The Value of Recovery Transformers in Protecting an Electric Transmission Grid Against Attack

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***Abstract*—This paper incorporates, as part of an attacker-defender (AD) model for an electric power transmission grid, an inventory of "recovery spares" for high-voltage transformers (HVTs). In this sequential-game model, an attacker first uses limited resources to disable grid components, seeking to maximize second-stage "total cost." This value corresponds to a defender who operates the damaged grid to minimize generating and economic load-shedding costs. The defender's problem includes 1) optimal power-flow solutions to represent post-attack operations, 2) optimized replacement of disabled HVTs with quickly installable recovery spares, and 3) longer-term replacements using new procurements. Global Benders decomposition, with a mixed-integer subproblem, solves the AD model; new enumerative techniques help solve the decomposition's master problem and subproblems. Computational tests demonstrate tradeoffs between the number of recovery spares and worst-case total cost, and show how the model could help guide an inventory strategy for recovery spares in an adversarial setting.**

***Index Terms*—Power system modeling, restoration and protection, security, substations, transformers.**

I. Introduction

HIGH-VOLTAGE transformers (HVTs) are key components in any electric power transmission grid. Electric power utilities do maintain spare transformers to protect power delivery against rare, age-related failures, but those spares could fail to protect against the simultaneous loss of multiple HVTs by the adversarial acts of terrorism or sabotage [1]. This paper develops a model to help guide an inventory strategy for special "recovery spares," which could supply the missing protection.

A report by the Congressional Research Service [2] points out why spare HVTs, beyond current inventories, are needed to protect against adversarial acts: 1) current inventories are small, because they are intended to protect against uncommon, random failures of equipment with large capital costs, 2) existing spares are often installed adjacent to operating HVTs in a substation, so an attack on a substation would likely destroy both spare and operating HVTs, and 3) long procurement times for replacements mean that the simultaneous loss of multiple HVTs from a coordinated attack on several substations could result in months of severely degraded service and substantial economic losses. Considering these points, and the fact that HVTs are difficult to transport because of weight and size, a committee of the National Academy of Sciences recommended the following: "[D]evelop and stockpile a family of easily transported high-voltage recovery transformers and other key equipment" [3]. In fact, at the time of that recommendation, the Department of Homeland Security had already funded a prototype, truck-mounted recovery spare for the common 345–138 kV HVTs [1].

Given the adversarial setting, this paper develops a game-theoretic interdiction model and associated solution methods to evaluate how an inventory of recovery HVTs can reduce the disruption caused by a worst-case, coordinated attack. Disruption means increased "total cost," as defined below.

We note that the Federal Energy Regulatory Commission has also addressed the adversarial problem with the Spare Transformer Equipment Program (STEP) [4]. Utilities that join the program may be required to acquire an inventory of spare HVTs [5]. Naturally, the idea is to share an economic burden in pursuit of a common benefit. (NERC's 2011 Spare Equipment Database (SED) Program [6] is also relevant.) Tests in Section V follow the STEP criteria, which support transmission HVTs in eight voltage classes, with high-side ratings up to 500 kV and low-side ratings down to 69 kV.

The threat of attacks on electric power substations motivates the development of a recovery-transformer program in the United States. But, an inventory of recovery HVTs could also help improve the resilience of a power grid in the face of natural disasters such as hurricanes; see [1] and the references therein. This is true because, similar to a coordinated attack, a natural disaster can cause the simultaneous loss of multiple HVTs. A detailed discussion of deliberate attacks *and* random disasters exceeds the scope of the current paper, but a short Appendix outlines how our methods extend to handle both.

II. Background: Attacker-Defender Models

An attacker-defender (AD) model is a two-stage sequential game that models 1) an intelligent *attacker* who first uses his limited resources to carry out a coordinated attack on the components of a given system, and 2) a *defender* who then operates the damaged system as best possible. An AD model can help identify a worst-case coordinated attack for a variety of infrastructure systems [7]. In the current paper, the attacker represents a terrorist group, while the defender represents the electric power utility company or companies associated with a grid, along with that grid's independent system operator. The model applies the following conservative assumptions to avoid the need to model terrorists' intentions and their ability to gather intelligence: the attacker knows the defender's optimal reaction to any coordinated

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attack, and will use that knowledge to maximize the defender's post-attack total cost. *Total cost* measures, over a given time horizon, all generating costs plus the economic cost of any load shedding. Without loss of generality, we ignore direct equipment replacement costs for the defender and, because achieving normal functionality may require many weeks or months after an attack, we also ignore short-term cascading outages.

Salmeron *et al.* [8] first describe and demonstrate an AD model for an electric transmission grid. This may be viewed as a type of network-interdiction model (e.g., [9]) that evaluates post-attack cost for the "network user" by solving a set of DC optimal power flow (OPF) models, which are linear programs (LPs) [10, p. 514]. The AD model is solved only heuristically in [8], but then in [11] the same authors develop global Benders decomposition (GLBD) to solve large-scale problems to optimality. (The algorithm does require one modeling assumption for correctness in an AC power grid, but extensive numerical tests on large-scale models find no violations of this assumption.) Additional work in this area includes mixed-integer programming (MIP) versions of the AD model (e.g., [12]–[14]), as well as component-ranking heuristics (e.g., [15]–[17]). We also see connections between the current paper's model and 1) models for optimal, resource-constrained recovery of a damaged transmission grid [18], and 2) AD models that use line switching for better recovery of such a grid after an attack [19], [20]. The defender's model in case 2) becomes a MIP as it does here.

To date, no optimizing AD model for a power grid has incorporated spare parts such as recovery HVTs (although Romero *et al.* [21] use heuristics in a related context). Rather, a fixed replacement ("repair") time is assumed for each piece of attack-disabled equipment, or repairs are irrelevant because only immediate, post-attack-but-pre-repair operations are modeled. Fixed replacement times are unrealistic, however, if one disabled HVT may be replaced quickly with a recovery spare, while a limited inventory of such spares means that another disabled HVT must wait for months for replacement through a new procurement. Assuming a deterministic replacement time in the latter case is reasonable, but note that this still involves a decision variable if the option exists to install a spare.

Adding a limited spare-parts inventory to the AD model makes the model much more difficult to solve, because the defender's problem now involves discrete decisions, not just the continuous ones from OPF LPs. Also, the model no longer separates by the time periods induced by component-replacement times. Despite these difficulties, we will show how to extend GLBD to incorporate the new defender's model, and show that GLBD can solve realistically sized problems.

The reader should note that our AD model only provides a tool for comparing different investment strategies for HVT recovery spares. That is, it makes no attempt to optimize a strategy directly. Identifying an optimal strategy would require definition and solution of a defender-attacker-defender (DAD) model [22], [23], in which the current paper's AD model could be viewed as a submodel. Solving a full-scale DAD model in this context exceeds our current computational capabilities. The number of potentially useful spare HVTs is small in our tests, however, so appropriate inventory levels could be identified by solving the AD model a modest number of times with varying inventory data. (See Section V.)

III. Attacker-Defender Model

A. Multi-Period Optimal Power Flow With Recovery HVTs

We first model the optimal functioning and restoration of an electrical power grid following a given coordinated attack. Let the vector $\hat{\boldsymbol{\delta}}$ denote a fixed *interdiction plan* (also *attack plan* or simply *attack*), where $\hat{\delta}_k = 1$ if grid component k is interdicted, and $\hat{\delta}_k = 0$, otherwise. *Grid component* corresponds to line, transformer, bus, generator or even a substation, which is itself a collection of other types of components. We assume that an interdicted component is disabled but, eventually, will be repaired or replaced.

The model OPF-T($\hat{\boldsymbol{\delta}}$), presented below, extends OPF($\hat{\boldsymbol{\delta}}$) from [11] to include installation of recovery spares for HVTs. Both the extended and original models approximate the optimal operation of a power grid after an attack $\hat{\boldsymbol{\delta}}$ using a set of DC optimal power flow models. (Wood and Wollenberg [10, p. 278] support the use of a DC model for "security analysis studies"; see also [22, p. 6].) For notational simplicity, we omit the details required to handle three features that our implementation does allow: 1) substations, 2) DC lines, and 3) customer classes. Salmeron *et al.* [11] show how to incorporate these and, in fact, all computational tests in the current paper do incorporate 1).

Indices and index sets:

$i \in \mathcal{I}$	buses, with i_0 as the reference bus;
$g \in \mathcal{G}$	generators; those connected to bus i are $\mathcal{G}_i \subset \mathcal{G}$;
$l \in \mathcal{L}$	AC transmission lines (includes transformers);
$\mathcal{L}^{\mathrm{HVT}} \subset \mathcal{L}$	HVTs for which a potential spare exists;
$o(l), d(l)$	origin and destination buses, respectively, for $l \in \mathcal{L}$;
$f \in \mathcal{F}$	transformer types with "limited spares" (see Note 1);
f_l	specific HVT type for $l \in \mathcal{L}^{\mathrm{HVT}}$;
$t \in \mathcal{T}$	time periods induced by component-replacement times (see further details below and also [11]);
$k \in \mathcal{K}$	system components; $\mathcal{K} = \mathcal{I} \cup \mathcal{G} \cup \mathcal{L}$.

Note 1: For modeling purposes, these situations are equivalent: (a) *No (recovery) spares* are available but a new procurement can replace any damaged HVT at time period t'', and (b) *unlimited spares* are available and can replace any damaged HVT at time $t' = t''$. By contrast, *limited spares* implies an inventory that can replace some HVTs at time $t' < t''$, with the remainder replaced by new procurements at time t''. ■

Data [units]:

h_g	generating cost for generator g [\$/MWh];
q_{it}	load-shedding cost at bus i in period t [\$/MWh] (Note: $\min_{i,t} q_{it} \approx 10 \max_g h_g$ in computational tests.);
$\bar{P}_g^{\mathcal{G}}$	maximum output from generator g [MW];
$\bar{P}_l^{\mathcal{L}}$	thermal capacity of line l [MW];
B_l	series susceptance of line l [Ω^{-1}];
d_{it}	load at bus i during period t [MW];

n_f	number of spares of type f available;
M_l	large constant for line l, e.g., $M_l = 2\bar{P}_l^{\mathcal{L}}$ [MW];
$\hat{\delta}_k$	1 if component k is attacked in $\hat{\boldsymbol{\delta}}$, and 0 otherwise; later, $\boldsymbol{\delta}$ denotes attacker-controlled variables.

Subsets and additional data (all given attack $\hat{\boldsymbol{\delta}}$):

$\mathcal{T}_g^{\mathcal{G}}(\hat{\boldsymbol{\delta}})$	time periods when generator g is available;
$\bar{\mathcal{T}}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}})$	time periods when line l is surely unavailable;
$\mathcal{T}_l^{\mathrm{HVT}}(\hat{\boldsymbol{\delta}})$	time periods when HVT $l \in \mathcal{L}^{\mathrm{HVT}}$ is available, but only if replaced by a spare;
$\mathcal{T}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}})$	time periods when line l is surely available; note that $\mathcal{T}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) \neq \mathcal{T} \setminus \bar{\mathcal{T}}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}})$ for $l \in \mathcal{L}^{\mathrm{HVT}}$;
λ_t	duration of period t [hours].

Variables [units]:

θ_{it}	phase angle at bus i during period t [radians];
$P_{lt}^{\mathcal{L}}$	power flow on AC line l during period t [MW];
$P_{gt}^{\mathcal{G}}$	generation from generator g during period t [MW];
S_{it}	load shed at bus i during period t [MW];
β_l	1 if a recovery transformer replaces interdicted transformer $l \in \mathcal{L}^{\mathrm{HVT}}$, and 0 otherwise.

Formulation of OPF-T($\hat{\boldsymbol{\delta}}$):

$$f(\hat{\boldsymbol{\delta}}) = \min_{\mathbf{P}^{\mathcal{G}}, \mathbf{P}^{\mathcal{L}}, \mathbf{S}, \boldsymbol{\theta}, \boldsymbol{\beta}} \sum_{t \in \mathcal{T}} \lambda_t \left(\sum_{g \in \mathcal{G}} h_g P_{gt}^{\mathcal{G}} + \sum_{i \in \mathcal{I}} q_{it} S_{it} \right) \quad (1)$$

subject to:

$$P_{lt}^{\mathcal{L}} = B_l(\theta_{o(l),t} - \theta_{d(l),t}) \quad \forall\, l \in \mathcal{L},\, t \in \mathcal{T}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) \quad (2)$$

$$-\bar{P}_l^{\mathcal{L}} \le P_{lt}^{\mathcal{L}} \le \bar{P}_l^{\mathcal{L}} \quad \forall\, l \in \mathcal{L},\, t \in \mathcal{T}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) \quad (3)$$

$$-M_l(1-\beta_l) \le P_{lt}^{\mathcal{L}} - B_l(\theta_{o(l),t} - \theta_{d(l),t}) \le M_l(1-\beta_l)\; \forall\, l \in \mathcal{L}^{\mathrm{HVT}},\, t \in \mathcal{T}_l^{\mathrm{HVT}}(\hat{\boldsymbol{\delta}}) \quad (4)$$

$$-\bar{P}_l^{\mathcal{L}}\beta_l \le P_{lt}^{\mathcal{L}} \le \bar{P}_l^{\mathcal{L}}\beta_l \;\forall\, l \in \mathcal{L}^{\mathrm{HVT}}, t \in \mathcal{T}_l^{\mathrm{HVT}}(\hat{\boldsymbol{\delta}}) \quad (5)$$

$$P_{lt}^{\mathcal{L}} = 0 \quad \forall\, l \in L,\, t \in \bar{\mathcal{T}}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) \quad (6)$$

$$\sum_{g \in \mathcal{G}_i} P_{gt}^{\mathcal{G}} - \sum_{l \in \mathcal{L}\,|\,o(l)=i} P_{lt}^{\mathcal{L}} + \sum_{l \in \mathcal{L}\,|\,d(l)=i} P_{lt}^{\mathcal{L}} = d_{it} - S_{it} \quad \forall\, i \in I,\, t \in \mathcal{T} \quad (7)$$

$$0 \le P_{gt}^{\mathcal{G}} \le \bar{P}_{gt}^{\mathcal{G}} \quad \forall\, g \in \mathcal{G},\, t \in \mathcal{T}_g^{\mathcal{G}}(\hat{\boldsymbol{\delta}}) \quad (8)$$

$$P_{gt}^{\mathcal{G}} = 0 \quad \forall\, g \in \mathcal{G},\, t \in \mathcal{T} \setminus \mathcal{T}_g^{\mathcal{G}}(\hat{\boldsymbol{\delta}}) \quad (9)$$

$$0 \le S_{it} \le d_{it} \quad \forall\, i,\, t \in \mathcal{T} \quad (10)$$

$$\theta_{i_0,t} = 0 \quad \forall\, t \in \mathcal{T} \quad (11)$$

$$\sum_{l \in \mathcal{L}^{\mathrm{HVT}} | f_l = f} \hat{\delta}_l \beta_l \le n_f \quad \forall\, f \in F \quad (12)$$

$$\beta_l \in \{0,1\} \quad \forall\, l \in \mathcal{L}^{\mathrm{HVT}}. \quad (13)$$

Note 2: All units above are converted into per-unit values for a base power of 100 MVA. ■

The objective function (1) minimizes total cost over the time horizon defined through $\mathcal{T}$. Constraints (2) are linearized admittance constraints for lines $l \in \mathcal{L}$ during time periods after an attack in which these lines are surely available (never interdicted, or interdicted but now repaired without the use of a recovery spare); constraints (3) are corresponding power-flow limits. Constraints (4) and (5) govern admittance and power during time periods when an interdicted HVT might be replaced by a recovery spare. (If the transformer is interdicted but not replaced by a spare, linearized admittance constraints (4) are "switched out," while constraints (5) force power-handling capacity to 0.) Constraints (6) force power flow on a line to 0 during time periods after an attack in which that line has no chance of functioning. (Note that the corresponding admittance constraints (2) are removed during these time periods, also. Specifically, compare "$t \in \bar{\mathcal{T}}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}})$" applied to constraints (6) and "$t \in \mathcal{T}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}})$" applied to constraints (2)). Constraints (7) balance power at the buses. Constraints (8) and (9) set the maximum output for each generator given that generator's availability, which depends on $\hat{\boldsymbol{\delta}}$. Constraints (10) ensure that load-shedding does not exceed demand. Constraint (11) sets the phase angle on the reference bus to 0. Constraints (12) and (13) limit the available recovery spares and establish decision-variable domains.

B. AD Model

To create a complete AD model, we posit an attacker who seeks to maximize the defender's minimized, post-attack, total cost. The minimization includes the assignment of available recovery spares to replace interdicted HVTs. A complete "power-flow interdiction model" (PFI) can then be stated.

Additional definitions:

D_k	resource required to interdict component k;
D	total interdiction resource available;
Δ	$\{\boldsymbol{\delta} \in \{0,1\}^{\lvert\mathcal{K}\rvert} \mid \sum_{k \in \mathcal{K}} D_k \delta_k \le D\}$ (logical constraints on the attacker's actions can be added [12]);
Δ'	$\{\boldsymbol{\delta} \in \Delta \mid \boldsymbol{\delta}$ is maximal for $\sum_{k \in \mathcal{K}} D_k \delta_k \le D\}$;
$\boldsymbol{\beta} \in \mathrm{B}(\boldsymbol{\delta})$	feasible assignments of spares (recovery HVTs) given interdiction plan $\boldsymbol{\delta}$; see constraints (12) and (13);
$\mathbf{p} \in \Pi(\boldsymbol{\delta}, \boldsymbol{\beta})$	feasible $(\mathbf{P}^{\mathcal{L}}, \mathbf{P}^{\mathcal{G}}, \mathbf{S}, \boldsymbol{\theta})$ given $\boldsymbol{\delta}$ and $\boldsymbol{\beta}$;
$\mathbf{p}^*(\boldsymbol{\delta}, \boldsymbol{\beta})$	optimal power flow given $\boldsymbol{\delta}$ and $\boldsymbol{\beta} \in \mathrm{B}(\boldsymbol{\delta})$ (Note: $\boldsymbol{\beta}$ need not be optimal for $\boldsymbol{\delta}$.);
$\bar{f}(\mathbf{p})$	objective (1) viewed as a function of $\mathbf{p}$.

Formulation of PFI:

$$z^* = \max_{\boldsymbol{\delta} \in \Delta} \overbrace{\min_{\boldsymbol{\beta} \in \mathrm{B}(\boldsymbol{\delta})} \min_{\mathbf{p} \in \Pi(\boldsymbol{\delta}, \boldsymbol{\beta})} \bar{f}(\mathbf{p})}^{\mathrm{OPF-T}(\boldsymbol{\delta})} \quad (14)$$

$$\equiv \max_{\boldsymbol{\delta} \in \Delta} \min_{\boldsymbol{\beta} \in \mathrm{B}(\boldsymbol{\delta})} \bar{f}(\mathbf{p}^*(\boldsymbol{\delta}, \boldsymbol{\beta})) \quad (15)$$

$$\equiv \max_{\boldsymbol{\delta} \in \Delta} \bar{f}(\mathbf{p}^*(\boldsymbol{\delta}, \boldsymbol{\beta}^*(\boldsymbol{\delta}))) \quad (16)$$

$$\equiv \max_{\boldsymbol{\delta} \in \Delta} f(\boldsymbol{\delta}). \quad (17)$$

The Appendix to this paper shows how to extend PFI to handle damage caused by probabilistically modeled natural disasters in addition to damage caused by a coordinated attack.

C. Algorithm 1: Global Benders Decomposition (GLBD)

GLBD as described in [11] will solve PFI. This algorithm resembles standard Benders decomposition, but does not require that $f(\boldsymbol{\delta})$ be concave in continuous $\boldsymbol{\delta}$. We present the algorithm here for later reference:

Algorithm 1

Begin

Step 0. Initialization

a. Select $\hat{\boldsymbol{\delta}}_1 \in \Delta$ as a feasible, coordinated attack.
b. Let $\ell \leftarrow 1$, $\hat{\Delta}_1 \leftarrow \{\hat{\boldsymbol{\delta}}_1\}$, $\hat{\boldsymbol{\delta}}^* \leftarrow \hat{\boldsymbol{\delta}}_1$, $\bar{z} \leftarrow \infty$, $\underline{z} \leftarrow 0$, and let $\epsilon \geq 0$ be the optimality tolerance.

Step 1. Solve the subproblem OPF-T($\hat{\boldsymbol{\delta}}$) **in iteration** ℓ:

Solve OPF-T($\hat{\boldsymbol{\delta}}_\ell$), and compute $f(\hat{\boldsymbol{\delta}}_\ell)$ and master-problem "cut" coefficients $\alpha_k(\hat{\boldsymbol{\delta}}_\ell)$ following Assumption 1, below.

If $\underline{z} < f(\hat{\boldsymbol{\delta}}_\ell)$, let $\underline{z} \leftarrow f(\hat{\boldsymbol{\delta}}_\ell)$ and $\hat{\boldsymbol{\delta}}^* \leftarrow \hat{\boldsymbol{\delta}}_\ell$.

If $\bar{z} - \underline{z} \leq \epsilon$, go to Step 3.

Step 2. Solve the master problem MP($\hat{\Delta}_\ell$) **in iteration** ℓ:

$$\bar{z}(\hat{\Delta}_\ell) = \max_{\boldsymbol{\delta} \in \Delta, z} z \tag{18}$$

$$\text{s.t. } z \leq f(\hat{\boldsymbol{\delta}}) + \sum_{k \in \mathcal{K}} \alpha_k(\hat{\boldsymbol{\delta}})\delta_k \;\forall\, \hat{\boldsymbol{\delta}} \in \hat{\Delta}_\ell. \tag{19}$$

Let $\hat{\boldsymbol{\delta}}_{\ell+1}$ denote the solution and let $\bar{z} \leftarrow \bar{z}(\hat{\Delta}_\ell)$.

Let $\hat{\Delta}_{\ell+1} \leftarrow \hat{\Delta}_\ell \cup \{\hat{\boldsymbol{\delta}}_{\ell+1}\}$ and $\ell \leftarrow \ell + 1$.

If $\bar{z} - \underline{z} \leq \epsilon$, go to Step 3, else go to Step 1.

Step 3. Print solution:

Print "The ϵ-optimal solution is", $\hat{\boldsymbol{\delta}}^*$.

End.

GLBD converges if 1) $\boldsymbol{\delta}$ is discrete and Δ is finite, 2) $f(\boldsymbol{\delta})$ can be computed correctly for any $\boldsymbol{\delta} \in \Delta$, and 3) $f(\boldsymbol{\delta}) \leq f(\hat{\boldsymbol{\delta}}) + \sum_{k \in \mathcal{K}} \alpha_k(\hat{\boldsymbol{\delta}})\delta_k$ for all $\hat{\boldsymbol{\delta}}, \boldsymbol{\delta} \in \Delta$. We obtain valid cuts (19) under the following assumption, verified in [8].

Assumption 1: Given optimal response $\mathbf{p}^* \equiv \mathbf{p}^*(\hat{\boldsymbol{\delta}}, \boldsymbol{\beta}^*(\hat{\boldsymbol{\delta}}))$ to interdiction plan $\hat{\boldsymbol{\delta}}$, the following definitions for $\alpha_k(\hat{\boldsymbol{\delta}})$ are valid: 1) If $\hat{\delta}_k = 0$, then $\alpha_k(\hat{\boldsymbol{\delta}}) \equiv$ (total energy flow through component k implied by $\mathbf{p}^*$) $\times (q_{\max} - h_{\min})$, where $q_{\max}$ is the system's maximum load-shedding cost, and $h_{\min}$ is the system's minimum generating cost; and 2) if $\hat{\delta}_k = 1$, then $\alpha_k(\hat{\boldsymbol{\delta}}) = 0$. ■

We require the defender to respond optimally to an attack in the AD model, so it is natural for Assumption 1 to ask for an optimal assignment of spares. That is too restrictive, however:

Assumption 2: Given $\hat{\boldsymbol{\delta}}$, a feasible assignment of recovery spares $\hat{\boldsymbol{\beta}}(\hat{\boldsymbol{\delta}}) \in \mathrm{B}(\hat{\boldsymbol{\delta}})$ and the corresponding power flow $\hat{\mathbf{p}} \equiv \mathbf{p}^*(\hat{\boldsymbol{\delta}}, \hat{\boldsymbol{\beta}}(\hat{\boldsymbol{\delta}}))$, let $\bar{f}(\hat{\boldsymbol{\delta}}) = \bar{f}(\hat{\mathbf{p}})$ and define $\bar{\alpha}_k(\hat{\boldsymbol{\delta}})$ as $\alpha_k(\hat{\boldsymbol{\delta}})$ is defined in Assumption 1, but with respect to $\hat{\mathbf{p}}$ rather than $\mathbf{p}^*$. Then, the following inequality is valid for MP($\hat{\Delta}_\ell$):

$$z \leq \bar{f}(\hat{\boldsymbol{\delta}}) + \sum_{k \in \mathcal{K}} \bar{\alpha}_k(\hat{\boldsymbol{\delta}})\delta_k. \quad ■ \tag{20}$$

The "approximate cut" (20) may not be as tight as (19), but it can be used to advantage, as described below.

IV. Computational Improvements

This section explains how the decomposition approach of Algorithm 1 can be modified for potentially faster solutions.

A. Algorithm 2: Approximate Subproblem Solutions by Truncated Branch-and-Bound

Algorithm 2 tries to spend less time solving the subproblem OPF-T($\hat{\boldsymbol{\delta}}_\ell$) by truncating the branch-and-bound solution procedure (B&B) if $\hat{\boldsymbol{\delta}}_\ell$ proves to be "sufficiently poor":

Algorithm 2

Proceed as in Algorithm 1, but with these modifications:
In Step 0, also define η, with $0 < \eta \leq 1$ (e.g., $\eta = 0.9$).
In Step 1, while solving OPF-T($\hat{\boldsymbol{\delta}}_\ell$) by B&B, if a feasible solution $\hat{\mathbf{p}}_\ell \equiv \mathbf{p}^*(\hat{\boldsymbol{\delta}}_\ell, \hat{\boldsymbol{\beta}}(\hat{\boldsymbol{\delta}}_\ell))$ is found with $\bar{f}(\hat{\mathbf{p}}_\ell) \leq \eta \underline{z}$ (i.e., $\hat{\boldsymbol{\delta}}_\ell$ is a sufficiently poor interdiction plan), stop B&B, and,

a. Add a solution-elimination constraint (SEC) to eliminate $\hat{\boldsymbol{\delta}}_\ell$. (See [11] and [24].)
b. Go to Step 2, but add the inequality (20) based on $\hat{\mathbf{p}}_\ell$ rather than a standard cut (19).

Algorithm 2 may accept a feasible power flow $\hat{\mathbf{p}}_\ell$ based on a non-optimal assignment of recovery spares. Assumption 2 lets us apply an approximate cut (20) in this case, but then an SEC must be added to ensure that $\hat{\boldsymbol{\delta}}_\ell$ does not repeat. Actually, SECs alone suffice for convergence, but we find that approximate cuts do accelerate convergence, in practice.

B. Algorithm 3: Enumeration to Solve All Subproblems

The number of recovery spares is likely to be small, so we may be able to solve the mixed-integer subproblem OPF-T($\hat{\boldsymbol{\delta}}_\ell$) by enumerating assignments of spares to replace disabled HVTs. This method's advantage is that a set of "simple," easy-to-solve LPs results from a fixed assignment $\hat{\boldsymbol{\beta}}$. By contrast, B&B applied to OPF-T($\hat{\boldsymbol{\delta}}_\ell$) must solve a sequence of large LPs, each linking the simple LPs through constraints (12).

Algorithm 3

Modify Algorithm 1 as follows: In Step 1, for each $\hat{\boldsymbol{\beta}} \in \mathrm{B}(\hat{\boldsymbol{\delta}}_\ell)$, solve the restricted version of OPF-T($\hat{\boldsymbol{\delta}}_\ell$) with $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$, and select the best restricted solution as the solution to OPF-T($\hat{\boldsymbol{\delta}}_\ell$).

C. Algorithm 4: Enumeration to Screen Subproblems

Solving OPF-T($\hat{\boldsymbol{\delta}}$) through enumeration of $\hat{\boldsymbol{\beta}} \in \mathrm{B}(\hat{\boldsymbol{\delta}})$ may become computationally prohibitive for large problems, but we can exploit "truncated enumeration" in a screening process that resembles the truncated B&B of Algorithm 2:

Algorithm 4

Proceed as in Algorithm 1, but with these modifications:
In Step 0, also define $0 < \eta \leq 1, 0 < \mu < 1$, (e.g., $\mu = 0.01$).
Replace Step 1 with the following:

a. Compute $N_{\text{tot}}(\hat{\boldsymbol{\delta}}_\ell)$, the number of maximal spares assignments given $\hat{\boldsymbol{\delta}}_\ell$, and let $ctr \leftarrow 1$, $\hat{\mathrm{B}} \leftarrow \emptyset$.
b. Identify $\hat{\boldsymbol{\beta}} \notin \hat{\mathrm{B}}$ that is maximal with respect to $\hat{\boldsymbol{\delta}}$ and let $\hat{\mathrm{B}} \leftarrow \hat{\mathrm{B}} \cup \{\hat{\boldsymbol{\beta}}\}$.
c. Given fixed $\hat{\boldsymbol{\beta}}$, solve OPF-T($\hat{\boldsymbol{\delta}}_\ell$) for $\hat{\mathbf{p}} = \mathbf{p}^*(\hat{\boldsymbol{\delta}}_\ell, \hat{\boldsymbol{\beta}})$.
   Let $f(\hat{\mathbf{p}})$ denote the solution's objective value.
d. If $f(\hat{\mathbf{p}}) \leq \eta \underline{z}$,
   Add a solution-elimination constraint to eliminate $\hat{\boldsymbol{\delta}}_\ell$.
   Go to Step 2, but apply (20) rather than (19).
e. Set $ctr \leftarrow ctr + 1$.

f. If $ctr < \mu N_{\text{tot}}(\hat{\boldsymbol{\delta}}_\ell)$, go to Step 1b.

g. Solve OPF-T($\hat{\boldsymbol{\delta}}_\ell$) by B&B.

The parameter μ reflects the computational effort we are willing to spend to determine that $\hat{\boldsymbol{\delta}}_\ell$ is non-optimal. If that level of effort is reached, the enumeration is halted, and B&B solves the subproblem exactly.

D. Algorithm 5: Solving Master Problems by Enumeration

Assumptions 1 and 2 imply that an optimal interdiction plan $\boldsymbol{\delta}^*$ will be maximal with respect to the interdiction-resource constraint, that is, $\boldsymbol{\delta}^* \in \Delta'$. When $|\Delta'|$ is modest, an enumerative method may solve MP(Δ_ℓ) more efficiently than B&B can. Let $\hat{\Delta}_\ell = \{\hat{\boldsymbol{\delta}}_1, \ldots, \hat{\boldsymbol{\delta}}_\ell\}$ denote the (maximal) master-problem solutions in the order generated, and define

$$\bar{z}_\ell(\boldsymbol{\delta}) = \min_{\ell'=1,\ldots,\ell} \left\{ f(\hat{\boldsymbol{\delta}}_{\ell'}) + \sum_{k \in \mathcal{K}} \alpha_k(\hat{\boldsymbol{\delta}}_{\ell'})\delta_k \right\} \forall \boldsymbol{\delta} \in \Delta'. \quad (21)$$

Given $\bar{z}_{\ell-1}(\boldsymbol{\delta})$ for all $\boldsymbol{\delta} \in \Delta'$ and given a new cut (20) in iteration ℓ, we can compute (21) recursively:

$$\bar{z}_\ell(\boldsymbol{\delta}) = \min \left\{ \bar{z}_{\ell-1}(\boldsymbol{\delta}),\ f(\hat{\boldsymbol{\delta}}_\ell) + \sum_{k \in \mathcal{K}} \alpha_k(\hat{\boldsymbol{\delta}}_\ell)\delta_k \right\} \forall \boldsymbol{\delta} \in \Delta', \quad \text{where } \bar{z}_0(\boldsymbol{\delta}) \equiv \infty\ \forall \boldsymbol{\delta} \in \Delta'. \quad (22)$$

Then, of course

$$\bar{z}(\hat{\Delta}_\ell) = \max_{\boldsymbol{\delta} \in \Delta'} \bar{z}_\ell(\boldsymbol{\delta}), \text{ and } \hat{\boldsymbol{\delta}}_{\ell+1} = \operatorname*{argmax}_{\boldsymbol{\delta} \in \Delta'} \bar{z}_\ell(\boldsymbol{\delta}). \quad (23)$$

Thus, MP($\hat{\Delta}_\ell$) can be solved by making only $O(|\Delta'|)$ cut-value calculations. For example, consider a model instance with this simple attack-resource constraint: "choose five targets to strike out of 100." In this case $|\Delta'| = C(100,5) \approx 7.5 \times 10^7$ and, if we store $z_\ell(\boldsymbol{\delta})$ as a simple array, the total number of additions and comparisons to compute $\bar{z}(\hat{\Delta}_\ell)$ may not exceed 10^9, a modest number.

Algorithm 5

Proceed as in Algorithm 4, with this modification: in Step 2, solve MP($\hat{\Delta}_\ell$) using enumeration; see (22) and (23).

V. Computational Results

A. Test Cases and Algorithms

We test variants of GLBD for solving PFI on 1) "RTS," the IEEE Two-Area 1996 Reliability Test System (see [25] and [26]), and on 2) "USRG," a realistic test system based on data for a "U.S. Regional Grid," derived from a reliability council at NERC. Tests in [8] and [11] use the same basic datasets; we use them here because an interested reader could reproduce our work using the RTS data, and because tests on the larger USRG dataset help demonstrate the computational capabilities and limitations of our methods. We also note that [14], [19], and [20] test interdiction models using the RTS data.

RTS has only one transformer type, a 230–138 kV HVT. A total of ten are distributed across four substations, with at most four HVTs in any substation. We assume that any disabled HVT may be replaced by a recovery spare, if one is available.

USRG has over 500 substations, with up to 15 HVTs in each; it also has 70 different HVT types. To simplify, we follow the STEP criteria [4], and model recovery spares only for HVTs having low-side ratings of at least 69 kV and high-side ratings of at most 500 kV. (No HTVs in USRG exceed the latter criterion.) Four transformer types fit these criteria, although we omit one type which occurs in only five, low-capacity substations: these substations are "invulnerable to attack" for purposes of testing. A recovery HVT must have the same voltage ratings as the HVT it replaces. USRG also has about 500 small HVTs with low-side ratings below 69 kV. For simplicity, we assume that these are replaceable quickly with new procurements, in unlimited numbers.

TABLE I
Summary of Test Cases

Component type	Number of Components RTS	Number of Components USRG	D_k	Num. of spares	Repair time (h) w/ spare	Repair time (h) w/o spare
Line	69	~5,000	1	0	n/a	48*
Bus	48	~5,000	2	0	n/a	168*
Transformer						
1: 230-138 kV	10	0	2	0-4, ∞	240	720
2: 345-138 kV	0	151		0-3, ∞	240	720
3: 230-69.6 kV	0	27		0, ∞	240	720
4: 230-69.4 kV	0	5		n/a	n/a	n/a
5: 138-69 kV	0	274		0-1, ∞	240	720
low side < 69 kV	0	~500		0	n/a	168*
Generator	66	~500	∞	0	n/a	n/a
Substation	4	~500	3	n/a	by comp.	by comp.

An attack on a substation disables all internal components and repair times are thus "by component" ("by comp."). Values indicated by "*" may also be viewed as reflecting unlimited spares. Transformer type f = 4 appears only in substations deemed "invulnerable to attack," and hence "n/a"appears in the corresponding row.

TABLE II
Summary of Algorithms by Test Case

Alg.	Solution methods: Subproblem	Solution methods: Master prob.	Results reported? RTS	Results reported? USRG
1	Standard B&B	B&B	No[1]	No[1]
2	Truncated B&B	B&B	Yes	No[2]
3	Full enumeration	B&B	Yes	No[3]
4	Screen by truncated enum.	B&B	No[4]	Yes
5	Screen by truncated enum.	Enumeration	No[5]	Yes

[1]Subproblems solve too slowly in "basic" GLBD (Alg. 1) to be competitive.
[2]For USRG, large LPs in the subproblem's B&B tree solve too slowly.
[3]Enumerating spares assignments in USRG is combinatorially prohibitive.
[4]RTS is small, so subproblems solve best by enumerating spares assignments.
[5]Enumeration will solve the RTS master problems, but standard B&B is fast on these small problems and obviates the need for any specialized method.

Table I summarizes the data and special assumptions for all test cases. General assumptions follow those in [11]. Note that generators cannot be attacked, and all planning horizons are 720 h, meaning that any interdicted component in the relevant system could be repaired or replaced by that time.

For simplicity, tests on RTS assume that the load at each bus is fixed at the peak value specified in bus-data table in [26]. In USRG, a three-segment load-duration curve approximates time-varying demand at each of about 3000 demand buses.

Table II summarizes GLBD and its variants, lists the combinations of algorithm and dataset that are tested, and indicates why certain tests are omitted.

B. Implementation

We implement all algorithms using, primarily, Xpress 7.4 [27], and run all tests on a Lenovo W520 computer with 16 GB of RAM and two Intel Core i7 processors running at 2.5 GHz. Branch-and-bound solutions of OPF-T($\hat{\boldsymbol{\delta}}$) use Xpress's

TABLE III
COMPUTATIONAL RESULTS FOR RTS GRID

D	n_1	Interdicted [Lines,HVTs Buses, Subs.]	Load shed (GWh)	Algorithm 2 Num. of iter.	Algorithm 2 Time (sec)	Algorithm 3 Num. of iter.	Algorithm 3 Time (sec)
8	0	[0,1,0,2]	751.3	7	0.6	7	0.6
	1	[0,1,0,2]	559.3	8	1.2	10	1.2
	2	[0,1,0,2]	379.3	23	**5.6**	33	6.8
	3+	[0,0,4,0]	307.9	89	56.0	96	**45.3**
10	0	[0,2,0,2]	1,019.5	7	0.6	6	0.6
	1	[0,2,0,2]	827.5	7	0.9	6	**0.6**
	2	[0,2,0,2]	635.5	9	**1.4**	20	2.9
	3	[0,2,0,2]	443.5	50	**20.7**	85	47.1
	4+	[0,0,2,2]	388.8	159	173.6	142	**98.4**
12	0	[0,2,1,2]	1,075.5	17	1.6	17	1.6
	1	[0,2,1,2]	883.5	22	3.4	18	**2.5**
	2	[0,2,1,2]	691.5	24	**4.5**	58	13.3
	3	[0,2,1,2]	499.5	106	**81.3**	179	185.0
	4+	[0,0,6,0]	437.5	333	**696.6**	388	890.0

D specifies the attacker's total resource; see Table I for resource required to interdict an individual component. n_1 specifies the number of type-1 transformers (230-138 kV) available as spares. "[Lines, HVTs, Buses, Subs.]" specifies the optimal number of interdicted lines, individual HVTs, buses and complete substations, respectively. A solution time listed in bold font is the shorter for the two algorithms.

MIP solver, with a Newton barrier algorithm applied at the root node. When full or partial enumeration applies to OPF-T($\hat{\boldsymbol{\delta}}$), Xpress solves the first OPF model with the Newton barrier algorithm, but solves subsequent OPFs using the dual simplex algorithm and exploiting advanced starting bases. Xpress manages its own multi-thread strategy. For Algorithms 1–4, Xpress generates the master problems, but CPLEX 12.3 [28] solves them using four parallel threads. (Solution times increase with more than four threads.) Algorithm 5 is coded entirely with Xpress, with no parallel processing in the enumeration.

C. Testing on IEEE Two-Area Reliability Test System Grid

Table III presents test results on RTS for a number of scenarios that differ by the total amount of attack resource D, and the number of recovery HVTs n_1. Any component of the listed types may be interdicted. Column three shows the number of lines, individual HVTs, buses and substations that are attacked in that row's solution. Column four, "Load shed," measures total unserved demand for electricity in GWh. This value is a surrogate for the true objective, which penalizes load shedding heavily compared to generating cost.

The defender enjoys increasing benefit as n_1 increases to 3 or 4, depending on the case, but beyond those values no additional benefit accrues. For example, when $D = 12$ and n_1 reaches 4, the attacker shifts from attacking substations, which do contain HVTs, to attacking individual buses only: more spares cannot help then. On the other hand, when $D = 10$ and $n_1 \geq 4$, the attacker continues to interdict two substations, even though recovery spares quickly replace key, disabled HVTs in those substations: the attacker must be satisfied with the disruption caused before recovery spares replace those HVTs. Wiring and load are crucial here, because replacing two of four disabled HVTs in either substation brings the substation back to full functionality. These results suggest that no simple, heuristic rules will consistently identify an optimal recovery-spare inventory.

TABLE IV
COMPUTATIONAL RESULTS FOR USRG

Number of spares (n_2,n_3,n_4,n_5)	Load shed (GWh)	n'	Alg. 4 (MP solved by B&B) Time to 5% gap (h)	Alg. 4 Time to 1% gap (h)	Alg. 4 Iter. to 5% gap	Alg. 5 (MP solved by enumeration) Time to 5% gap (h)	Alg. 5 Time to 1% gap (h)	Alg. 5 Iter. to 5% gap
(0,0,0,0)	873.5	500+	1.21	2.30	480	n/a	n/a	n/a
(1,0,0,1)	620.7	74	22.26	DNF	1,500	**3.94**	**5.64**	2,099
		18	0.67	0.95	437	**0.59**	**0.74**	550
(2,0,0,1)	558.6	74	15.30	DNF	1,382	**4.15**	**5.92**	2,573
		18	**0.86**	**1.18**	514	0.92	1.23	841
(3,0,0,1)	514.6	74	**5.62**	DNF	846	6.36	**9.70**	3,917
		18	**0.43**	**0.78**	252	0.73	1.87	667
$(\infty,\infty,\infty,\infty)$	514.6	500+	1.45	2.25	54	n/a	n/a	n/a

Cases have attack resource $D = 15$. The number of spares of type f, $f \in \{2, \ldots, 5\}$denoted by n_f. (See Table I.) The number of vulnerable substations is n'. Both algorithms use $\eta = 1.0$ and $\mu = 0.01$ for enumerative screening of subproblems. A 24-hour time limit applies to all test runs; entries marked "DNF" did not finish within that limit; "n/a" indicates that combinatorial requirements make this test impossible. Bold solution times indicate the faster algorithm for "Time to 5% gap" and for "Time to 1% gap."

D. Testing on U.S. Regional Grid

Table IV displays results for USRG assuming $D = 15$. We test only Algorithms 4 and 5, as explained under Table II. Given the values for D_k and the large number of substations, the attacker will interdict exactly five substations.

Algorithm 4 solves the baseline, no-spares case for USRG to within 1% of optimality in 2.30 h. We cannot solve a model instance that include all types of spares and all 500+ transformers in less than 24 h, however. To reduce solution times in practice, we apply a heuristic criterion to reduce the attacker's potential target set: 1) a substation is deemed "vulnerable to attack" if it appears in the m most disruptive interdiction plans observed while solving the baseline case. For $m = 25$, this criterion gives $n' = 18$ vulnerable substations, and for $m = 50$ it gives $n' = 35$. However, to create a more computationally challenging test, we add a second criterion for $m = 50$: 2) a substation is also deemed vulnerable if its total transformer capacity exceeds 4 GW. This produces $n' = 74$, with 64 substations having spare-replaceable HVTs. The total number of these HVTs is 116, with up to four in any substation.

The numbers above may seem modest, but an attacker interdicting exactly five of 74 substations still has over 16 million choices. And even the most-restrictive selection criterion appears to be a good one, because 1) near-optimal interdiction plans for $n' = 18$ and $n' = 74$ are identical in all cases, 2) solutions only target substations with spare-replaceable HVTs, 3) for $(n_2, n_3, n_4, n_5) = (1, 0, 0, 1)$, the unrestricted problem (i.e., all substations are vulnerable) yields a solution within 5% of optimality (in about 50 h) that is identical to the restricted problem's solution, and because 4) sufficiently large inventories of recovery spares in the restricted model lead to the solution obtained from the unrestricted model given $(n_2, n_3, n_4, n_5) = (\infty, \infty, \infty, \infty)$. (We simulate the last case by using a fixed replacement time for each HVT of 240 h.) Table IV shows load-shedding results for USRG that are qualitatively similar to the results for RTS: a small inventory of recovery HVTs reduces load-shedding significantly, but no additional benefit accrues after modest, "maximally useful inventory levels" are reached.

TABLE V
MAXIMALLY USEFUL INVENTORIES OF RECOVERY HVTs FOR USRG

Interdicted Substation	Recovery HVTs $f=2$ $n_{ft}^{\rm tot}$	$f=2$ $n_{ft}^{\rm rep}$	$f=3$ $n_{ft}^{\rm tot}$	$f=3$ $n_{ft}^{\rm rep}$	$f=4$ $n_{ft}^{\rm tot}$	$f=4$ $n_{ft}^{\rm rep}$	$f=5$ $n_{ft}^{\rm tot}$	$f=5$ $n_{ft}^{\rm rep}$
s_1	2	0	0	0	0	0	0	0
s_2	3	2	0	0	0	0	0	0
s_3	3	0	0	0	0	0	0	0
s_4	2	1	0	0	0	0	0	0
s_5	2	0	0	0	0	0	1	1
Total	12	3	0	0	0	0	1	1
n_f		3		0		0		1
$\bar{n}_f$		16		4		0		11

This table reports detail of a near-optimal solution to PFI given $n'=74$ interdictable substations, and a maximally useful inventory of recovery spares n_f for HVT types $f\in\{2,\ldots,5\}$. The solution interdicts five substations, $s_1,\ldots,s_5$, each having $n_{fs}^{\rm tot}$ spare-replaceable HVTs of type f (all of which are disabled). The value $n_{fs}^{\rm rep}$ specifies how many of the disabled HVTs are replaced by recovery spares. For reference, $\bar{n}_f$ gives the maximum number of HVTs of type f that could be disabled by an attack on five substations, with each f considered individually.

Those levels are $(n_2,n_3,n_4,n_5)=(3,0,0,1)$, here. (Results for the unrestricted problem, reported in the last row of Table IV, corroborate this.) Table V gives details on the worst-case attack when $(n_2,n_3,n_4,n_5)=(3,0,0,1)$ and $n'=74$. These details make the point more starkly: total cost after a worst-case attack does not improve by increasing the inventory of recovery HVTs beyond four, but that attack disables $\sum_{f,s} n^{\rm tot}=13$ HVTs.

While it is certainly possible for an optimal inventory of spares to exceed the number required to recover from a worst-case attack, this does not happen here. In fact, the single type-5 recovery spare used appears in an interdicted substation that also contains a higher-voltage type-2 HVT. Thus, it appears that, for USRG, the spare-replaceable HVTs that handle the highest voltages are the critical ones.

In summary, we see that, given assumed component-replacement times, relatively small inventories of recovery HVTs serve to protect USRG optimally against a comparatively severe attack. Naturally, other transmission grids might prove to be less robust, and a more detailed study should be executed before implementing any model-based recommendations.

As a final point on the USRG tests, we wish to note that Algorithm 5 shows more promise than the reported solution times indicate. This algorithm, which solves its master problem by enumeration, is substantially faster than Algorithm 4 in some cases, yet is slower in others. But, we have programmed the enumeration using Xpress's algebraic modeling language Mosel, which is a scripting language not intended for intensive numerical calculations. Preliminary tests show that a C++ implementation would reduce the listed solution times listed by at least half. Furthermore, simple enumeration executes easily on parallel processers, and this should reduce solution times further. Less promising are the extra iterations that Algorithm 5 requires compared to Algorithm 4. This issue requires further study.

VI. CONCLUSION

This paper has described an attacker-defender (AD) model for an electric power transmission grid that maintains an inventory of high-voltage transformers (HVTs) for quick replacement; these are special "recovery transformers" or "recovery spares." Solutions of this model could help guide the related inventory policy for a utility company or a group of utility companies facing an adversarial threat. For example, results for a realistic problem show that small inventories of recovery spares for only two HVT types suffice to protect the grid as best possible, even though much larger numbers of those types could be disabled in a worst-case attack.

The new AD model extends earlier work that asserts a fixed repair time for each disabled grid component. A solution optimally assigns a given inventory of recovery spares to minimize total cost, measured as generating cost plus economic cost for load shedding. A deterministic replacement time applies to any piece of equipment that requires new procurement or for which spares exist in large numbers.

GLBD solves the AD model. Given a coordinated attack on a set of components, a mixed-integer subproblem within GLBD optimizes assignments of recovery spares using an implicit evaluation of a set of standard optimal power flows for every possible assignment. We note that the technique could be applied to other major equipment with limited numbers of spares.

Successful solution of realistic model instances depends on several new algorithmic techniques. The fastest algorithms screen a potential interdiction plan for non-optimality by partially enumerating assignments of spares and comparing results against the incumbent plan. If non-optimality of the plan is proven quickly, the algorithm generates a solution-elimination constraint and a valid inequality (i.e., an "approximate cut") for the master problem. If not, branch and bound solves the subproblem to completion and generates a standard cut. Another promising technique is the enumeration of feasible interdiction plans to solve the decomposition's master problem. Even an implementation in a scripting language reduces solution times substantially for some model instances.

APPENDIX
SPARE HVTs FOR ATTACKS AND NATURAL DISASTERS

Based on OPF-T($\hat{\boldsymbol{\delta}}$), this Appendix outlines a model for analyzing the value of an inventory of recovery HVTs to improve the expected response of a transmission grid to both deliberate attacks and probabilistically modeled natural disasters such as hurricanes. We assume that a simulation (e.g., [29], [30]) has identified a finite set of natural-disaster damage scenarios, along with their probabilities of occurrence.

Additional notation:

$s\in\mathcal{S}$	natural-disaster scenarios;
$\hat{\boldsymbol{\delta}}_s$	damaged equipment for scenario s; $(\hat{\boldsymbol{\delta}}_s)_k=1$ if component k is disabled in scenario s, and 0 otherwise;
p'_s	probability that damage scenario s occurs during a given year;
$p_{\rm A}$	probability that an attack occurs during a given year (see [3]);
$\mathbf{n}$	inventory of spare parts (e.g., $\mathbf{n}=(n_2,n_3,n_4,n_5)$ as in Section V-D);
$f_{\mathbf{n}}(\hat{\boldsymbol{\delta}})$	same as $f(\hat{\boldsymbol{\delta}})$, but with $\mathbf{n}$ added to emphasize inventory;
$z^*_{\mathbf{n}}$	expected total cost, with inventory vector $\mathbf{n}$ noted.

Because attacks and natural disasters are "low-frequency events" [31], we assume that all such events are mutually exclusive. Also, for clarity, we have added explicit notation to represent the inventory of spares. The new model, measuring expected total cost per year, has the following form:

$$z^*_{\mathbf{n}} = p_{\mathrm{A}} \max_{\boldsymbol{\delta} \in \Delta} f_{\mathbf{n}}(\boldsymbol{\delta}) + \sum_{s \in \mathcal{S}} p'_s f_{\mathbf{n}}(\hat{\boldsymbol{\delta}}_s). \tag{24}$$

The first term in (24) uses PFI, as described in the body of the paper, to evaluate the expected, yearly, total cost associated with attacks. The second term uses PFI's subroblem OPF-T($\hat{\boldsymbol{\delta}}$) to evaluate the analogous cost for natural disasters. A discounted, expected total cost could be computed for a range of values of $\mathbf{n}$ and an appropriate inventory selected. We note that, in the course of solving PFI, our decomposition algorithms often compute $f_{\mathbf{n}}(\hat{\boldsymbol{\delta}}_\ell)$ for more than a thousand "attack damage scenarios" $\hat{\boldsymbol{\delta}}_\ell$. Thus, evaluating a thousand or more random damage scenarios $\hat{\boldsymbol{\delta}}_s$ to calculate the second term in (24) should pose no computational difficulties.

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