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On the Use of Ordinal Data in Data Envelopment Analysis

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In many problems involving efficiency analysis using DEA, certain factors may be measurable only on an ordinal scale. Specifically, it may be possible only to rank order the DMUs according to a factor, rather than being able to assign a specific numerical value of that factor to each DMU. To illustrate this, we examine a problem involving the evaluation of new technology installations. The presence of qualitative factors in such an environment motivates the need to investigate how such factors can be incorporated into existing efficiency measurement models. In particular, a procedure is presented for incorporating an ordinal factor into the DEA structure, with the resulting formulation being a particular form of cone ratio model. The model is then applied to the technology installation efficiency problem.

Key words: data envelopment analysis, ordinal data, new technology, efficiency

INTRODUCTION

The Data Envelopment Analysis (DEA) methodology developed by Charnes *et al.*[1] has been utilized to evaluate efficiency in a wide variety of applications. The usual setting for such applications involves a set of similar decision making units (DMUs), for each of which there is an observable and measurable set of inputs and outputs.

One case application, for example, is discussed in Cook *et al.*[2] where the DMUs are a set of (246) maintenance crews located throughout a Canadian province. To evaluate the relative efficiency with which the crews perform, a set of measures were selected for study. These measures are of two types—outputs or measures of accomplishment, and inputs or measures representing resources available to the crews and circumstances within which the crews operated. Typical output measures used are total surface area of pavement maintained and average performance rating received by the pavements. Input measures include maintenance budget spent in a given analysis year and the climatic conditions existing in that year.

Numerous applications of the DEA technique are reported in the literature. These include analysis of courts, school districts, airforce maintenance units, power plants, and so on. A complete bibliography is available in Seiford[3].

A distinguishing feature of all applications of DEA reported to date is the specific assumption that all input and output factors involve measurable, i.e. cardinal data. In some applications of DEA, however, one or more factors believed to be relevant to the analysis may be measurable only on an ordinal scale. Specifically, it may only be possible to rank the DMUs relative to such factors. Consider the problem of analysing the relative efficiency of a set of possible new technology installations for factories. Each installation can be considered as a DMU. Suppose the factors believed to be relevant measures of output are:

- time to install the technology;
- % of time the finished product is serviceable;
- a management satisfaction score.

The factors considered to be strong influences (inputs) on these outputs are:

- complexity of the installation;
- previous experience of project team;
- novelty of the application;
- urgency of the technology.

Suppose that all factors except for urgency are observable and measurable. Specifically, assume

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that urgency is expressible only in terms of a ranking of the projects. That is, we can provide only a prioritization of the DMUs (project No 1 is more urgent than project No 2 which is more urgent than project No 3, and so on). The problem to be addressed is how to incorporate properly such a factor into the DEA structure.

The presence of ordinal factors in efficiency evaluation and project prioritization settings is a common phenomenon. That is certainly the case, for example, in R&D settings. Here, important but often intangible factors such as the contribution of a project to national profile, are critical to the priority setting exercise. Such a factor could generally only be measured on an ordinal scale. Ordinal factors have been present in other studies, but have either been only *implicitly* included or have been *quantified* whether quantification was practical or not. A strong argument can be made, however, that only those factors that are truly measurable should be quantified. Factors that are qualitative, i.e. that appear in preference format, should be dealt with as such.

In the present paper an approach is presented for incorporating ordinal data into the DEA structure. Specifically, a factor is incorporated relative to which the DMUs have been rank ordered. To motivate the presentation, the next section presents a brief description of a case application where ordinal data appears in a natural setting. The section after develops an appropriate representation in terms of worth vectors for ordinal data in the DEA framework. In the presence of this representation it is necessary to define the restrictions which must be imposed on the resulting worth vectors. This is given in the fourth section. Conclusions and discussion follow.

IMPLEMENTATION EFFICIENCY IN NEW TECHNOLOGY ADOPTION

The impact of new technology in the manufacturing environment has been a subject of considerable interest and investigation over the past decade. This interest is due in large part to the globalization of markets, and the need to compete in the international arena. Significant effort has been expended on investigating those factors which influence adoption, and the successful usage of new technology. Better knowledge of such influences can assist implementors/management in understanding which types of technology work well, what forms of supplier commitments are most efficient, . . ., etc.

To illustrate the application of DEA in this area, and more specifically to demonstrate the use of ordinal variables, consider the installation of 31 robotics systems of various types in manufacturing plants. The installations involved a wide range of plant sizes, system complexity levels, and outcomes in the form of installation time and performance level when in operation. Although arc welding applications and vehicle component manufacturing dominated the sample, these categories included widely varying types of installations. The vehicle component installations ranged from light stamping to heavy welding on military vehicles.

In a previous study (Cook *et al.*[4]) a full description of the factors used to carry out a DEA analysis is given. There, ordinal inputs such as urgency were not explicitly incorporated into the DEA structure but, rather, were treated as control parameters. To facilitate discussion of the application, the description of the variables is recreated here. The data for these factors were collected through on-site interviews with engineering managers, and reflect a consensus of opinions within the organizations examined.

While many different parameters were collected, for analysis purposes three output variables and four input variables are of interest.

Outputs

Time to install the technology—this was the number of weeks required to take the technology from physical installation to the point where it was in full operation within the production process.

Uptime—an estimated percentage of the total time (when production could take place) when the technology was in service.

Management satisfaction (MSAT)—this is a measure on a five-point scale representing the degree to which the project met management expectations.

Inputs

System complexity—this measure is a combination (sum) of four factors pertaining to robotic systems; namely (a) number of machines controlled by the system; (b) the number of part numbers requiring programming; (c) the number of robots, and (d) the number of operations performed by the robots.

Previous experience with technology (PREVEX)—this measure is also a sum of four factors: (a) the number of years that programmable equipment has been operated; (b) maintenance capability with programmable equipment; (c) the number of years of experience with systems powered by the same methods; and (d) maintenance expertise with similar mechanical systems.

Novelty of the application (NEQPT)—this is based on a five-point scale for each major component, with each scale representing the component's innovativeness ranging from the purchase of standardized off-the-shelf equipment to customized components.

Urgency—this measure is on a four-point scale, based on the reporting managers assessment of the urgency connected with the project.

While much of the data is cardinal in nature, certain factors which could have a strong influence on implementation are ordinal. One of these measures, *urgency* of the project, was given only on a four-point scale. Specifically, each installation was *classified* as having an urgency 'rating' of 1, 2, 3 or 4 (1-low urgency, 4-high urgency). Table 1 displays the data for the seven factors across the 31 sites.

In the previous study by Cook *et al.*[4] only those factors whose data were considered as being numerically reliable were used directly in the DEA analysis. (Arguably, management satisfaction should be treated as an ordinal variable in the same way urgency is treated. The MSAT variable was, however, seen as more immediately observable, i.e. a more direct declaration of satisfaction was available than was true of urgency.) Urgency was treated in Reference 4 as a control parameter. In order to incorporate classification or rank order factors such as urgency directly into the DEA structure, an appropriate representation must be developed. This is the subject of the following sections.

TABLE 1. *List of factors for sites*

Site No	TIME	UPTIME	MSAT	COMPLEX	PREVEX	NEQPT	URGENCY
1	197	78	4	39	11	11	2
2	184	95	2	27	20	4	3
3	175	85	4	39	6	14	2
4	150	78	4	33	18	13	1
5	188	90	5	38	4	11	3
6	176	78	4	40	9	11	1
7	194	85	3	42	16	16	1
8	180	97	3	36	9	10	2
9	188	80	4	34	12	8	4
10	176	78	3.5	34	19	9	3
11	176	100	5	39	18	5	2
12	144	78	4	14	15	4	3
13	176	95	5	34	18	11	3
14	199	99	4	38	16	13	1
15	188	78	3	42	13	16	1
16	184	65	1	38	11	15	3
17	188	78	3	28	11	15	3
18	188	90	4	31	11	2	1
19	196	78	4	42	14	17	2
20	190	78	2	36	12	9	1
21	185	78	4	22	11	2	2
22	135	78	2	2	16	8	1
23	100	78	3	34	4	19	1
24	195	80	3	38	4	11	2
25	152	65	4	25	6	5	1
26	189	83	2	35	14	15	2
27	179	80	3	34	11	3	1
28	198	78	5	25	4	10	2
29	145	78	4	36	18	5	2
30	165	78	2	32	16	10	1
31	174	40	4	28	4	9	3

REPRESENTATION OF ORDINAL DATA IN DEA

The DEA Structure

The original Data Envelopment Analysis model of Charnes *et al.*[1] (denoted CCR) presented a method for evaluating the relative efficiency of each member of a set of n decision making units. Letting Y_{r_k} denote the amount of output of type r produced by DMU k, and X_{i_k} the amount of input of type i used by k, the ratio DEA model is designed to determine for each DMU 'o' the best positive set of input multipliers $\{v_{io}\}_{i=1}^{I}$ and output multipliers $\{u_{ro}\}_{r=1}^{R}$ such that the ratio of total weighted outputs to total weighted inputs is maximized. This is done subject to the constraints that the corresponding ratio for each DMU k (including the one in question) does not exceed 1 or 100%. Stated in mathematical programming terms, one solves for each DMU 'o' the fractional linear programming problem

$$e_o = \max \sum_r u_{ro} Y_{ro} / \sum_i v_{io} X_{io}$$

$$\text{subject to: } \sum_r u_{ro} Y_{rk} / \sum_i v_{io} X_{ik} \leqslant 1 \quad \text{for all k} \tag{1}$$

$$u_{ro} \geqslant \varepsilon, \quad v_{io} \geqslant \varepsilon, \quad \text{for all } r, i.$$

As shown by Charnes and Cooper,[5] this fractional problem can be reduced to a linear programming problem by way of a change of variables. Specifically, (1) is equivalent to the linear problem (in vector notation)

$$e_o = \max \mu Y_o$$

$$\text{subject to: } \nu X_o = 1$$

$$\mu Y_k - \nu X_k \leqslant 0 \quad \text{for all } k \tag{2}$$

$$\mu_r \geqslant \varepsilon, \; \nu_i \geqslant \varepsilon, \quad \text{for all } r, i.$$

Thus, the original problem of maximizing the ratio of weighted outputs to inputs can be viewed as a problem of maximizing weighted outputs in the presence of normalized weighted inputs. We now turn to the problem of representing ordinal data within this DEA structure.

Ordinal data

For purposes of exposition, consider the example cited in the second section in which the ordinal input *urgency* is presented in the form of a ranking of the n DMUs. With no loss of generality, let DMU No 1 (technology installation No 1) have the highest urgency, No 2 the second highest, . . . etc. More particularly, let the DMUs be ranked according to L ($\leqslant n$) rank positions (ties at the same rank position are possible).

Define the variable w_l to be the value or *worth* associated with the lth rank position. This will be discussed later. Further, for each DMU k define the L-dimensional unit vector $\delta_k = (\delta_{kl})$ where

$$\delta_{kl} = \begin{cases} 1 & \text{if DMU } k \text{ is ranked in the } l\text{th position} \\ 0 & \text{otherwise.} \end{cases}$$

Zero data values can, in general, present a technical complication in DEA. However, several ways have been suggested in the literature for handling such values (see, for example, Ali and Seiford[6] and Charnes *et al.*[7]). In solving the example presented at the end of this paper, the 0s in the δ_k vectors were replaced by small non-zero values.

The CCR model (in the primal LP format) incorporating the rank order input can now be written in the form:

$$\max \mu Y_o$$

$$\text{subject to: } \nu X_o + W\delta_o = 1$$

$$\mu Y_k - \nu X_k - W\delta_k \leqslant 0 \text{ for all } K \tag{3}$$

$$\begin{aligned}
&\mu_r \geqslant \varepsilon \quad \text{for all } r\\
&\nu_i \geqslant \varepsilon \quad \text{for all } i\\
&W \in \Psi,
\end{aligned}$$

where $W = (w_1, w_2, \ldots, w_L)$, and Ψ is the *permissible set* of worth vectors with $\Psi \subseteq \mathscr{R}^L_+$. It is noted that if DMU k is ranked lth, then $W\delta_k = w_l$.

In the section to follow, we examine appropriate structures for the permissible set Ψ. For the structures examined, it is shown that (3) is reducible to the standard CCR format, and hence is solvable by existing software.

THE PERMISSIBLE SET OF WORTH VECTORS W

Strict ordinal relations on the w_l

The worth variables $\{w_l\}_{l=1}^{L}$ associated with the L rank positions should satisfy certain regularity conditions. At a minimum the lth rank position must be deemed to be worth at least as much as the $(l + 1)$th position. In the context of the above example of technology urgency, rank position No 1 would pertain to the most urgent installation, rank position No 2 to the second level of urgency, etc.

Consider, then, the case where Ψ is defined as

$$\Psi = \{W = (w_l) \mid w_l - w_{l+1} \geqslant \varepsilon, \quad l = 1, \ldots, L-1 \quad \text{and} \quad w_L \geqslant \varepsilon\} \tag{4}$$

Theorem 1

Problem (3) with Ψ defined by (4) can be converted to standard CCR format.

Proof

If (3) is written in the form

$$\begin{aligned}
\max \quad & \mu Y_o \\
\text{subject to:} \quad & \nu X_o + W\delta_o = 1 \\
& \mu Y_k - \nu X_k - W\delta_k \leqslant 0 \\
& -\mu_r \leqslant -\varepsilon \\
& -\nu_i \leqslant -\varepsilon \\
& -w_l + w_{l+1} \leqslant -\varepsilon \\
& -w_L \leqslant -\varepsilon,
\end{aligned} \tag{5}$$

its dual becomes

$$\begin{aligned}
\min \quad & \theta + \vec{0}\lambda - \vec{\varepsilon}S^+ - \vec{\varepsilon}S^- - \vec{\varepsilon}\gamma \\
\text{subject to:} \quad & 0\theta + \textstyle\sum_k \lambda_k y_{rk} - s_r^+ && = y_{or}, \quad \forall r \\
& x_{io}\theta - \textstyle\sum_k \lambda_k x_{ik} && -s_i^- = 0, \quad \forall i \\
& \delta_{o1}\theta - \textstyle\sum_k \lambda_k \delta_{k1} && -\gamma_1 = 0 \\
& \delta_{o2}\theta - \textstyle\sum_k \lambda_k \delta_{k2} && +\gamma_1 - \gamma_2 = 0 \\
& \vdots && \vdots \\
& \delta_{oL} - \textstyle\sum_k \lambda_k \delta_{kL} && +\gamma_{L-1} - \gamma_L = 0 \\
& && \theta, \lambda_k, s_r^+, s_i^-, \gamma_l \geqslant 0.
\end{aligned} \tag{6}$$

Now, perform simple row operations on those constraints involving the δ_{kl} by replacing the second constraint by the sum of the first two constraints, the third by the sum of the first three constraints, ... etc. Problem (6) can then be rewritten as

$$\begin{array}{lll} \min & \theta + \vec{0}\lambda - \vec{\varepsilon}S^+ - \vec{\varepsilon}S^- - \varepsilon\gamma & \\ \text{subject to:} & 0\theta + \sum_k \lambda_k x_{rk} - s_r^+ & = y_{or}, \quad \forall r \\ & x_{io}\theta - \sum_k \lambda_k x_{ik} & -s_i^- = 0, \quad \forall i \\ & \bar{\delta}_{ol}\theta - \sum \lambda_k \bar{\delta}_{kl} & -\gamma_l = 0, \quad \forall l \\ & \theta, \lambda_k, s_r^+, s_i^-, \gamma_l \geqslant 0, & \end{array} \tag{7}$$

where $\bar{\delta}_{kl} = \Sigma_{j=1}^{l} \delta_{kj} = \delta_{k1} + \delta_{k2} + \ldots + \delta_{kl}$.

If the primal version of this problem is now set up, one obtains:

$$\begin{array}{lll} \max & \mu Y_o & \\ \text{subject to:} & \nu X_o + \bar{W}\bar{\delta}_o & = 1 \\ & \mu Y_k - \nu X_k - \bar{W}\bar{\delta}_k & \leqslant 0 \\ & -\mu_r & \leqslant -\varepsilon, \forall r \\ & -\nu_i & \leqslant -\varepsilon, \forall i \\ & -\bar{w}_l & \leqslant -\varepsilon, \forall l. \end{array} \tag{8}$$

Problem (8) is now in the standard CCR format.

Example

Assume there are four DMUs that are ranked according to urgency, with DMU No 1 ranking highest, No 2 and No 3 in the second rank position and No 4 in the lowest or third rank position. Thus, the $(\delta_k) = (\delta_{kl}) = (\delta_{k1}, \delta_{k2}, \delta_{k3})$ are given by

$$\begin{array}{ll} \delta_1 = (1,0,0) & \Rightarrow \bar{\delta}_1 = (1,1,1) \\ \delta_2 = \delta_3 = (0,1,0) & \Rightarrow \bar{\delta}_2 = \bar{\delta}_3 = (0,1,1) \\ \delta_4 = (0,0,1) & \Rightarrow \bar{\delta}_4 = (0,0,1) \end{array}$$

With Ψ defined according to (4), problem (8) possesses a certain feature which may be undesirable in many situations. This feature is pointed out by the following theorem.

Theorem 2

In the solution of (5), DMU 'o' is evaluated only against those DMUs k whose rank position l_k is at or above that of 'o'; i.e. for DMUs k where $l_k \geqslant l_o$.

Proof

From the above example, it is clear that for any DMU k in rank position l_k, the constraint $\mu Y_k - \nu X_k - W\bar{\delta}_k \leqslant 0$ takes the form $\mu Y_k - \nu X_k - \Sigma_{l=l_k}^{L} w_l \leqslant 0$. Therefore, when DMU k_o is being evaluated, variables $w_1, w_2, \ldots, w_{l_{k_o}-1}$ can be set arbitrarily large, without influencing the constraint $\nu X_{k_o} + W\bar{\delta}_{k_o} = 1$. In so doing, however, those DMUs k in rank positions 1, 2, ..., $l_{k_o} - 1$ are prevented from being in the efficient reference set for k_o.

In this respect, the L rank positions behave as *categories* in the sense of Banker and Morey's[8] definition. However, in their use of categorical variables, the authors were concerned with DMUs in any one category being compared *only* with those DMUs in the same or less advantaged categories. There, this *nesting* feature (Theorem 2) was desirable.

In general, however, it is required to make a comparison among all DMUs, without having this nesting phenomenon present. To facilitate this, it is necessary to restrict the range of W. This aspect is examined in the following subsection.

Ratio scale bounds on the w_l

The issue of defining appropriate restrictions on the worth variables w_l associated with ordinal data, involves describing the status of any given DMU relative to its peers. Some insight can be obtained if the relative positions of two DMUs are examined in terms of a cardinal factor. Specifically, if $x_{k,i}$ denotes the amount of some cardinal input i available to DMU k, $\nu_i x_{k,i}$ is the credited worth of that input to k. This quantity is analogous to w_{l_k} for an ordinal factor. Furthermore, $\nu_i x_{k,i}$ and $\nu_i x_{k+1,i}$ define the status of DMUs k and $k+1$ in terms of input i. A somewhat trivial, but useful observation is that the ratio $r_{k,i} = \nu_i x_{k,i} / \nu_i x_{k+1,i}$ is a fixed quantity.

This statement is true regardless of the size of ν_i, and regardless of any scale changes of a multiple nature (e.g. multiplying $x_{k,i}$ by a constant factor) which one might care to impose*. This obvious 'fixed ratio' property inherent to cardinal inputs and outputs is absent in the case of an ordinal factor. The ratio w_l / w_{l+1} for an ordinal input or output, (which is analogous to $\nu_i x_{k,i} / \nu_i x_{k+1,i}$ for a cardinal factor), is clearly a variable quantity. It, therefore, seems reasonable that if restrictions are to be imposed on the w_l, these should be specified in terms of this ratio. In that regard, the restrictions will be in the form of bounds on w_l / w_{l+1}. Specifically, define fixed limits h_l, $g_l \geqslant 1$ such that

$$h_l \geqslant w_l / w_{l+1} \geqslant g_l,$$

hence, define

$$\begin{aligned} \Psi = \{ W = (w_1, \ldots, w_L) &: w_l - g_l w_{l+1} \geqslant 0 \\ \text{and } - w_l + h_l w_{l+1} \geqslant 0, \quad l &= 1, 2, \ldots, L-1, \quad w_L \geqslant \varepsilon \} . \end{aligned} \tag{9}$$

Problem (1) can then be written as:

$$\begin{array}{lll} \max & \mu Y_o & \\ \text{subject to:} & \nu X_o + W\delta_o & = 1 \\ & \mu Y_k - \nu X_k - W\delta_k & \leqslant 0, \forall k \\ & w_l - g_l w_{l+1} & \geqslant 0,\ l = 1, \ldots, L-1 \\ & -w_l + h_l w_{l+1} & \geqslant 0,\ l = 1, \ldots, L-1 \\ & w_L \geqslant \varepsilon, \mu_r \geqslant \varepsilon, \nu_i \geqslant \varepsilon \ \ \forall r, i. & \end{array} \tag{10}$$

Problem (10) is a form of the cone ratio model as described by Charnes *et al.*[7] This form of the model provides a very natural setting for constraining the worth vector for an ordinal variable, in view of the fixed ratio arguments made above regarding cardinal data. Furthermore, since Ψ is a type of polyhedral cone, its negative polar cone can be used to transform (10) into a standard CCR model (Charnes *et al.*[1]).

Referring to the technology adoption problem of the second section, it is clear that an appropriate structure for incorporating such factors as urgency is that suggested above. In that regard, we use the notation w_1 to denote the importance to be attached to high-urgency projects (rating level of 4); w_2 to the next to highest urgency (rating level of 3); and so on. Further, we impose bounds on the ratios w_l / w_{l+1}, $l = 1, 2, 3$ such that any urgency rating is worth at least 50% ($g_l = 1.5$) more than the next highest level (larger l), and no more than 300% ($h_l = 3.0$) of the next lower level.

Each urgency factor was expressed as a vector δ as described in the third section. Two different analyses were carried out — one for model (8) where no upper and lower bounds h_l, g_l are present, and one for model (10) containing such bounds. Table 2 displays the results.

Clearly, the model (10) efficiency scores are generally lower than those for model (8) (no score can be higher than in (8)), and provide a more realistic picture of the relative status of the DMUs. While the choice of h_l, g_l, values was arbitrary in this case, an analysis of various combinations could provide some insights as to the sensitivity of the ratings to these bounds.

As an example of how bounds in model (10) influence ratings, consider the cases of sites 3 and 5. In the case that no ratio bounds are imposed, site No 3 is compared only with other sites of the

*Clearly, although the relative status of two DMUs is well defined in a ratio sense, such is not true if one uses the difference $\nu_i x_{k,i} - \nu_i x_{k+1,i}$ to describe the comparative status of these DMUs.

TABLE 2. *Efficiency scores for models (8) and (10)*

Site No	Efficiency score Model (4.5)	Efficiency score Model (4.7)	Site No	Efficiency score Model (4.5)	Efficiency score Model (4.7)
1	0.98	0.77	17	0.94	0.68
2	1.00	0.82	18	1.00	1.00
3	0.97	0.91	19	0.97	0.70
4	0.77	0.77	20	0.93	0.93
5	1.00	1.00	21	1.00	1.00
6	1.00	1.00	22	1.00	1.00
7	0.96	0.92	23	1.00	1.00
8	1.00	0.93	24	1.00	1.00
9	0.92	0.67	25	1.00	1.00
10	0.86	0.57	26	0.94	0.74
11	1.00	0.86	27	0.94	0.92
12	1.00	1.00	28	1.00	1.00
13	1.00	0.73	29	0.82	0.72
14	1.00	1.00	30	0.84	0.84
15	0.89	0.89	31	0.82	0.82
16	0.91	0.63			

same urgency (level 2) and lower urgency (level 1). That is, in the optimization process sites of urgency levels 3 and 4 get disqualified in a sense from influencing the standings of sites with urgency level 2. The rating obtained by site No 3 is 0.97 or 97%. Upon introduction of the ratio bounds in model (10), the rating of site No 3 drops to 91%. That site is now compared with additional sites including site No 5. Note that site No 5 completely dominates (has higher outputs and lower inputs than) site No 3.

CONCLUSIONS AND SUMMARY

This paper has presented an approach for handling ordinal data factors within the DEA structure. By imposing upper and lower bounds on the ratios w_l/w_{l+1} of the worths attached to rank positions l and $l+1$, a form of cone ratio DEA model arises. The approach is illustrated using data pertaining to new technology installations.

The idea of treating non-quantifiable factors directly as ordinal data, (rather than quantifying the non-quantifiable) adds an important new dimension to DEA. In this paper, we have examined only the inclusion of a single ordinal variable, consequently, only part of the problem has been resolved. Issues involving multiple ordinal factors and the relative importance and/or fuzziness aspects pertaining to such factors, entail considerations beyond what has been investigated here. This will be the subject of later research.

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