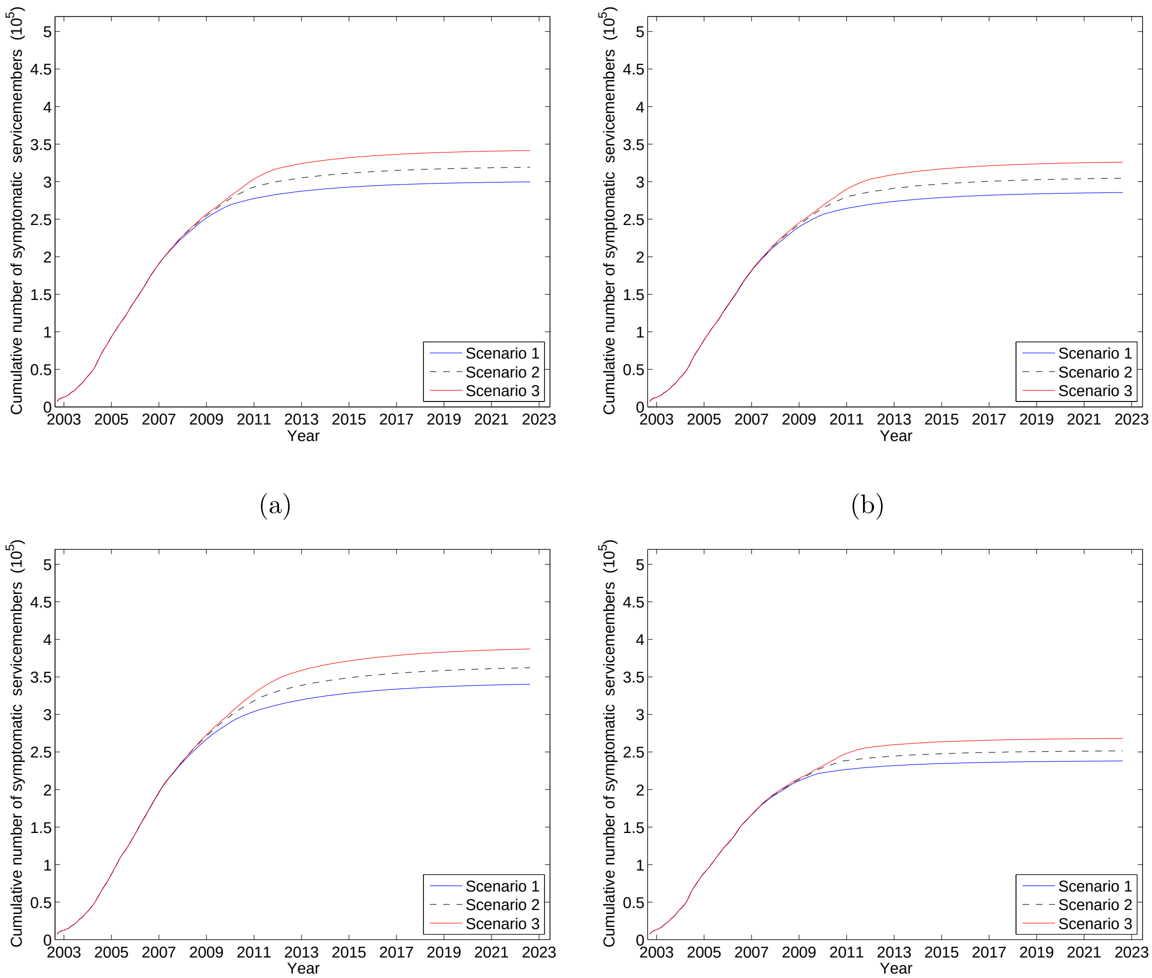


(c)



(d)

Figure 10:



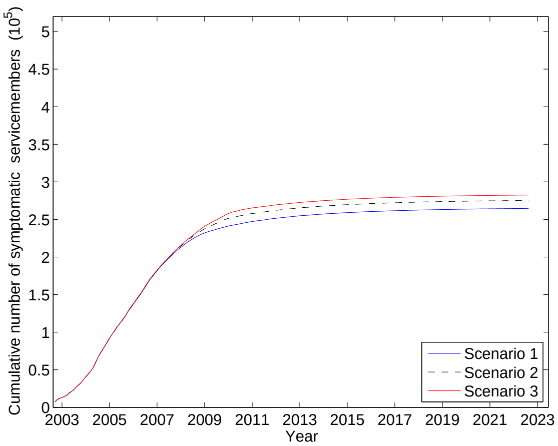
(a)

(b)

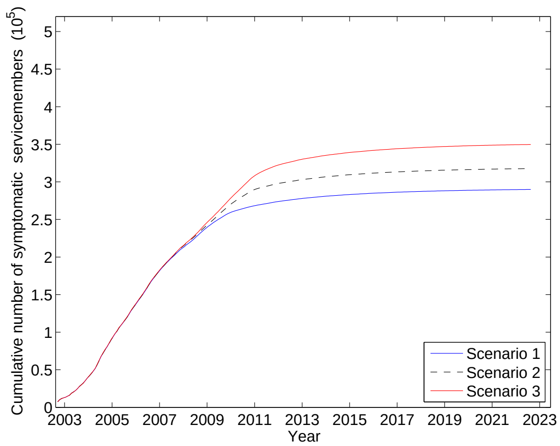
(c)

(d)

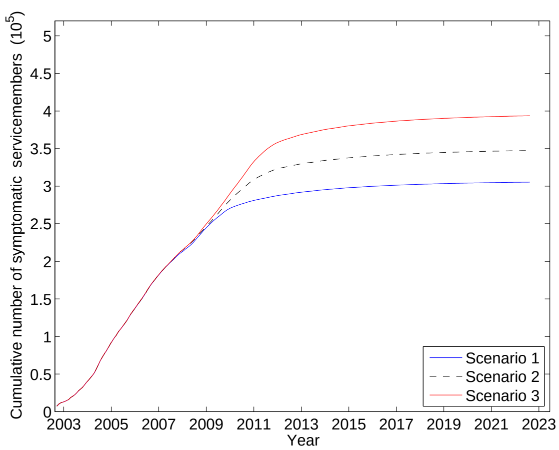
Figure 11:



(a)



(b)



(c)

Figure 12: