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Data Envelopment Analysis in the Presence of Both Quantitative and Qualitative Factors

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This paper presents a framework for incorporating ordinal data factors into the standard ratio DEA model. An application involving the prioritization of R&D projects is presented as a case in point where such ordinal factors appear in a natural way. Two different models for incorporating ordinal data are developed; one in which the ordinal factors are ranked and one where they are not. Finally, the issue of selecting a lower bound ε on factors is addressed.

Key words: data envelopment analysis, discrimination

INTRODUCTION

The Data Envelopment Analysis (DEA) method of Charnes, Cooper and Rhodes[1] (CCR) has been applied to a wide range of real world problems as shown in Seiford[2]. In addition to the standard applications involving relative efficiency measurement, this analysis tool has also been utilized for ranking alternatives (for example, projects) and for examining general multiple criteria decision situations. A distinguishing feature of the DEA methodology is its capability to handle multiple inputs and outputs. Because of the types of not-for-profit settings to which DEA has been applied, the corresponding analysis factors are very often non economic in nature. This being the case, the inputs and outputs often represent qualitative factors, which have been quantified as a convenience to accommodate the DEA structure. In many instances, however, this quantification is superficial, particularly if the data is actually a categorization or is simply an ordinal ranking of the DMUs.

In an earlier paper by Cook, Kress and Seiford[3] a structure was presented which permitted the inclusion of a single ordinal input within the standard CCR model framework. This input can be viewed as an ordinal ranking of the DMUs. In the present paper we extend this idea to the more general case where a mix of qualitative and quantitative inputs and outputs are present.

As a practical setting, the next section briefly discusses an application involving project selection in the area of research and development. The third section develops a generalization of the CCR model to include ordinal data. By imposing ratio constraints on the multipliers of such factors, a model somewhat resembling the cone ratio model of Charnes *et al.*[4], but more properly an assurance region model as per Thompson *et al.*[5] arises. In the fourth section we examine the issue of replacing the infinitesimal ε by an actual lower bound, viewing the role of ε as a discrimination parameter. A discussion follows in the final section.

ORDINAL DATA IN R&D PROJECT SELECTION: AN EXAMPLE

Consider the problem of selecting R&D projects in a major public utility corporation with a large research and development branch. Research activities are housed within several different divisions, for example, thermal, nuclear, electrical, and so on. In a budget constrained environment in which such as organization finds itself, it becomes necessary to make choices among a set of potential research initiatives or projects that are in competition for the limited resources. To

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evaluate the impact of funding (or not funding) any given research initiative, two major considerations generally must be made. First, the initiative must be viewed in terms of more than one factor or criterion. Second, some or all of the criteria that enter the evaluation may be qualitative in nature. Even when clearly quantitative factors are involved, such as long term saving to the organization, it may be extremely difficult to obtain even a crude estimate of the value of that factor. The most that one can do in many such situations is to classify the project (according to this factor) on some scale (high/medium/low or say a 5-point scale).

Let us assume that for each qualitative criterion each initiative is rated on a 5-point scale, where the particular point on the scale is chosen through a consensus on the part of executives within the organization. Table 1 presents an illustration of how the data might appear for 10 projects, 3 qualitative output criteria (benefits), and 3 qualitative input criteria (cost of resources).

In the actual setting examined, a number of potential benefit and cost criteria were considered as displayed in Tables 2a and 2b.

It is pointed out that we use the convention that a rating of 1 is 'best' and 5 'worst' in the case of outputs, while for inputs the reverse is true. This convention is needed since 'more' is better in the case of outputs while 'less' is better with inputs.

Regardless of the manner in which such a scale rating is arrived at, the existing DEA model is capable only of treating the information as if it has cardinal meaning (e.g. something which

TABLE 1. *Ratings by criteria*

Project No.	Outputs			Inputs		
	1	2	3	4	5	6
1	2	4	1	1	4	5
2	1	1	4	3	1	4
3	1	1	1	5	4	5
4	3	3	3	2	3	4
5	4	3	5	1	5	2
6	2	5	1	5	4	4
7	1	4	1	1	2	3
8	1	5	3	3	3	3
9	5	2	4	2	4	1
10	5	4	4	1	1	1

TABLE 2a. *Potential benefits*

Criteria	Subcriteria or Interpretation
1. Enhancement of energy efficiency	—development of high yield technologies —initiatives which will reduce energy demand —development of technologies for utilizing residues
2. Enhancement of diversification/ alternative energy sources	—initiatives which provide or strive for new energy sources —provide for flexibility in or adaptability of existing and new facilities
3. $ Saved internal to organization	—cost reduction devices —new technology to replace obsolete equipment
4. Impact on environment	—reduction of emissions into water and atmosphere —reduction of risk of nuclear accidents
5. Enhancement to internal technical capability and research profile	—provides training and develops expertise —provides technical resources (software, equipment, etc.) —builds linkages to external research community.
6. Enhancement to research profile as viewed by the external community	—impact on research status among other utility companies —impact on profile abroad
7. Economic impact on external community	—job creation outside organization —$ savings to public and industry created by energy efficiency devices
8. Impact on nuclear performance	—influence on nuclear station maintenance, etc.

TABLE 2b. *Potential costs*

Criteria	Subcriteria or Interpretation
1. Technical expertise available internally	
2. Technical expertise available externally	—consultants
	—other research centres
3. Technology available	—equipment
	—software

receives a score of 6 is evaluated as being twice as important as something that scores 3). There are a number of problems with this approach. First and foremost, the projects' original data in the case of some criteria may take the form of an ordinal ranking of the projects. Specifically, the most that can be said about two projects i and j is that i is preferred to j. In other cases it may only be possible to classify projects as say 'high', 'medium' or 'low' in importance on certain criteria. When projects are rated on, say, a 5-point scale, it is generally understood that this scale merely provides a relative positioning of the projects. In a number of agencies investigated (for example, hydro electric and telecommunications companies), 5-point scales are common for evaluating alternatives in terms of qualitative data, and are often accompanied by interpretations such as

1 = Extremely important
2 = Very important
3 = Important
4 = Low in importance
5 = Not important,

which are easily understood by management. While it is true that market researchers often treat such scales in a numerical (i.e. cardinal) sense, no one seriously believes that an 'extremely important' classification for a project should be interpreted literally as meaning that this project rates three times better than one which is only classified as 'important'. The key message here is that many, if not all criteria used to evaluate R&D projects are qualitative in nature, and should be treated as such. The model presented in the following sections extends the DEA idea to an ordinal setting, hence accommodating this very practical consideration.

ORDINAL AND CARDINAL FACTORS IN DEA

Representation of ordinal data in the DEA structure

The problem setting of the previous section represents a situation in which the factors involved are qualitative rather than quantitative. To cast the R&D project selection problem in a general format, consider the situation in which a set of N projects or DMUs ($n = 1, 2, \ldots, N$) are to be evaluated in terms of a mix of numerical (quantitative) and ordinal (qualitative) factors. Specifically, assume there are R_1 numerical outputs, R_2 ordinal outputs, I_1 numerical inputs, and I_2 ordinal inputs. Let the R_1-dimensional vector of numerical outputs and the I_1-dimensional vector of numerical inputs for project n be denoted Y_n and X_n, respectively. In regard to the ordinal factors, define the L-dimensional unit vectors $(\gamma_{r1}(n), \ldots, \gamma_{rl}(n), \ldots, \gamma_{rL}(n))$ and $(\delta_{i1}(n), \ldots, \delta_{il}(n), \ldots, \delta_{iL}(n))$ where

$$\gamma_{rl}(n) = \begin{cases} 1 & \text{if project } n \text{ is rated in } l\text{th place on the } r\text{th ordinal output} \\ 0 & \text{otherwise} \end{cases}$$

and

$$\delta_{il}(n) = \begin{cases} 1 & \text{if project } n \text{ is rated in } l\text{th place on the } i\text{th ordinal input} \\ 0 & \text{otherwise.} \end{cases}$$

For example, if $L = 5$ and if project 1 is rated in third place (i.e., in the third category among the five categories), on output criterion 5, then $\gamma_{53}(1) = 1$ and $\gamma_{5l}(1) = 0$ for $l = 1, 2, 4, 5$. The choice of L seems to be rather arbitrary in most practical situations. Likert scales used by market researchers are commonly taken as 5, 7 or 9. The use of a 5 point scale seems to be fairly common in many settings since particular meanings can be attached to the various levels as indicated earlier. For purposes herein we assume the use of a common scale for all criteria.

Define the decision variable or worth w_{rl}^1 associated with being rated lth relative to the rth output factor. Similarly, w_{il}^2 denotes the worth variable associated with being rated lth relative to the ith input criterion.

In the usual formulation of the CCR model with only numerical factors, the contribution of the rth output criterion, say, to the overall efficiency ratio of the nth project, is $\mu_r y_{rn}$. For an ordinal output criterion, the contribution will be $\sum_{l=1}^{L} w_{rl}^1 \gamma_{rl}(n)$. Using vector notation (e.g. W_r is the L-dimensional vector of variables w_{rl}), we define the mixed numerical/ordinal DEA ratio model (in the primal LP format) by:

$$\max \mu Y_o + \sum_{r \varepsilon \mathrm{ORD}} W_r^1 \gamma_r(o) \tag{1}$$

subject to:

$$vX_o + \sum_{i \varepsilon \mathrm{ORD}_2} W_i^2 \delta_i(o) = 1 \tag{2}$$

$$\mu Y_n + \sum_{r \varepsilon \mathrm{ORD}_1} W_r^1 \gamma_r(n) - vX_n - \sum_{i \varepsilon \mathrm{ORD}_2} W_i^2 \delta_i(n) \leqslant 0, \qquad n = 1, \ldots, N, \tag{3}$$

$$\mu_r \geqslant \varepsilon, \qquad r \varepsilon \mathrm{CARD}_1 \tag{4}$$

$$v_i \geqslant \varepsilon, \qquad i \varepsilon \mathrm{CARD}_2 \tag{5}$$

$$\{W_r^1, W_i^2\} \varepsilon \Psi. \tag{6}$$

The notation Y_o and $\gamma_r(o)$ are used to signify the particular DMU being considered at the time. Here, ORD_1 and ORD_2 represent the sets of ordinal outputs and inputs, respectively, CARD_1 and CARD_2 are the sets of numerical outputs and inputs. Ψ denotes the set of allowable or *admissible* worth vectors. It must be noted that the use of the same ε for different input and output factors within the DEA structure has met with objections from several quarters over the past several years. Since two inputs might, for example, be 'labour hours' and 'available computer technology', the scales for the respective data values $x_{i_1 n}$ and $x_{i_2 n}$ might be very different. In this case the likely sizes of v_{i_1} and v_{i_2} may similarly differ. Thrall[6] has suggested a mechanism for correcting for this scale differential through the use of a penalty vector G which can be used to augment ε. By choosing G appropriately one can effectively reduce all factors to a form of common unit. For simplicity of presentation and notation herein we assume the cardinal scales are similar in dimension and that G is the unit vector. The more general case would proceed in a similar manner.

We now examine various forms which Ψ might take.

Strict ordinal relations on the components of the worth vectors

While in the case of pure numerical criteria, it may not be necessary to restrict the multiplier space (for μ and v) other than through the normalization constraints (2), ratio bound restrictions (3), and lower limits (4) and (5), this is not true in the case of an ordinal factor. Because w_{rl}^1, for example, is meant to represent the weight or worth of being rated lth on a criterion, at a minimum we require that $w_{rl}^1 > w_{rl+1}^1$. That is, it is more important to be rated in the lth category on a given criterion than in the $l + 1$st category. Imposing a strict ordinal ranking on the components of each worth vector we therefore define

$$\Psi = \{(W_r^1, W_i^2) \mid w_{rl}^1 - w_{rl+1}^1 \geqslant \varepsilon, w_{rL}^1 \geqslant \varepsilon, w_{il}^2 - w_{il+1}^2 \geqslant \varepsilon, w_{iL}^2 \geqslant \varepsilon, \text{ for all } r, i, l\}, \tag{7}$$

where ε is a small positive scalar. Defining Ψ in this manner thus ensures that a strictly higher weight is given to projects that rate in position l than is true of projects rated in position $l + 1$.

ε could be made dependent upon l (i.e. replace ε by ε_l), and all results below would still follow in a similar fashion. For convenience here, we use a common ε. in Ψ, ε could be augmented by an appropriate penalty vector G since now we are using the same ε for both ordinal and cardinal factors. The choice of an appropriate penalty vector G to 'standardize' all factors here should, however, be no different than in the case where all factors are numerical.

Problem (1)–(6) where Ψ is defined by (7), is then a modification of the usual CCR structure, in that it contains additional ordinal relations on the worth variables $\{w_{rl}^1, w_{rl}^2\}$. The following theorem shows, however, that this model can be reduced to the standard CCR format.

Theorem 1

Problem (1)–(7) can be expressed in standard CCR format.

Proof

Writing constraints (4)–(7) in the form

$$-\mu_r \leqslant -\varepsilon$$
$$-v_i \leqslant -\varepsilon$$
$$-w_{rl}^1 + w_{rl+1}^1 \leqslant -\varepsilon$$
$$-w_{il}^2 + w_{il+1}^2 \leqslant -\varepsilon,$$

the dual of (1)–(7) becomes

$$\min \theta + \vec{0}\lambda - \vec{\varepsilon}S^+ - \vec{\varepsilon}S^- - \sum_{r\varepsilon\mathrm{ORD}_1} \vec{\varepsilon}\beta_r - \sum_{i\varepsilon\mathrm{ORD}_2} \vec{\varepsilon}\rho_i \tag{8}$$

subject to:

$$0\theta + \sum_{n=1}^{N} \lambda_n y_{rn} - s_r^+ = y_{or}, \qquad \text{for all } r\varepsilon\mathrm{CARD}_1 \tag{9}$$

$$x_{io}\theta - \sum_{n=1}^{N} \lambda_n x_{in} - s_i^- = 0, \qquad \text{for all } i\varepsilon\mathrm{CARD}_2 \tag{10}$$

$$\left.\begin{array}{l} 0\theta + \sum_{n=1}^{N} \lambda_n \gamma_{r1}(n) - \beta_{r1} = \gamma_{r1}(0) \\ 0\theta + \sum_{n=1}^{n} \lambda_n \gamma_{r2}(n) + \beta_{r1} - \beta_{r2} = \gamma_{r2}(0) \\ \vdots \\ 0\theta + \sum_{n=1}^{N} \lambda_n \gamma_{rL}(n) + \beta_{rL-1} - \beta_{rL} = \gamma_{rL}(0) \end{array}\right\} \text{for all } r\varepsilon\mathrm{ORD}_1 \tag{11}$$

$$\left.\begin{array}{r} \delta_{i1}(o)\theta - \sum_{n=1}^{N} \lambda_n \delta_{i1}(n) - \rho_{i1} = 0 \\ \delta_{i2}(o)\theta - \sum_{n=1}^{N} \lambda_n \delta_{i2}(n) + \rho_{i1} - \rho_{i2} = 0 \\ \vdots \\ \delta_{iL}(o)\theta - \sum_{n=1}^{N} \lambda_n \delta_{iL}(n) + \rho_{iL-1} - \rho_{iL} = 0 \end{array}\right\} \text{for all } i\varepsilon\mathrm{ORD}_2 \tag{12}$$

$$\lambda_n, \quad s_r^+, \quad s_r^-, \quad \beta_{rl}, \quad \rho_{il} \geqslant 0. \tag{13}$$

In (8) S^+ and S^- are R_1 and R_2-dimensional vectors respectively (e.g. $S^+ = (s_1^+, s_2^+, \ldots, s_{R_1}^+)$). Similarly β_r and ρ_i are L-dimensional vectors of dual variables.

Here notation such as $\vec{\varepsilon}$ denotes the vector $\vec{\varepsilon} = (\varepsilon, \varepsilon, \ldots, \varepsilon)$.

Now perform simple row operations on the first L rows (i.e., for $r = 1$) of (11), by replacing the second constraint by the sum of the first two constraints, the third by the sum of the first three constraints, ..., etc. The resulting set of L constraints becomes

$$0\theta + \sum_{n=1}^{N} \lambda_n \bar{\gamma}_{rl}(n) - \beta_{rl} \geqslant \bar{\gamma}_{rl}(0), \tag{14}$$

where

$$\bar{\gamma}_{rl}(n) = \sum_{k=1}^{l} \gamma_{rk}(n) = \gamma_{r1}(n) + \gamma_{r2}(n) + \ldots + \gamma_{rl}(n). \tag{15}$$

This exercise is then repeated for each $r\varepsilon\mathrm{ORD}_1$. Doing the same thing with each set of L constraints in (3.12) (i.e., for each i), we get the modified constraints

$$\bar{\delta}_{il}(0)\theta - \sum_{n=1}^{N} \lambda_n \bar{\delta}_{il}(n) - \rho_{il} \geqslant 0, \qquad \text{for all } l, i, \tag{16}$$

where $\bar{\delta}_{il}(n)$ is defined in a manner analogous to $\bar{\gamma}_{rl}(n)$ of (15). Replacing (11) and (12) by (14) and (16), the primal version of the resulting problem is:

$$\max \mu Y_o + \sum_{r\varepsilon\mathrm{ORD}_1} \bar{W}_r^1 \bar{\gamma}_r(o) \tag{17}$$

subject to:

$$vX_o + \sum_{r\varepsilon\mathrm{ORD}_1} \bar{W}_i^2 \bar{\delta}_i(o) = 1 \tag{18}$$

$$\mu Y_n + \sum_{r\varepsilon\mathrm{ORD}_1} \bar{W}_r^1 \bar{\gamma}_r(n) - vX_n - \sum_{i\varepsilon\mathrm{ORD}_2} \bar{W}_i^2 \bar{\delta}_i(n) \leqslant 0 \tag{19}$$

$$\mu_r \geqslant \varepsilon, \quad v_i \geqslant \varepsilon, \quad \bar{w}_{rl}^1 \geqslant \varepsilon, \quad \bar{w}_{il}^2 \geqslant \varepsilon. \tag{20}$$

Problem (17)–(20) is now in standard CCR format.

When Model (1)–(6) is applied to the data of Table 1, the efficiency scores obtained are shown in Table 3.

TABLE 3. *Efficiency scores (non-ranked criteria)*

Project	1	2	3	4	5	6	7	8	9	10
Score	0.76	0.73	1.00	0.67	1.00	0.82	0.67	0.67	0.55	0.37

Here, projects 3 and 5 turn out to be 'efficient', while all other projects are rated well below 100%. In this particular analysis, ε was chosen as 0.03. In another run (not shown here) where $\varepsilon = 0.01$ was used, projects 3, 5 and 6 received ratings of 1.00, while all others obtained somewhat higher scores than those shown in Table 3. When a very small value of $\varepsilon(\varepsilon = 0.001)$ was used, all except one of the projects was rated as efficient.

Clearly this example demonstrates the same degree of dependence on the choice of ε as is true in the standard DEA model. See Ali and Seiford[7].

From the data in Table 1 it might appear that only project 3 should be efficient since 3 dominates project 5 in all factors except for the second input where project 3 rates fourth while project 5 rates fifth. As is characteristic of the standard ratio DEA model, a single factor can produce such an outcome. In the present case this situation occurs because $w_{25}^2 = 0.03$ while $w_{24}^2 = 0.51$. Consequently, project 5 is accorded an 'efficient' status by permitting the gap between w_{24}^2 and w_{25}^2 to be (perhaps unfairly) very large. Actually, the set of multipliers which render project 5 efficient also constitute an optimal solution for project 3.

Criteria importance

There are many decision situations involving multiple criteria where management may wish to impose an *a priori* ranking of those criteria. It is particularly true that when the criteria under consideration are ordinal and are expressed on a common L-point scale, but note that while it may be desirable to allow different scales for different criteria, this would be problematic if we want to incorporate criteria importance. In (21) for example some pairs of criteria may be non comparable. A ranking of those criteria will be provided.

The model (1)–(7) of the previous section, as it presently stands, does not contain any provision for incorporating the relative importance of the various criteria.

To incorporate such a feature here, the admissible set Ψ of worth vectors $\{W_r^1, W_i^2\}$ must be redefined. Assume that the output worth variables w_{rl}^1 have been prioritized (with no loss of generality, assume they are already numbered in descending order of importance), specifically,

$$w_{rl}^1 - w_{r+1\,l}^1 \geqslant \varepsilon, \quad r = 1, \ldots, R_1 - 1, \qquad \text{for all } l \tag{21}$$
$$w_{R_1 l}^1 \geqslant \varepsilon, \qquad \text{for all } l.$$

Similarly, let the input worth variables be prioritized

$$w_{il}^2 - w_{i+1\,l}^2 \geqslant \varepsilon, \quad i = 1, \ldots, R_2 - 1, \qquad \text{for all } l \tag{22}$$
$$w_{R_2 l}^2 \geqslant \varepsilon, \qquad \text{for all } l.$$

Now define $\bar{\Psi}$ by

$$\bar{\Psi} = \{(W_r^1, W_i^2)/w_{rl}^1 - w_{rl+1}^1 \geqslant \varepsilon, \quad w_{rL}^1 \geqslant \varepsilon; \quad w_{il}^2 - w_{il+1}^2 \geqslant \varepsilon, \quad w_{iL}^2 \geqslant \varepsilon;$$
$$\text{and satisfying (21) and (22)}\} \tag{23}$$

The DEA structure given by (1)–(6), together with (23), is then a modified version of the original CCR model. It must be pointed out, however, that this structure cannot be reduced to the standard CCR format as was true of the formulation of the previous subsection. The ranking of the variables along two separate dimensions prohibits this. This modified model has a structure somewhat reminiscent of the cone ratio model of Charnes *et al.*[4], although the idea of deriving factor weights based on pure ordinal data was introduced by Cook and Kress[8] in the context of a general multi criteria decision framework. There, issues of discrimination functions in regard to criteria importance as well as criteria clearness are addressed.

For a full discussion of ordinal data issues see Cook and Kress[8].

Applying the revised model (with Ψ defined by (23)), the relative positioning of the projects changes as shown by the efficiency scores of Table 4.

Here most ratings dropped slightly, yet projects 3 and 5 still retain their efficient status.

TABLE 4. *Efficiency scores (ranked criteria)*

Project	1	2	3	4	5	6	7	8	9	10
Score	0.71	0.72	1.00	0.60	1.00	0.80	0.62	0.63	0.50	0.35

CHOOSING ε

As discussed earlier, the value of ε selected in the DEA model does influence the sizes of the multipliers and the efficiency scores that result. See Ali and Seiford[7]. Arguably, a larger value of ε is preferable to a smaller value in that once a set of analysis factors is chosen, it would seem unreasonable to credit the worth of any factor with negligible importance ($\varepsilon \approx 0$). Geometrically, the larger the value of ε used, the greater is the distance of the efficient frontier (isoquant) from the axes, at least in terms of the numerical factors with multipliers μ and v. Furthermore, ε also plays the role of a minimum discrimination parameter in Ψ (e.g., it is the minimum difference between the importance attached to the lth and $(l + 1)$st rank positions). It is reasonable to distinguish between such different rank positions to the greatest extent possible.

Consider the n problems.

$$\hat{\varepsilon}_o = \max \varepsilon \tag{24}$$

subject to:

$$\text{(2)–(6)} \tag{25}$$

TABLE 5. *Efficiency scores using ε_{max}*

Project	1	2	3	4	5	6	7	8	9	10
Score	0.68	0.69	0.93	0.52	1.00	0.73	0.62	0.59	0.52	0.31

with ε being treated now as a variable rather than as a fixed parameter. Define

$$\varepsilon_{max} = \min_{i=1,\ldots,n} \{\hat{\varepsilon}_i\}. \tag{26}$$

Proof of the following theorem is straightforward.

Theorem 2

There exists a feasible solution to each of the n problems (24)–(25) if and only if $\varepsilon \leqslant \varepsilon_{max}$.

In the sense of Theorem 2, ε_{max} is the largest feasible value of ε for purposes of evaluating the relative efficiency of the various DMUs (projects). As indicated earlier, any solution found using ε_{max} is considered to be the most discriminating in terms of the relative importance attached to ordinal inputs and outputs. It is argued, therefore, that an appropriate choice for ε is ε_{max}.

Using the data provided earlier, ε_{max} was derived and then problem (1)–(6) was resolved for each project. The resulting ratings are as shown in Table 5.

Clearly, the values of the various efficiency scores are lower (except for the single efficient project 5) than was the case above. This is to be expected, given the more restrictive nature of the optimization problem. It is pointed out that the concept of ε_{max} is related to $\bar{\varepsilon}$-stable introduced in Lang *et al.*[10]

DISCUSSION

Many DEA applications can involve qualitative or ordinal data factors. Nowhere is this more prevalent than in problems pertaining to R&D projects, where a major difficulty of quantification stems from the uncertainty of the outcomes that are likely to flow from research. This fact necessitates an alteration to the standard DEA structure to accommodate such qualitative information.

The ability to be able to model both qualitative and quantitative factors in the DEA structure extends the usefulness of the tool to a broader range of problems. Moreover, since DEA was initially developed as an efficiency measurement tool for not-for-profit situations, and since such situations commonly exhibit 'soft factors', the capability to handle such factors becomes a necessity.

While we have not addressed directly such issues as fuzziness of criteria, it is worth investigating how this aspect of multicriteria analysis could be incorporated into the DEA structure. Similarly, it may be useful to examine how discrimination factors can be incorporated to reflect varying levels of importance of one factor versus another. These and other issues are topics for later research.

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