Hierarchy of Alternatives: The Case of the Project Objective Memorandum (POM)

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ABSTRACT

Some government and corporate decisions are hierarchical in two dimensions: a hierarchy of alternatives and a corresponding hierarchy of decision makers. An example of such a hierarchical decision process is the U.S. Department of Defense Program Objective Memorandum (POM), which sets development and acquisition plans within a given budget. At the top level of the hierarchy, senior leaders set directions for those acquisition and development plans, directions that can be viewed as or translated into families of portfolios called henceforth *Programs*. Programs comprise projects that are the eventual fundable entities. Although "hierarchy" is a core feature in this decision-making setup, it does not comply with the well-known analytic hierarchy process, where decision alternatives are at the bottom level of a hierarchy that also includes goals and criteria. In this article, we propose a modeling framework of a different type, where the hierarchy only comprises alternatives; the criteria, which may be alternatives dependent, are "orthogonal" to the levels of the hierarchy. We develop a methodology for handling such a decision setup and demonstrate its application in reference to the POM. The multicriteria-decision-analysis part of the methodology hinges on the widely used concept of least squares.

INTRODUCTION

We consider a hierarchy of decision alternatives, such as investment opportunities, development programs, and acquisition decisions, that are interconnected across the levels of the hierarchy. Specifically, an alternative at a certain mid-level of the hierarchy is associated with one or more alternatives at a higher level ("parent alternatives") and with one or more alternatives at a lower level ("child alternatives"). Except for the lowest level, each alternative is associated with a subset of child alternatives, and except for the highest level, each alternative is associated with a subset, which may comprise a singleton, of parent alternatives. This hierarchical structure of decision alternatives is motivated by the U.S. Department of Defense (DoD) Planning, Programming, Budgeting, and Execution (PPBE) process (Blickstein et al., 2016), and in particular, the Program Objective Memorandum (POM) (Paden, 2018). However, the proposed hierarchical structure can apply to decision

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APPLICATION AREAS:
Decision Theory, Analysis of Alternatives Cost Analysis

OR METHODS:
Linear/Integer Programming, Nonlinear Programming

processes in any large organization or corporation in which interconnected strategic, operational, and tactical decisions are taken by different levels of leadership and management.

While the term "hierarchy" here is shared with the well-known analytic hierarchy process (AHP) (Saaty, 1990), the decision setting here is profoundly different; our hierarchy only comprises decision alternatives, typical to a certain level of decision making, which are interconnected and shaped into a hierarchy. In this hierarchy, there are no criteria, subcriteria, etc., which are part of the hierarchy in the AHP. Criteria and subcriteria are applied at any level of the alternatives' hierarchy, where a multiple criteria analysis is performed on appropriately defined subsets of alternatives. This structure is described and demonstrated later.

Developing the PPBE and composing the POM are decision processes that fall under the general label of multiple criteria decision analysis (MCDA) (Koksalan et al., 2011; Goicoechea et al., 1992). These two decision processes also involve group decision making (Saaty and Peniwati, 2013; Hwang and Lin, 1986) and consensus formation among stakeholders.

The process producing the POM is a fiscally constrained prioritization exercise in which the DoD Services (Army, Navy, etc.) generate and/or change the contents of their defense programs to reflect updated planning priorities and budget constraints. The POM is a part of the PPBE process, which is the DoD's resource allocation mechanism for allocating defense budget. The POM is created annually with a five-year outlook, adjusting the existing force structure to achieve strategic goals in both the near and long terms (Blickstein et al., 2016).

There is an embedded hierarchy of decisions in the POM process, which is generally aligned with the chain of command. Senior leadership of the defense establishment sets general *Thrusts* for their respective Services. They give future directions for planning and actions, and prioritize among Program sets. Within a Thrust, flag officers set priorities among Programs related to that Thrust and determine programs of record. A Program may serve more than one Thrust. Senior staff officers within the relevant branches of the services determine priorities among *Projects* within Programs. Based on preferences among Thrusts at the top level, preferences among Programs associated with a certain Thrust (or several Thrusts) at the middle level, and preferences among Project within a Program, at the lowest level, the objective is to determine the best set of Projects to be funded within the budget constraints. This hierarchical structure motivates the model proposed in this paper.

MODEL

While the hierarchy of decision alternatives can have any number of levels, we assume here, without loss of generality, a three-level hierarchy, which is consistent with the levels in the DoD's POM. The model setting comprises two parallel hierarchies: *alternatives* and *decision makers*. As described earlier, the three levels of alternatives are *Thrusts, Programs*, and *Projects*. Top leadership of a military Service (or board of directors in a corporation) prioritizes Thrusts, flag officers (senior executive officers of a corporation) prioritize Programs, and senior staff officers (top management of a corporation) prioritize Projects. Obviously, each level of decision making can offer input to a higher level and can intervene in the prioritization taking place at a lower level.

The three levels in the hierarchy also differ in the resolution in which the decision alternatives are specified. Thrusts, prioritized by Service leadership (e.g., U.S. Navy's Chief of Naval Operations [CNO]), are typically characterized in broad-brush terms such as "readiness," "capacity," and "capabilities" (Chief of Naval Operations, 2022). Thrusts point at areas of importance in view of current and future threats and may be manifested in statements such as "increase readiness of surface forces," "enhance capabilities of undersea warfare," or "focus on air warfare." A Thrust is connected with one or more Programs, which affect the realization and effect of that Thrust. For example, various Programs of aircraft development, such as fighter aircrafts, unmanned aerial vehicles (UAVs), and helicopters, serve the Air Warfare Thrust. Cyber defense and cyber attack Programs may serve a cyberwafare Thrust. A Program may comprise several Projects that need to be prioritized. Figure 1 presents an example of such a hierarchy comprising

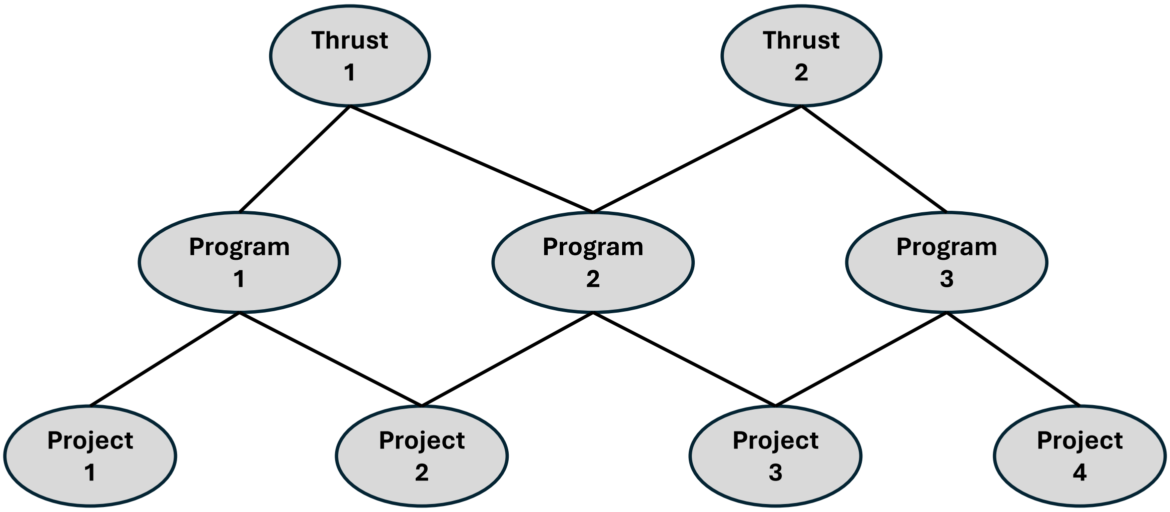

Figure 1. Decision alternatives hierarchy.

two Thrusts, three Programs, and four Projects. As we can see, the interrelations are general; a Thrust may have more than one associated Program, a Program may serve more than one Thrust, and similar interrelations may apply to Programs and Projects.

Next, we formalize the interrelations in the alternatives' hierarchy and show how we evaluate the Projects, which are the fundable entities. Define T, P, and E as the sets of Thrusts, Programs, and Projects, respectively. We say that Thrust $T_i \in \mathrm{T}$ is *associated* with Program $P_j \in \mathrm{P}$ if the former is a "parent alternative" of the latter. Similarly, we define association between a Program $P_j \in \mathrm{P}$ and Project $E_k \in \mathrm{E}$. In other words, two alternatives in two adjacent levels are associated if they form a "parent-child" relation. For example, a possible Thrust alternative "Cyber Warfare" is associated with Programs "Firewall" and "Cyber Offense," and the Program "Firewall" comprises the Projects "Iron Curtain," "Steel Fence," and "Rocky Passage." The Program "Firewall" may also be associated with the Thrust "Information Security." Obviously, this example, and its associated Projects' names, are made up just for illustration.

Let T_i^C denote the set of Programs associated with Thrust T_i (set of children of T_i) and P_j^F the set of Thrusts associated with Program j (set of parents of P_j). Similarly, we define the set P_j^C of children $E_k \in \mathrm{E}$ of Program j, and the set E_k^F of Program parents of Project k. We assume that each Thrust has at least one Program child, each Program has at least one Thrust parent and at least one Project child, and each Project has at least one parent (see Figure 1). Note that $\bigcup_{T_i \in \mathrm{T}} T_i^C = \bigcup_j P_j = \mathrm{P}$, $\bigcup_{P_j \in \mathrm{P}} P_j^F = \bigcup_{T_i \in \mathrm{T}} T_i = \mathrm{T}$, $\bigcup_{P_j \in \mathrm{P}} P_j^C = \bigcup_{E_k \in \mathrm{E}} E_k = \mathrm{E}$, and $\bigcup_{E_k \in \mathrm{E}} E_k^F = \bigcup_{P_j \in \mathrm{P}} P_j = \mathrm{P}$. Referring to Figure 1, we have the following: $T_1^C = \{P_1,P_2\}$, $T_2^C = \{P_2,P_3\}$, $P_1^F = \{T_1\}$, $P_2^F = \{T_1,T_2\}$, $P_3^F = \{T_2\}$, $P_1^C = \{E_1,E_2\}$, $P_2^C = \{E_2,E_3\}$, $P_3^C = \{E_3,E_4\}$, $E_1^F = \{P_1\}$, $E_2^F = \{P_1,P_2\}$, $E_3^F = \{P_2,P_3\}$, $E_4^F = \{P_3\}$.

The alternatives in the set of Thrusts and in each of the subsets of Programs and Projects are evaluated by a combined MCDA and group decision model described later. At the end of the evaluation stage, we obtain ratings of the alternatives for each of the following sets of alternatives: (i) set of Thrusts T, (ii) each subset of Programs T_i^C, and (iii) each subset of Projects P_j^C. Note that the MCDA evaluation is performed for each aforementioned subset separately, with respect to specially tailored set of criteria, and perhaps by different sets of stakeholders and decision makers.

Let $u_i > 0$ denote the value of Thrust $T_i \in \mathrm{T}$, $v_{ij} > 0$ denote the value of Program $P_j \in T_i^C$ when evaluated with respect to Thrust $T_i \in \mathrm{T}$, and $w_{jk} > 0$ denote the value of Project $E_k \in P_j^C$ when evaluated with respect to Program $P_j \in P$. Note that a certain Program P_j may belong to two or more Thrusts, that is, the subsets P_j^F may not be mutually exclusive. For example, in Figure 1, the subsets

P_1^F and P_2^F contain the mutual alternative Thrust T_1. The same relation may also occur for the subsets of Projects E_k^F with respect to Programs. In the next section we describe how we obtain the values of u_i, v_{ij} and w_{jk}.

Suppose there are 100 units of "value" to be distributed among the alternatives at each level of the hierarchy. That is, $\sum_i u_i = \sum_{i,j} v_{ij} = \sum_{j,k} w_{jk} = 100,\ \ u_i, v_{ij}, w_{jk} > 0$. The value u_i of Thrust T_i is distributed among its child Programs in T_i^C such that

$$\sum_{P_j \in T_i^C} v_{ij} = u_i. \tag{1}$$

The overall value of Program P_j is

$$V_j = \sum_{T_i \in P_j^F} v_{ij}. \tag{2}$$

Similarly, we distribute the value V_j of Program P_j among its child Projects such that

$$\sum_{E_k \in P_j^C} w_{jk} = V_j. \tag{3}$$

The overall value of Project E_k is

$$W_k = \sum_{P_j \in E_k^F} w_{jk}. \tag{4}$$

It is easily seen that by this construction, $\sum_i u_i = \sum_j V_j = \sum_k W_k = 100$.

In addition to incorporating the relative standing of a Project within its subset of Projects, the value W_k also reflects the values of the alternatives—Programs and a Thrusts—up the alternatives' hierarchy.

For example, consider Project 2 (E_2 in our notation) in Figure 1. Its "lineage" includes Programs 1 and 2, and both Thrusts. It supports two parent Programs—Program 1 (P_1 in our notation) and Program 2 (P_2 in our notation). Program 1 supports only one Thrust, T_1, but Program 2 supports both Thrusts. From Equations (1) and (2), we have that:

$$W_2 = w_{12} + w_{22} = V_1 + V_2 - (w_{11} + w_{23}), \tag{5}$$

where $V_1 = v_{11}$ and $V_2 = u_1 - v_{11} + v_{21}$. So we can see that the full lineage is manifested in the value of Project 2.

The objective is to maximize the overall values of the funded Projects subject to a budget constraint and possibly other constraints, as discussed in the "Optimization" section.

OBTAINING ALTERNATIVES' RATINGS

We first describe the general model for obtaining ratings in a multiple criteria multiple-stakeholders setting, and then we apply it to our hierarchy of alternatives to obtain the values u, V, and W. Consider a set of N alternatives evaluated with respect to L criteria by a group of S stakeholders. Similarly to the AHP method (Saaty, 1990), we assume that stakeholders express their preferences by ratio-scale matrices (Golany and Kress, 1993). A ratio-scale matrix is a square matrix that comprises positive entries r_{ij} such that $r_{ij} = r_{ji}^{-1}$. Each stakeholder conveys their preferences by L $(N \times N)$ ratio-scale matrices that provide pairwise comparisons of the N alternatives—one for each criterion—and one $(L \times L)$ matrix for the pairwise comparisons of the criteria's weights.

Let $r_{ij}^l(s)$ denote the extent alternative i is preferred to alternative j, $\ i, j = 1, \ldots, N$, with respect to criterion l, $l = 1, \ldots, L$ by stakeholder $s, s = 1, \ldots, S$. Similarly, we define $d_{lm}(s)$ as the extent criterion l is considered more important than criterion m by stakeholder s. There are several ways to obtain the ratings for alternatives, and weights for criteria, from ratio-scale matrices (Golany and Kress, 1993). For example, The AHP model obtains these values and weights by computing the principal eigenvectors of the ratio-scale matrices (Saaty, 1990). Here we propose a different approach based on extremal principles, namely, least squares. It has been shown that the least squares method performs as well as other scaling methods (Golany and Kress, 1993) and it is widely used in other contexts, for example, regression analysis in statistics. Most importantly, it is intuitive and naturally addresses scaling of preferences by multiple decision makers and obtaining group decision consensus. The least squares model is a quadratic separable optimization model where the objective is to find the set of ratings that are closest, in the Euclidean distance sense, to the pairwise comparisons given by the stakeholders. This model is easily solved, for realistic-size problems, by minimal computational resources.

Specifically, for a given criterion l, $l = 1, \ldots, L$, we solve the following nonlinear optimization problem for the ratings of the alternatives with respect to the lth criterion:

$$Min \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{s=1}^{S} \left(\frac{x_i^l}{x_j^l} - r_{ij}^l(s) \right)^2, \tag{6}$$

s.t.

$$\sum_{i=1}^{N} x_i^l = 1, \;\; x_i^l \geq 0, \; i = 1, \ldots, n. \tag{7}$$

The variables x_i^l, $i = 1, \ldots, N$, represent the derived least squares ratings for the alternatives with respect to criterion l. The optimal values of these variables are closest, in the Euclidean (L_2) metric, to the pairwise-comparison evaluations provided by the S stakeholders. In a sense, these ratings represent a formal "mathematical" consensus among the stakeholders regarding the evaluations of the alternatives with respect to criterion l. Equation (7) is just a normalizing constraint. We discuss the meaning and usefulness of these derived consensus ratings later. Note that problem (6, 7) is solved L times, once for each criterion.

A similar consensus-formation procedure applies to determining the criteria weights. The minimization problem in this case is

$$Min \sum_{l=1}^{L} \sum_{m=1}^{L} \sum_{s=1}^{S} \left(\frac{y_l}{y_m} - d_{lm}(s) \right)^2, \tag{8}$$

s.t.

$$\sum_{l=1}^{L} y_l = 1 \,. \;\; y_l \geq 0, \; l = 1, \ldots, L. \tag{9}$$

Note that this model can be generalized to account to weighted opinions by stakeholders. The opinion of some stakeholders, say, a subject matter expert, on a certain set of alternatives and/or criterion, may have a heavier weight than other stakeholders when forming the consensus in (6) and (8). Specifically, (6) may be replaced by

$$Min \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{s=1}^{S} a_s \left(\frac{x_i^l}{x_j^l} - r_{ij}^l(s) \right)^2, \tag{10}$$

where $a_s > 0$ is the weight of the opinion of stakeholder s. No particular scaling restrictions are required for these coefficients. The same generalization may apply to (8).

If $\hat{x}_i^l$, $i = 1, \ldots, N, l = 1, \ldots L$, denote the optimal evaluation ratings of the alternatives (optimal solutions of running (6, 7) L times), and $\hat{y}_l$, $l = 1, \ldots L$, are the optimal criteria weights obtained as the solutions of (8, 9), then the final overall rating of alternative i is

$$\hat{X}_i = \sum_{l=1}^{L} \hat{y}_l \hat{x}_i^l. \tag{11}$$

By definition, $\hat{X}_i \in (0,1)$ and it is easily verified that $\sum_{i=1}^{n} \hat{X}_i = 1$.

The least squares models (6, 7) and (8, 9) are applied on the set T of Thrusts and on all the subsets T_i^C of Programs and all the subsets P_j^C of Projects, as defined earlier. For the set of alternative Thrusts $T_i \in$ T, we define a set of, say, L_T criteria and a set of stakeholders. Each stakeholder generates L_T pairwise-comparison ratio-scale $(|\mathrm{T}| \times |\mathrm{T}|)$ matrices for evaluating the alternative Thrusts, and one $(L_T \times L_T)$ matrix for the criteria. The resulting $\hat{X}_i$ ratings in (11), multiplied by 100, are the desired u_i values. Next we apply (6)–(9) and (11) to each subset T_i^C of Programs and multiply the resulting $\hat{X}_{ij}$ ratings for that subset by u_i to obtain v_{ij} and eventually, via (2) and (3) the values of V_j and w_{jk}. Finally, we apply (6)–(9) and (11) to each subset P_j^C of Projects, and multiply the resulting $\hat{X}_{jk}$ ratings for that subset by V_j to obtain w_{jk} and eventually, via (4), the values W_k. In the "Example" section, we demonstrate the process described above.

If a Program (Project) is associated with more than one Thrust (Program), its rating $\hat{X}$ in the various subsets to which it belongs may differ substantially. There may be several reasons for such likely discrepancies. First, the contexts may be completely different; the value of a program in, say, an aerial context may be completely different than in a surface warfare context. Second, criteria weights, and even the set of criteria, may be different in different subsets. For example, the criterion "Timeliness" (regarding a Program) may have different weights in the two Thrusts: "Future Force Structure" and "Power Projection in Region Z." Third, the evaluation of the (common) Program itself with respect to the same criteria may be different in the two subsets T_1^C and T_2^C because the Program is evaluated relative to different sets of alternatives, for example, P_2 in Figure 1 is evaluated relative to P_1 in T_1^C and relative to P_3 in T_2^C. The same arguments apply to Projects with regard to Programs. Thus, for all practical purposes, we can assume independence of ratings across subsets.

Finally, note that the cardinality of subsets within a certain level—Programs and/or Projects—may vary. Some Thrusts may generate more Programs than others, and some Programs may cover more Projects than others. For example, the number of Programs associated with the Thrust "Surface Warfare" may be different from the number of Programs in the "Cyber Warfare" Thrust. In such situations, for the same value of their respective parent alternatives, a large subset has a smaller average rating than a small subset because more alternatives share value of the parent alternative. This potential discrepancy may wrongly affect the values because highly valued Projects or Programs may receive low ratings just because they are members in a larger subset than other, less valued, subsets of alternatives. Although it looks like an issue of concern, it actually is not. First, the variability among subsets' size in a certain level of the hierarchy may not be that large. It is quite unlikely that one Thrust will have, say, two Programs, and another will have 10. Even if such a situation happens, one could redefine Thrusts and Programs (e.g., split a Thrust into "sub-Thrusts") such that the cardinalities will be compatible. Second, one could address this issue by defining the cardinality of a subset—the "number of children"—as a criterion in the MCDA process where Programs are compared; more Project in a Program will award that Program more value. Finally, recall that the main objective is to discriminate among alternatives and prioritize them so that ultimately a subset of Projects is selected for funding (if the budget is sufficient to fund all Projects there is no need for the POM in the first place). Tying the values of alternatives will not attain this objective because it will be impossible to identify the desired subset of fundable Projects. Stakeholders of a larger subset of Projects will be incentivized to prioritize the alternatives, as opposed to tying them, to avoid splitting the parent Program's value over too many Projects and thus "wasting" value. An example can explain this point.

Suppose Program 1 controls two Projects, E_1 and E_2, and Program 2 controls four projects, $E_3, E_4, E_5,$ and E_6.. That is, $|P_1^C| = 2, |P_2^C| = 4$. Let $V_1 = 30,\ V_2 = 40$ be the obtained values of Programs P_1 and P_2, respectively. If, following the preference analysis, E_1 is tied with E_2 in P_1^C, and $E_3, E_4, E_5,$ E_6 are tied in P_2^C, then Projects E_1 and E_2 will get a value $W_1 = W_2 = 30/2 = 15$, while $W_3 = W_4 = W_5 = W_6 = 40/4 = 10$. If all Projects are of equal costs and only two Projects can be funded, then the two Projects in Program 1 will be funded and none of Program 2, despite the fact the Program 2 is more valuable. If there is a clear winner in the first set, say, $\hat{X}_{E_1} = 0.8$, and a clear winner in the second set, say, $\hat{X}_{E_3} = 0.7$, then $W_1 = 0.8 \times 30 = 24$, $W_3 = 0.7 \times 40 = 28$, and the rating of E_3 reflects its worth. In summary, if one of the subsets of alternatives is larger than others in its level, the stakeholder needs to work harder to identify those alternatives at the top. Finally, one could always control the number of Projects selected from each Program, regardless of its cardinality, by imposing additional constraints in the resource allocation model as discussed in the next section. See (14) and (15) in the next section.

OPTIMIZATION

The ratings W_k, cost figures C_k for the various Projects, and a budget limit B, are inputs to the optimization model, which is a standard knapsack problem: the goal is to identify the set of Projects that maximize the total ratings' value subject to the budget constraint. Let z_k be a binary variable that takes the value 1 if Project k is selected for funding and 0 otherwise. The optimization model is:

$$Max \sum_{E_k \in \mathrm{E}} W_k z_k, \tag{12}$$

s.t.

$$\sum_{E_k \in \mathrm{E}} C_k z_k \leq B, \quad z_k \in \{0,1\}. \tag{13}$$

A natural and simple heuristics to rank order the Projects for funding is to order the ratios $\frac{W_k}{C_k}$ in descending order, $\left(\frac{W_{(1)}}{C_{(1)}} \geq \frac{W_{(2)}}{C_{(2)}} \geq \ldots\right)$, and to select the set of Projects $\left(E_{(1)}, \ldots, E_{(k*)}\right)$ where k^* is such that $\sum_{i=1}^{k*} C_{(i)} \leq B$, but $\sum_{i=1}^{k*+1} C_{(i)} > B$.

The basic knapsack model can be modified to account for additional constraints associated with Programs and Thrusts. For example, it may be decided that the budget allocation to Program $P_j \in \mathrm{P}$ should be at least a fraction α of the budget. This requirement will result in the constraint:

$$\sum_{E_k \in P_j^C} C_k z_k \geq \alpha B. \tag{14}$$

Another possible requirement is that the budget allocation for Program $P_j \in \mathrm{P}$ should be at least as high as the budget allocation to Program $P_{j'} \in \mathrm{P}$. In that case, we add to (12, 13) the constraint:

$$\sum_{E_k \in P_{j'}^C} C_k z_k - \sum_{E_k \in P_j^C} C_k z_k \leq 0. \tag{15}$$

EXAMPLE

We demonstrate the methodology on a made-up and oversimplified POM process in the Navy. While the example does not give any analytical insights regarding a real POM process (part of which may be classified), it highlights and narrates all the modeling and structural aspects of the proposed method.

Navy leadership have identified two Thrusts to be considered: "Air" and "Surface." Following the identification of several criteria, such as "capability gaps" and "future needs," an MCDA model, as described earlier, is set up and solved. The stakeholders at this level are Navy leadership. After some discussions, including sensitivity analysis for the (judgmental) inputs of the MCDA model, it is agreed that at the Thrust level Air gets the rating of 0.3 and Surface the rating of 0.7. That is, $u(\text{Air}) = 30$ and $u(\text{Surface}) = 70$. Next, the two sets of Programs, corresponding to the two Thrusts, are considered: T^C_{Air}, which comprises "Air Defense" (AD) and "Aircraft" (AC), and $T^C_{Surface}$, which comprises "AD" and "Ships." Note that the Program "AD" belongs to the two Thrusts as it affects both surface security and air superiority. The stakeholders involved in prioritizing the Programs in T^C_{Air} reached a consensus that the ratings of AC and AD are 0.4 and 0.6, respectively. The stakeholders of $T^C_{Surface}$ reached a consensus of 0.2 and 0.8, for Programs AD and Ships, respectively. Further down the alternatives' tree there are three subsets of Projects: P^C_{AC} comprising Projects Proj1 and Proj2, P^C_{AD} comprising Projects Proj2 and Proj3, and P^C_{Ships} comprising Projects Proj3, Proj4, and Proj5. Note that Proj2 supports both Programs "AC" and "AD," and Proj3 supports Programs "AD" and "Ships." Also notice the uneven cardinality of the Projects' subsets: while $|P^C_{AC}| = |P^C_{AD}| = 2$, $|P^C_{Ships}| = 3$.

Next, we demonstrate the process for obtaining the ratings of Proj3, Proj4, and Proj5 within the Program "Ships." Similar processes obtained the values for the Thrusts and Programs presented above, and the values of the rest of the Projects.

Suppose there are four stakeholders who rate the three Projects in P^C_{Ships} according to five criteria. This results in 20 3×3 ratio-scale matrices of pairwise comparisons of the three alternatives—one for each stakeholder and criterion. Solving (6, 7) for each one of the five criteria, we obtain the consensus ratings $x^l_{Ships,k}$, $k = \text{Proj3}, \text{Proj4}, \text{Proj5}$, for criteria $l = 1, \ldots, 5$. Next, the four stakeholders evaluate the criteria importance and submit four 5×5 ratio-scale matrices of pairwise comparisons of the five criteria. Solving now (8, 9), we obtain the consensus weights y_l for the criteria. The relative values of the alternatives are obtained as a linear functional: $X_{Ships,k} = \sum_{l=1}^{5} y_l x^l_{Ships,k}$, $k = \text{Proj3}, \text{Proj4}, \text{Proj5}$. The ratings for the alternatives obtained from the MCDA stage in the various subsets are summarized in Table 1.

Table 1, along with Figure 2, provide all the information needed for obtaining the values of the Projects. First, the values of the Thrusts are $u_{Air} = 100 \times X_{Air} = 100 \times 0.3 = 30$, $u_{Surface} = 100 \times X_{Surface} = 100 \times 0.7 = 70$. Next, we compute the values of the Programs: $V_{AC} = v_{Air,AC} = u_{Air} \times X_{Air,AC} = 30 \times 0.4 = 12$, $V_{AD} = v_{Air,AD} + v_{Surface,AD} = u_{Air} X_{Air,AD} + u_{Surface} X_{Surface,AD} = 30 \times 0.6 + 70 \times 0.2 = 32$, and $V_{Ships} = v_{Surface,Ships} = u_{Surface} X_{Surface,Ships} = 70 \times 0.8 = 56$. Finally, we obtain the values of the Projects, which are fed into the resource allocation optimization model.

Table 1. Relative ratings of alternatives.

Set	Ratings
T	$X_{Air} = 0.3$; $X_{Surface} = 0.7$
T^C_{Air}	$X_{Air,AC} = 0.4$; $X_{Air,AD} = 0.6$
$T^C_{Surface}$	$X_{Surface,AD} = 0.2$; $X_{Surface,Ships} = 0.8$
P^C_{AC}	$X_{AC,\text{Proj1}} = 0.6$; $X_{AC,\text{Proj2}} = 0.4$
P^C_{AD}	$X_{AD,\text{Proj2}} = 0.3$; $X_{AD,\text{Proj3}} = 0.7$
P^C_{Ships}	$X_{Ships,\text{Proj3}} = 0.6$; $X_{Ships,\text{Proj4}} = 0.1$; $X_{Ships,\text{Proj5}} = 0.3$

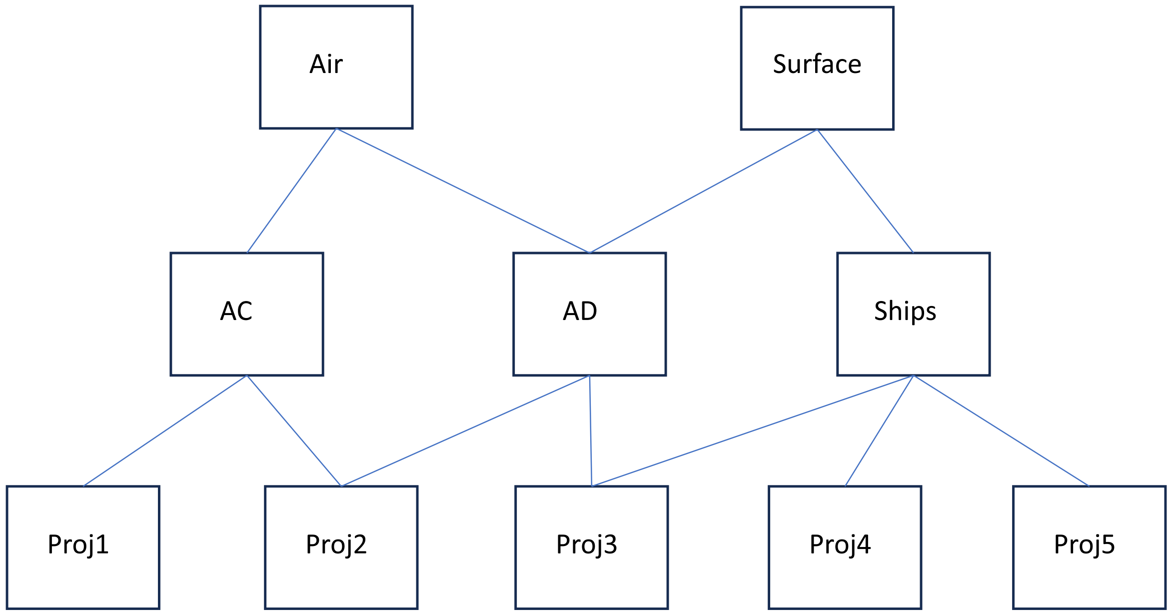

Figure 2. A POM-related example.

$$W_{\text{Proj1}} = w_{AC,\,\text{Proj1}} = V_{AC} \times X_{AC,\,\text{Proj1}} = 12 \times 0.6 = 7.2,$$
$$W_{\text{Proj2}} = w_{AC,\,\text{Proj2}} + w_{AD,\,\text{Proj2}} = V_{AC} \times X_{AC,\,\text{Proj2}} + V_{AD} \times X_{AD,\,\text{Proj2}} = 12 \times 0.4 + 32 \times 0.3 = 14.4,$$
$$W_{\text{Proj3}} = w_{AD,\,\text{Proj2}} + w_{AD,\,\text{Proj2}} = V_{AD} \times X_{AD,\,\text{Proj3}} + V_{Ships} \times X_{Ships,\,\text{Proj3}} = 32 \times 0.7 + 56 \times 0.6 = 56,$$
$$W_{\text{Proj4}} = w_{Ships,\,\text{Proj4}} = V_{Ships} \times X_{Ships,\,\text{Proj4}} = 56 \times 0.1 = 5.6,$$
$$W_{\text{Proj5}} = w_{Ships,\,\text{Proj5}} = V_{Ships} \times X_{Ships,\,\text{Proj5}} = 56 \times 0.3 = 16.8.$$

We verify that indeed $\sum_{k=1}^{5} W_{\text{Proj}k} = 100$, and see that Proj3 has a relative high value (56) due to its high desirability in two valuable Programs—AD and Ships. Suppose the total costs of the Projects (in billions) are: 1, 3, 7, 4, 2 for projects Proj1, …, Proj5, respectively, and the budget is \$12B. The resource allocation problem is a standard knapsack model:

$$Max\, 7.2z_1 + 14.4z_2 + 56z_3 + 5.6z_4 + 16.8z_5,$$

s.t.

$$\begin{aligned} &z_1 + 3z_2 + 7z_3 + 4z_4 + 2z_5 \leq 12, \\ &z_i \in \{0,1\},\ i = 1, \ldots, 5. \end{aligned} \tag{16}$$

The optimal solution of (16) is $z_2 = z_3 = z_5 = 1$, $z_1 = z_4 = 0$. Thus, the fundable Projects are Proj2, Proj3, and Proj5. One could impose additional constraints to reflect priorities such as "at least one Project from Program AC must be funded," which translates into $z_1 + z_2 \geq 1$, or "the budget allocation for Program Ships must be at least 40% of the total budget," which adds the constraint $7z_3 + 4z_4 + 2z_5 \geq 4.8$ to (16), and so on.

SUMMARY AND CONCLUSIONS

We have developed a decision aid methodology for dealing with a multilevel decision, where alternatives form a natural hierarchy among themselves. The methodology resembles the AHP model but is different from it in two major aspects. First, the hierarchy only comprises alternatives; the criteria, which may be different for different sets of alternatives, are "orthogonal" to the levels of the hierarchy; they are not part of it. Moreover, the desirability (value) of a Project depends not only on its relative ratings compared to other Projects within its subset, but also on the desirability of alternatives up the alternative chain in the hierarchy—in our case, Programs and Thrusts. Also, Projects that support more Programs and Thrusts (have more parents) benefit by obtaining higher ratings. In other words, the model balances off between the value of a Project within its peers and its "popularity" among Programs and Thrusts. Second, the consensus ratings of the alternatives in the various sets are obtained by a simple and theoretically sound extremal method where the Euclidian distance from a desired consensus is minimized. This approach is widely used in statistics (least squares regression) and is intuitively appealing.

The final note is a word of caution. The methodology described in this article is by no means a "black box" that produces the "correct" set of Projects to be selected, as is, into the POM. It is a decision aid, not a decision tool. Its purpose is to introduce some structure into a decision process concerning multiple levels of alternatives and decision makers, which is typical to the POM process.

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