Reshaping the US Army: Brigade Combat Team Optimization

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APPLICATION AREA: Readiness
OR METHOD: Mixed-Integer Linear Programming

Following 9/11, our Army began its most significant reorganization since World War II to ensure that the formations of all components are fully manned, equipped, and trained.

—GEN Peter Schoomaker, US Army Chief of Staff, December 2006

ABSTRACT

In 2003, the United States Army began one of the most extensive transformations in recent history by transitioning from a division-centric to a brigade combat team (BCT)-centric force. In 2010, with lengthy counterinsurgency campaigns in Iraq and Afghanistan winding down, the US Army needed to assess the impact of that transformation on its ability to conduct future campaigns. Specifically, the US Army needed to determine the ability of BCT mixes and designs to meet projected operational demands. The US Army Training and Doctrine Command Analysis Center was tasked to conduct this study. In this article, we discuss the optimization model that we developed to assess the best *force mix* to meet those demands. The model takes into account the missions that BCTs must accomplish, the Army Force Generation process, and active and reserve components, as well as other relevant factors. Along with other studies, this study informed the Army's Force Modernization Review/Program Objective Memorandum 13-17, and Total Army Analysis 14-18.

INTRODUCTION

During the first decade of the 21st century, the US Army fought two major counterinsurgency campaigns in Iraq and Afghanistan. In 2010, as the Army drew down from Iraq and focused on surging in Afghanistan, the US Army Training and Doctrine Command (TRADOC) Analysis Center (TRAC) was tasked with a series of force design studies to assist the Chief of Staff of the Army in understanding if the US Army had the proper force mix to address future operational demands. These force-design studies helped form a larger body of analysis, the *Total Army Analysis* (US Army, 2014), which examined the Army's ability to meet operational demands in the years 2014–2018. At the core of the study is determining the best mix of brigade combat teams (BCTs) to meet operational demands. In this article we develop a mixed-integer, linear optimization model called the BCT assignment optimization (BCTAO) model. The BCTAO model allows analysts to examine how to best assign BCTs to missions over time in order to maximize the ability to meet operational demands across the force.

BACKGROUND

Context

In September 2003, TRADOC established *task force modularity* to develop organizational constructs in response to a Chief of Staff of the Army direction to provide modular, capable Army units to combatant commanders. TRADOC's construct included brigade-level modular formations called BCTs. These BCTs were designed to meet the projected challenges of ongoing operations in both Iraq and Afghanistan. They were meant to be as capable as their predecessors, easier to deploy, and allow the Army to build additional maneuver brigades within current Army end-strength, providing strategic depth. This was the beginning of a campaign of comprehensive analyses (Figure 1) that continues today, fundamentally reshaping the Army to be prepared for both current and future contingencies in an uncertain world. The analysis we describe here is the first of three critical studies examining the trade space between keeping a larger number of BCTs that have predominantly two maneuver battalions, and creating a smaller number of BCTs that all have three maneuver battalions.

As a result of the transformation begun in 2004, the US Army had 73 BCTs in 2010. The mix of those BCTs was: 24 heavy BCTs (HBCTs) equipped with two maneuver battalions of M1A2 Abrams tanks and M2A3 Bradley infantry fighting vehicles; 40 infantry BCTs (IBCTs) equipped with two maneuver battalions of dismounted infantry; and nine Stryker BCTs (SBCTs) equipped with three maneuver battalions of Stryker, medium-weight, fighting vehicles. Additional support brigades (which are deployed with the BCTs according to the mission requirements) include artillery, engineer, military police, and aviation brigades.

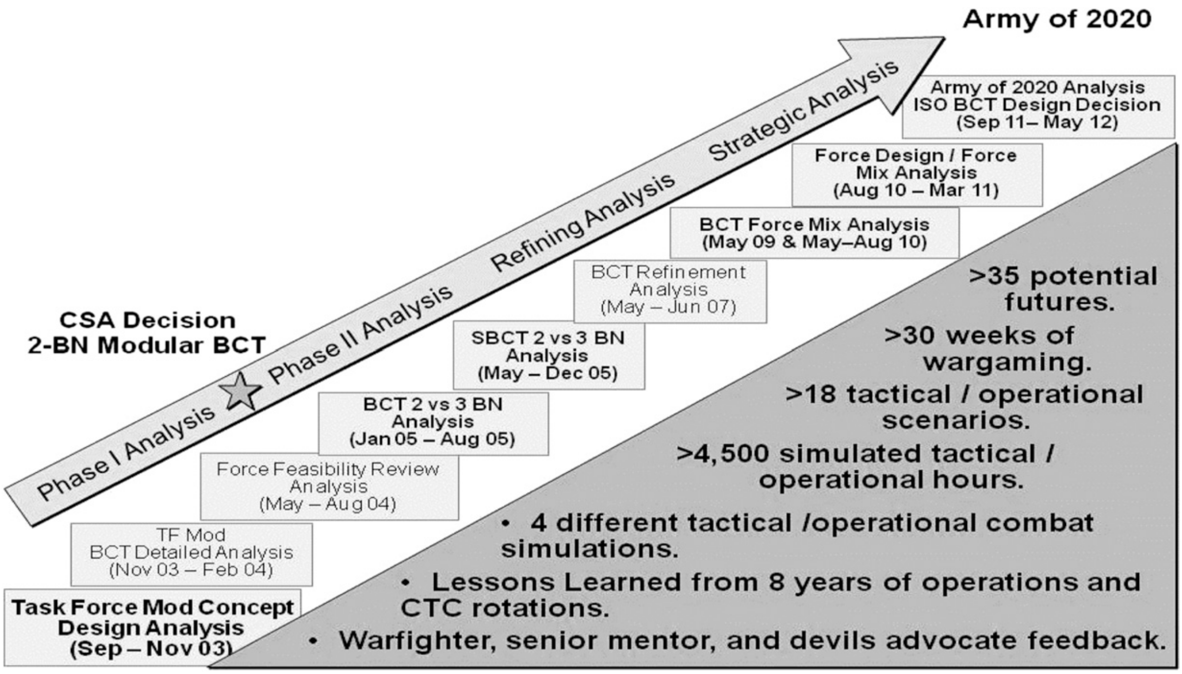

Figure 1. Campaign of Comprehensive Analysis, 2003 to Present (TRAC, 2013).

Study Focus and Elements

The study seeks the mix of IBCTs, HBCTs, SBCTs, and/or their variants that maximizes sufficiency and effectiveness over time and meets operational demands with minimal risk, consistent with the Army Force Generation (ARFORGEN) process. The study focuses on the two-battalion and three-battalion BCT configurations.

Each BCT type has a different organization of personnel and equipment that makes it more or less suitable (or ''preferred'') for specific missions. The Army Affordable Modernization Strategy proposed a three-maneuver battalion design for the HBCTs and IBCTs, resulting in 61 BCTs: 19 HBCTs, 9 SBCTs, and 33 IBCTs. The study team examined options from 57 to 73 BCTs when assessing the three-battalion conversion for the HBCT and IBCT. This study a) identified the optimal BCT mix by quantity and type, and b) assessed the benefit of adding a third maneuver battalion to the current two-battalion IBCT and HBCT designs, in order to assist leaders in making resource decisions for 2012–2018. The analysis had to account for force sufficiency (percent demands met over a seven-year timeline) and force effectiveness (percent demands satisfied by the most-preferred BCT types available). The study assessed BCT mix variations tied to force modernization (addition of the Brigade Engineer Battalion and protected mobility capability) and the addition of a third maneuver battalion to heavy and infantry BCTs. To properly assess sufficiency and effectiveness, TRAC used a range of operations across three different integrated security constructs (ISCs).

Integrated Security Construct Scenarios and Demand

TRAC obtained three variations of ISCs from the Army G-3/5/7. Each ISC contains a mix of one-time scenarios and/or phases of operations (all referred to as scenarios for the remainder of this article). Threats varied from conventional, paramilitary, insurgent, terrorist, or criminal to a hybrid mix of several of these threat types. Operations varied, including major combat operations (MCO), counterinsurgency (COIN), combating terrorism, foreign internal defense, peacekeeping and peace enforcement, noncombatant evacuation, foreign humanitarian assistance, consequence management, security assistance, and counter-drug missions. Each scenario also had an associated duration (usually seven years), start and finish time, and requirement for numbers of BCTs.

A ''demand'' in a given scenario is identified as a situation that requires the commitment of one or more BCTs to conduct any of the above operations to reduce or eliminate the threat. ISC scenarios provide demand signals for two-battalion BCTs. Demand signals for the three-battalion scenarios are derived by adjusting the scenario demands after accounting for increased structure and capability of the three-battalion design. For example, a demand for three two-maneuver-battalion BCTs would be equivalent to a demand of two three-battalion BCTs.

ISC ''A'' had the lowest mission demand, consisting of one MCO scenario in addition to the other types of operational commitments. ISC ''B'' contained two MCOs but with a smaller BCT requirement for each MCO, resulting in only a slightly higher demand over the seven-year planning horizon. ISC ''C'' had a COIN focus with no MCOs; however, ISC ''C'' had the highest demand of BCTs required over time.

Figure 2 depicts notional demands of each BCT type in our baseline scenario for ISC ''A.'' Demands are represented in quarterly increments, which are deemed to be sufficient resolution for this study. That is, we assume our demand signals are an accurate representation of forces required, and that the forces assigned are adequate for mission accomplishment. Operational planning will require more refined timelines when establishing a BCT's actual training and deployment plans.

To simplify our computations, we do not model the ''Relief in Place/Transfer of Authority'' (RIP/TOA) process explicitly. (This process establishes that for a given mission, ideally, units would have a short overlap period to ensure a seamless transition.) RIP/TOA is a crucial enabler for mission success and must be factored in when creating the final deployment and redeployment timelines for each individual BCT. However, we assume its exclusion from the BCTAO model will not significantly affect the comparison among several BCT mixes to fit the given missions over the extended timeline.

BCT mission suitability, which we use as a surrogate for operational effectiveness, is derived from current and former BCT commanders' and selected general officers' input through a process called the ''Warfighter Input Collection.'' The study team converted inputs from warfighters into quantifiable distinction coefficients, before applying optimization. The specific suitability inputs required warfighters

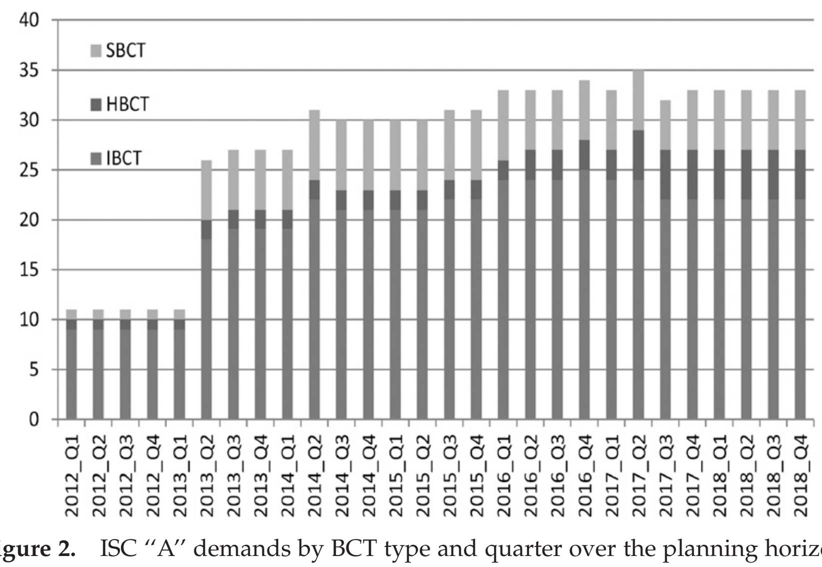

Figure 2. ISC ''A'' demands by BCT type and quarter over the planning horizon.

to include their assessments on the ability of each BCT design to meet demands associated with scenarios drawn from ISCs.

Modernization and Reorganization

The Army's Program Objective Memorandum (POM) is a document that indicates how the Army should allocate resources over the next six years to achieve its objectives. To ensure an Army modernization strategy is incorporated in the POM for 2014–2018, TRAC was tasked with evaluating a range of strategic choices using future projections based on a foundation force design and a force-mix analysis. Although these strategic choices vary, they all abide by the following planning guidance:

- Each individual BCT can become a "modernized" BCT or remain "unmodernized" (i.e., unchanged). The modernization of an individual BCT results from a change in equipment during the course of the study. In our study, when the timeline begins, all BCTs are "unmodernized."
- IBCTs and HBCTs may be "reorganized" by transitioning from two to three maneuver battalions. SBCTs have three maneuver battalions at all times.
- Once a BCT is modernized and/or reorganized, it cannot revert to its previous state.

It is important to note that our optimization model does not explicitly recommend a modernization or reorganization timeline. We use the model to assess the effectiveness of a number of modernization or reorganization timelines proposed by planners. Specifically, we analyze three BCT mixes:

- *Baseline* uses current organization of 73 BCTs, as described earlier.
- *Baseline enhanced* uses the same 73 BCTs as in the baseline case, but adds a brigade engineering battalion to each HBCT, SBCT, and IBCT, and adds protected mobility to IBCTs. The term "protected mobility" for the IBCT is centered on capability, not a specific system, such as the mine resistant ambush protected vehicle.
- *Reorganized over time* starts with the baseline organization, but after a certain number of years (depending on the BCT), two-battalion BCTs transition to a three-battalion design for the rest of the planning horizon. The transition reduces the number of BCTs from 73 to as few as 57 over the seven years of the planning horizon.

Army Force Generation

The ARFORGEN (Department of the Army, 2008) process is "a cyclic training and readiness strategy that synchronizes strategic planning, prioritization and resourcing to generate trained and ready modular expeditionary forces tailored to joint mission requirements." More specifically, ARFORGEN is a model used to manage the readiness of BCTs so that the appropriate number of trained BCTs is available to meet deployment requirements. To sustain trained BCTs available to deploy during multiyear operations, BCTs transition in cycles through a progression in three sequential readiness pools:

- *Reset* pool: Units enter this pool after returning from a deployment or during modernization or reorganization. They are available to support civil authorities for national emergencies.
- *Train/Ready* pool: Units conduct mission preparation and collective training for anticipated future missions. They are eligible for deployment to unanticipated contingencies or other operational requirements.
- *Available* pool: Units are available for worldwide deployment. At any time, available units are either deployed (assigned to satisfy a mission demand) or nondeployed.

Figure 3 depicts the ARFORGEN cycle with the possible transitions between these pools.

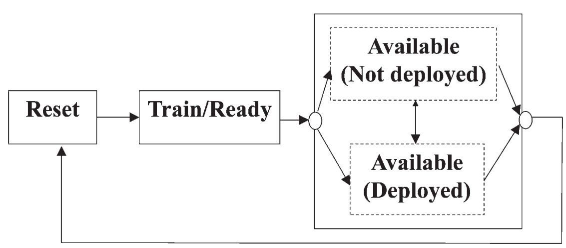

Figure 3. ARGFORGEN transitions.

MARATHON and BCTAO Models

In January 2009, a model called MARATHON, developed by the US Army Center for Army Analysis, was used to complete the 2009 BCT force-mix study. To complete the 2010 force-mix study, both MARATHON and BCTAO were used. Both models seek to maximize scenario demands satisfied over time by preferred BCTs while minimizing operational risk subject to the number and types of BCTs available by quarter, BCT preference, and ARFORGEN requirements.

Because BCTAO was a new model, it was important to benchmark BCTAO results with a model that had produced the 2009 BCT mix analysis. MARATHON output provided data that validated the results from the BCTAO model runs. The minor differences that did occur were explainable by the features and constraints of the two models. Once it had been verified that the BCTAO model and MARATHON provided consistent results, the BCTAO model was used to produce the 2010 analysis results.

There were two key features of the BCTAO model that led to its use for the 2010 study. First, the BCTAO model accounts for risk better than the MARATHON model because it matches BCTs to mission demands including a substitution ranking when a preferred BCT is not available (or, when a need for that BCT is anticipated for an upcoming mission). In MARATHON, if the preferred BCT was not available, the next-preferred BCT was selected, without any indication of the risk associated with not selecting the preferred BCT. Second, in MARATHON, demands were satisfied on a first-come, first-served basis, and prioritizing missions for limited BCT assets was not an option. Thus, even if a lesser contingency operational demand appeared one day prior to a critical MCO demand, the contingency demand was filled first. This shortcoming is corrected by the BCTAO model that optimizes all assignments simultaneously with priorities as deemed by the planners. (Note: The MARATHON model was updated in 2011 and now employs a prioritization scheme.)

As a consequence, the BCTAO model was selected and used by TRADOC to produce the results that were briefed to senior Army staff.

RELATED LITERATURE

Workforce assignment approaches aim to determine the staffing needed to cover a certain demand, usually over a period of time, with variants that include flexible starting times for the workers, shift length, break placements, and overtime (see, e.g., Thompson [1995]). On the other hand, the BCTAO model seeks to maximize fit assignment values and minimize unmet demand for a given BCT mix. Examples that penalize unmet demand include Bard and Wan (2008), who determine the optimal size and composition of a workforce to meet workstation needs. Webster (2011) finds optimal weekly schedules for airport security officers, minimizing gaps between passenger and bag demands and screening capacity in half-hour intervals. Castillo et al. (2009) use multiobjective optimization to combine several measures (such as labor cost and service quality) in the objective.

These models are formulated as MIPs, but they often do not scale up well enough to be solved with general-purpose MIP software. Alternatively, Jaumard et al. (1998) solve a nurse scheduling problem through column generation. A network flow version of staff scheduling problems can also be found in Balakrishnan and Wong (1990), where nodes represent the beginning and end of each day, and arcs represent a work period assigned to a specific shift or to a rest period. Sriram and Haghani (2003) deal with the problem of scheduling aircraft maintenance in compliance with aviation regulations, and then reassigning those to flight segments with the goal of minimizing cost. They use both formal optimization and a heuristic approximation for large, practical instances. This problem resembles our problem in that BCTs also need periodic "maintenance" (time in the reset and ready/train pools) in compliance with the ARFORGEN cycle before they can be assigned to a mission.

Previous force-mix studies have been carried out but no model so far was capable of meeting the desired capabilities for the study that TRAC intended to undertake. For example, the abovementioned MARATHON model (Brantley and Stoddard, 2005) uses discrete event simulation and explicitly accounts for factors such as the ARFORGEN cycle. Some extensions of the MARATHON model include a heuristic optimization interface seeking to maximize scenario demands satisfied over time by preferred BCTs: demands are satisfied on a first-come first-served basis, along with some prioritization rules for BCT assets.

MATHEMATICAL MODEL DEVELOPMENT

Specifications

The BCTAO model can be used with any period of study (time horizon) of choice. For this analysis, we used a seven-year time horizon divided into quarterly (three-month) periods. This allowed us to represent the scenarios' demand signals and ARFORGEN cycles adequately. We assume that all the below data involved in our model are deterministic.

BCTAO assigns BCTs to missions over time to maximize the "fit" of those assignments across the force. The term "fit" corresponds to the operational effectiveness of the BCT to perform a potential mission. The result that BCTAO provides is the BCT-mission fit over time that best meets operational demands. With this, planners may assess specific aspects of the assignment, such as sufficiency (i.e., if the mission demands are met) and effectiveness (i.e., the suitability of BCTs to perform their assigned mission). Each BCT has a suitability value (determined by subject matter experts in warfighter input collection) that represents that BCT's ability to perform the assigned mission. These values may be discounted over time to prioritize engagements in the shorter and medium terms. There exists a possibility that some missions are assigned to less suitable BCTs, given limited availability of the most suitable BCTs. Additionally, some missions may be left unmet, that is, there are not enough BCTs to assign a BCT for a particular mission. If a mission is unmet, a large penalty is incurred.

The particular BCT mix to assign is an input. Planners may assess the effect on the optimized fit of using BCT mixes where some BCTs transition over time, such as those that are modernized, or those converted from two- to three-maneuver battalions (HBCTs and IBCTs).

While in the ARFORGEN *available* pool, each BCT can be assigned to one mission demand at a time, or to no mission at all, so the BCT will be *available deployed* or *available nondeployed*, respectively. Each BCT is either in the active component (AC) or reserve components (RC), and thus will only be assigned to one ARFORGEN pool at any given time. In each of the pools, BCTs must spend between a minimum and maximum time that depends on the BCT component (see Table 1). In the example shown, AC BCTs have less flexibility in the time they must spend in the reset pool than RC BCTs; however, they have more flexibility in the other pools. In addition to adherence to the ARFORGEN cycle, the solution provided by the optimization must be consistent with the initial status of each BCT at the beginning of the planning horizon, and the time they have already been in that pool.

Planners may also consider "boots on the ground" (BOG):dwell ratio restrictions by component type or at other levels of aggregation. For AC units, the ratio is the amount of time deployed to the amount of time not deployed. For RC units, the ratio is measured as time mobilized to time not mobilized. The Army's goals for BOG:dwell and mobilized:not mobilized are

Table 1. ARGFORGEN time (in quarters) by readiness pool and component.

Pool	Component	Minimum time (quarters)	Maximum time (quarters)
Reset	Active	4	4
Reset	Reserve	8	12
Train/Ready	Active	3	5
Train/Ready	Reserve	4	4
Available	Active	3	5
Available	Reserve	4	4

1:3 and 1:5 for AC and RC units, respectively. Mission demands may require more than one type of BCT. We call these ''parts'' of the mission. For instance, a specific mission may require one IBCT and two HBCTs, that is, this mission would have a total of three parts requiring two different BCT types. In the scenarios used for this analysis, missions were comprised of up to five parts, often requiring different BCT types. For a given mission, each part that requires the same BCT type is rated at a certain fit level with respect to the different BCT types that can perform the mission. For example, Table 2 displays a set of missions and parts, along with fit (suitability) values for different BCT types that could perform each mission part. In this example, mission ''M15'' is comprised of five parts: one three-battalion HBCT and four three-battalion IBCTs. The fit values show the suitability of each type of BCT for a given mission part. The BCT types considered (see heading row in the table) range from a regular (two-battalion) IBCT to a three-battalion HBCT, with some variants (not all included). ''B'' indicates modernized BCTs, ''#B'' indicates modernized and reorganized BCTs. The unmet value is the penalty paid in any period (quarter) that a mission part is not accomplished by any BCT.

Note that each part is treated separately with respect to the overall mission. If a part cannot be met by any BCT, BCTAO will still attempt to allocate all other parts of that mission.

Because BCTAO does not consider a RIP/TOA process, there are never two BCTs assigned to the same part of a mission. However, BCTAO outputs can be slightly adjusted by operational planners, for example, to allow some arriving units to do so shortly before their start date, and/or some leaving units to do so shortly after their finish date.

Formulation

BCTAO can be stated as a mixed-integer, linear programming model. Notation and formulation follows.

Indexed sets (units).

- $t \in T$: set of time periods, e.g., quarters in $T = \{2011Q1, 2011Q2, \ldots, 2017Q4\}$. This set is assumed to be ordered and their elements will be referred to as $t = 1, 2, \ldots, |T|$
- $b \in B$: set of brigade combat teams (BCTs), e.g., $B = \{BCT1, \ldots, BCT73\}$
- $c \in C$, set of component types, $C = \{\text{Active}, \text{Reserve}\}$
- B_c^{Comp}: subset of BCTs in component c
- $p \in P$: set of BCT types, also used to define types of mission parts, e.g., $P = \{\text{Stryker}, \text{Infantry}, \text{Heavy}\}$
- B_p^{Type}: subset of BCTs of type p
- $a \in A$: set of readiness statuses as in the Army Force Generation process (ARFORGEN): Reset, Train/Ready, and Available, accordingly, $A = \{\text{R}, \text{TR}, \text{Av}\}$
- B_a^{Read}: subset of BCTs that start in readiness status a. Remark: This characterization is independent of the BCT availability. Each BCT should be included in one of the B_a^{Read} subsets, regardless of when it becomes available.
- A_a: subset of readiness statuses that allow a transition into readiness status a: $A_{\text{R}} = \{\text{Av}\}$, $A_{\text{Av}} = \{\text{TR}\}$, $A_{\text{TR}} = \{\text{R}\}$
- $m \in M$, set of missions

Table 2. Mission parts sample and fit values.

Mission	BCT type	No. of parts	IBCT	SBCT	HBCT	BSBCT	BIBCT	BHBCT	3BIBCT	3BHBCT	Unmet
M15	3BHBCT	1	0.004	0.101	0.086	0.148	0.006	0.095	0.012	0.154	−10
M15	3BIBCT	4	0.087	0.066	0.012	0.075	0.094	0.016	0.238	0.031	−10
M16	3BIBCT	1	0.071	0.093	0.022	0.133	0.093	0.024	0.153	0.035	−10
M16	BSBCT	2	0.004	0.118	0.080	0.155	0.005	0.094	0.010	0.153	−10
M17	BSBCT	1	0.003	0.187	0.045	0.270	0.004	0.063	0.010	0.102	−3
M18	3BIBCT	1	0.071	0.093	0.022	0.123	0.093	0.024	0.133	0.035	−3

Note: ''BCT type'' describes the type of BCT that provides the best fit (most preferred) for the mission part, as shown by the highlighted fit values.

Parameters (units).

- $t_b^{\text{BIni}}, t_b^{\text{BEnd}}$: initial and final periods when BCT b is available (period index)
- t_b^{B0}: number of periods that BCT b has been in its initial ARFORGEN readiness level (periods)
- n_{mp}: number of BCTs parts of type p required by mission m (BCTs)
- t_m^{MIni}: initial period for mission m (period index)
- d_m: duration of mission m (periods)
- f_{bmp}: fit for assigning BCT b to do part type p of mission m (fit units)
- q_{mp}: (negative) penalty for each part p of mission m that is left unmet in any time period (fit units)
- $\underline{t}_{ac}^{\text{Read}}, \bar{t}_{ac}^{\text{Read}}$: minimum and maximum number of consecutive periods, respectively, in readiness a for BCTs in component c (periods)
- w_t: discount factor for risk assessment in period t (fraction). Discount factors used in our testing include constant factors ($w_t = 1\ \forall t$) and linearly decreasing factors, by year [$w_t = 1 - 0.1(t-1)\ \forall t$].
- $\underline{d}_c, \bar{d}_c$: minimum and maximum BOG:dwell limits, respectively, for component c. Remarks: These limits are on the aggregate (i.e., average) for all BCTs together; the limit is calculated as time deployed (i.e., assigned to a mission) divided by total time the BCT is in the system for AC units and time mobilized to total time the BCT is in the system. (fraction)

Derived sets.

- T_b^B: subset of periods for BCT b. Calculated as $T_b^B = \{t_b^{\text{BIni}}, t_b^{\text{BIni}} + 1 \ldots, t_b^{\text{BEnd}}\}$.
- T_b^{B1}: subset of periods for BCT b excepting the first one. Calculated as $T_b^{B1} = \{t_b^{\text{BIni}} + 1, \ldots, t_b^{\text{BEnd}}\} = T_b^B \backslash \{t_b^{\text{BIni}}\}$
- T_m^M: subset of periods for mission m. Calculated as $T_m^M = \{t_m^{\text{MIni}}, t_m^{\text{MIni}} + 1, \ldots, t_m^{\text{MIni}} + d_m - 1\}$
- A_b^{Ini}: subset of readiness where BCT b may start, including transitions. Calculated as $A_b^{\text{Ini}} = \{a | b \in R_a^{\text{Read}}\} \cup \{a | a \in A_{a'} \wedge b \in B_{a'}^{\text{Read}}\}$

Decision variables.

- X_{bmpt}^B: one if BCT b is assigned to a part of type p in mission m in period t; zero otherwise
- U_{mpt}: number of unmet parts of type p in mission m in period t
- S_{abt}: one if BCT b starts in readiness a in period t; zero otherwise
- Y_{abt}: one if BCT b is in readiness a in period t; zero otherwise
- z^{BCTAO}: objective function of the BCTAO model (fit units)

Constraints.

$$\sum_a Y_{abt} = 1 \quad \forall b, t | t \in T_b^B \tag{1}$$

$$Y_{abt} \le Y_{ab,t-1} + \sum_{a' \in A_a} Y_{a'b,t-1} \quad \forall a, b, t | t \in T_b^{B1} \tag{2}$$

$$Y_{abt_b^{\text{BIni}}} = 0 \quad \forall a, b | a \notin A_b^{\text{Ini}} \tag{3}$$

$$\sum_{\substack{t' | t' \in T_b^B \\ t \le t' \le t + \bar{t}_{ac}^{\text{Read}}}} Y_{abt'} \le \bar{t}_{ac}^{\text{Read}} \quad \forall a, b, c, t | b \in B_c^{\text{Comp}}, t \in T_b^B \tag{4}$$

$$\sum_{\substack{t | t \in T_b \\ t_b^{\text{BIni}} \le t \le t_b^{\text{BIni}} + \bar{t}_{ac}^{\text{Read}} - t_b^{B0}}} Y_{abt} \le \bar{t}_{ac}^{\text{Read}} - t_b^0 \quad \forall a, b, c | b \in B_c^{\text{Comp}} \cap B_a^{\text{Read}} \tag{5}$$

$$S_{abt} \ge Y_{abt} - Y_{ab,t-1} \quad \forall a, b, t | t \in T_b^{B1} \tag{6}$$

$$S_{abt} \le 1 - Y_{ab,t-1} \quad \forall a, b, t | t \in T_b^{B1} \tag{7}$$

$$S_{abt} \le Y_{abt} \quad \forall a, b, t | t \in T_b^B \tag{8}$$

$$Y_{abt'} \ge S_{abt} \quad \forall a, b, c, t, t' | b \in B_c^{\text{Comp}}, t' > t, t' \le t + \underline{t}_{ac}^{\text{Read}} - 1, t \in T_b^B, t' \in T_b^B \tag{9}$$

$$Y_{abt} = 1 \quad \forall a, b, c, t | b \in B_c^{\text{Comp}} \cap B_a^{\text{Read}}, t_b^{\text{BIni}} \le t \le t_b^{\text{BIni}} + \underline{t}_{ac}^{\text{Read}} - t_b^0 - 1, t \in T_b^B \tag{10}$$

$$\sum_{\substack{m, p | \\ n_{mp} > 0, t \in T_m^M}} X_{bmpt}^B \le Y_{\text{"Av"}bt} \quad \forall b, t | b \in B^{BN}, t \in T_b^B \tag{11}$$

$$\sum_{b \in B | t \in T_b^B} X_{bmpt}^B + U_{mpt} = n_{mp} \quad \forall m, p, t | n_{mp} > 0, t \in T_m^M \tag{12}$$

$$\underline{d}_c \le \left(\sum_{b \in B_c^{\text{Comp}}} |T_b^B| \right)^{-1} \sum_{b \in B_c^{\text{Comp}}} \sum_{\substack{m,p,t| \\ n_{mp} > 0, t \in T_m^M}} X_{bmpt}^B \le \bar{d}_c \quad \forall c \tag{13}$$

$$X^B, X^H, Y, S \text{ binary } \{0,1\} \quad U \ge 0 \tag{14}$$

$$z^{\text{BCTAO}} = \sum_{m,p|n_{mp}>0} \sum_{t \in T_m^M} w_t \left(q_{mp} U_{mpt} + \sum_{b \in B | t \in T_b^B} f_{bmp} X_{bmpt}^B \right) \tag{15}$$

BCTAO model.

$$\text{BCTAO: Maximize } z^{\text{BCTAO}}$$

subject to constraints (1)–(15).

Constraint (1) ensures that, at any time, each BCT is in only one pool. Constraints (2) and (3) are used to enforce transitions between pools according to ARFORGEN process specifications. Constraint (4) guarantees these transitions follow time limits (consecutive periods in the same readiness process). The same condition, but at the beginning of the horizon (where we must account for time the BCT has already been in a given readiness status) is accounted for in constraint (5).

Constraints (6)–(8) identify the starting time in a new pool. Specifically, we force the "start" S variable to become one if there is an increase from zero to one in variable Y from period $t-1$ to period t. We also force S to become 0 if there is no change in Y or if it decreases from 1 to 0.

Constraints (9)–(10) are similar to constraints (4)–(5) for the minimum time a BCT must remain in a given pool.

Constraint (11) allows the assignment of each BCT to one mission if its readiness status is available. Constraint (12) ensures each demand signal (i.e., mission-part required in a given period) is assigned a BCT or flagged as unmet. Constraint (13) calculates and limits the fraction of the time all BCTs in each component deploy, with respect to the total time the BCTs exist in the model. Constraint (14) specifies the types of decision variables. Constraint (15) states the objective function consisting of positive "fit" values, and negative weights (i.e., penalties) for unmet demand, which TRADOC planners typically refer to as "risk."

Adjustments

The BCTAO model seeks to achieve the maximum fit, but there are no constraints to enforce continuity in the assignment of BCTs to missions. That is, a BCT may be assigned to a certain mission part in period t, and then be reassigned to a different part in period $t+1$. Although there may be real-life situations that require such an assignment, there are many instances where such a reassignment makes no operational sense. To mitigate this problem, we create a second, adjusted BCTAO (ABCTAO) model.

ABCTAO's foundation is still the BCTAO model. In fact, the BCTAO model is run first, and its output recorded. We then define an "unforced change" for each occurrence where a BCT is assigned to two consecutive, but different, parts, for parts occurring during those periods. For example, an unforced change would occur if "BCT #1" is assigned to a part "1" of "Mission 1" in period $t = 1$, and then it is assigned to part "2" of that same mission, or a part of any other mission while "Mission 1" is still ongoing in period $t = 2$. Additional notation and modifications to the BCTAO model follow.

Sets.

- T_b^{AV}: subset of periods where BCTAO assigns BCT b to be in available readiness. Calculated as $t \in T_b^{\text{AD}} \Leftrightarrow Y_{\text{"Av"},b,t} = 1$

Data.

- q^{Cont}: (negative) noncontinuity penalty for each unforced change (fit units)

Decision variables.

- Δ_{bt}: one if an unforced change for BCT b in period t occurs; zero otherwise
- z^{ABCTO}: objective function of the ABCTO model (fit units)

Constraints.

$$\sum_{\substack{m,p| \\ n_{mp}>0,\, t\in T_m^M}} X_{bmpt}^B \leq 1 \quad \forall b,t|b \in B, t \in T_b^{\text{Av}} \tag{16}$$

$$X_{bmpt}^B = 0 \quad \forall b,m,p,t|b \in B^{BN}, n_{mp} > 0, t \notin T_b^{\text{AV}}, t \in T_m^M \cap T_b^B \tag{17}$$

$$\Delta_{bt} \geq X_{bmpt}^B - X_{bmp,t+1}^B \quad \forall b,m,p,t|b \in B^{BN}, n_{mp} > 0, t \in T_m^M \cap T_b^{\text{Av}}, t+1 \in T_b^{\text{Av}} \tag{18}$$

$$\Delta_{bt} \geq X_{bmp,t+1}^B - X_{bmpt}^B \quad \forall b,m,p,t|b \in B^{BN}, n_{mp} > 0, t \in T_m, t \in T_b^{\text{Av}}, t+1 \in T_b^{\text{Av}} \tag{19}$$

$$\Delta \text{ binary } \{0,1\} \tag{20}$$

$$z^{\text{ABCTAO}} = \sum_{m,p|n_{mp}>0} \sum_{t\in T_m^M} w_t \left(q_{mp} U_{mpt} + \sum_{b\in B|t\in T_b^B} f_{bmp} X_{bmpt} \right) + \sum_b \sum_{t\in T_b^B} q^{\text{Cont}} \Delta_{bt} \tag{21}$$

ABCTAO model.

$$\text{ABCTAO}: \text{ Maximize } z^{\text{ABCTAO}}$$

subject to (12) and (16)–(21)

First, we keep constraint (12) to account for unmet missions in the ABCTAO model. It ensures that, if BCTs are reallocated from the original solution, they are still used in the given missions. New constraints (16)–(17) adhere to the existing schedule in the available pool derived from the BCTAO solution. Constraints (18)–(19) track unforced changes. Constraint (20) specifies decision variable types, and (21) establishes the (adjusted) objective function to maximize, which implicitly minimizes unmet changes while keeping the same deployed units as in the BCTAO output. The difference $z^{\text{BCTAO}} - z^{\text{ABCTAO}}$ depends on the relative value of the unmet mission penalties, q_{mp}, and the fit values, f_{bmp}, with respect to the noncontinuity penalty q^{Cont}.

ABCTAO Example

To illustrate the value of the adjustment model, we describe the example shown in Table 3. We begin in the first quarter of year 2012. Suppose every mission part is perfectly matched with one exception: there is an HBCT mission part (''M12'') that spans the four quarters of 2012 but for the first three quarters, there is only one unit available, and it is an SBCT, ''SBCT12.'' Clearly, for the first three quarters, the SBCT must take on the HBCT mission part, for there are no other units. At the end of the fourth quarter of 2012, this mission part ends. However, a new mission part begins in fourth quarter of 2012 (''M13''). This new mission part requires an SBCT, and it will last for four total quarters. ''HBCT13'' is coming out of its ARFORGEN training pool, and is available for deployment in the fourth quarter of 2012 for the next four quarters.

The BCTAO model, seeking to maximize fit, would replace SBCT12 with HBCT13 to complete the last quarter of mission part M12, and move SBCT12 for its last quarter of deployment to M13, the first of four quarters of an SBCT mission. In the next quarter, HBCT13 would then have to move to backfill SBCT12, as its four quarters of deployment have been completed, and it must be sent home for reset. This maximizes fit, but in most cases, would not be operationally sound.

ABCTAO would assign a penalty for moving SBCT12 into the M13 mission part because it breaks continuity. If the continuity penalty, q^{Cont}, for moving SBCT12 were less than the reduction in fit incurred by leaving SBCT12 in an HBCT mission part for one quarter and assigning HBCT13 to an SBCT mission for one quarter,

Table 3. Continuity example.

Mission-Part	BCT type	2012_Q1	2012_Q2	2012_Q3	2012_Q4	2013_Q1	2013_Q2	2013_Q3
M12	HBCT	SBCT12	SBCT12	SBCT12	?			
M13	SBCT				?	HBCT13	HBCT13	HBCT13

ABCTAO would leave SBCT12 to finish mission part M12 and have HBCT13 begin the SBCT mission M13 in the last quarter of 2012.

MODEL DETAILS

Computational Implementation

The computational experience has been carried out on a 2.60 GHz Dell Precision M6300 dual-processor laptop (but using only one processor), with 3.50 Gbytes of RAM, running under Windows XP. The mathematical models have been implemented in GAMS (Brooke et al., 1999) and solved using CPLEX (IBM, 2009) with default settings. A typical BCTAO model contains an average of approximately 72,000 binary decision variables, and 64,000 constraints, which poses some challenges in terms of solvability. If an instance does not converge to near optimality (under 1 percent of relative gap) within several hours, we run an alternative approximation by using a ''rolling horizon'' approach (e.g., solving two or more separate blocks of periods in sequence).

Example Model Outputs

BCTAO produces several outputs. At the micro-level, planners may analyze two main categories of results:

ARFORGEN process. A key output for planners is to realize how each BCT evolves in and adheres to the ARFORGEN process. Table 4 shows this for a subset of BCTs. For example, ''BCT20'' begins in train/ready readiness in year 2012 and eventually becomes available in 2013 (when it performs mission ''M15,'' see Table 5); finally, it returns to the reset pool.

BCT assignment. Table 5 depicts a notional example of which BCTs are assigned to the required mission parts by time period. The excerpt shows that all five parts in mission ''M15'' are assigned to some BCT during the given period (i.e., there are not unmet mission parts). We also observe persistence in the BCTs assigned over time.

In actuality, there will be changes in mission requirements over time and a variety of other factors affecting the individual units, such as building in RIP/TOA, actual equipment and personnel readiness, and limits on the gradual deployment of units geographically collocated. Therefore, the above assignment specifics are not intended to be implemented exactly in the long term. However, this schedule helps planners to gain insights into an ''ideal'' optimal assignment from which they can refine actual schedules that meet operational needs.

At a macro level, decision makers can use overall BCTAO outputs to identify which BCT mix best fits a given mission set, assuming the available information at the time of conducting the analysis. This, in turn, helps them to

Table 4. Notional output of BCTAO (excerpt): Evolution of BCTs in the ARFORGEN cycle.

	2012_Q1	2012_Q2	2012_Q3	2012_Q4	2013_Q1	2013_Q2	2013_Q3	2013_Q4
BCT20	TR	TR	TR	TR	Av	Av	Av	R
BCT21	TR	TR	TR	Av	Av	Av	R	R
BCT22	TR	Av	Av	Av	Av	Av	R	R
BCT23	R	R	TR	TR	TR	TR	Av	Av
BCT24	TR	TR	TR	TR	TR	Av	Av	Av
BCT25	TR	TR	TR	TR	Av	Av	Av	Av

Table 5. Notional output of BCTAO (excerpt): Assignment of BCTs to missions. Showing BCTs assigned to five parts of mission "M15" over time.

Mission	BCT type	2012_Q1	2012_Q2	2012_Q3	2012_Q4	2013_Q1	2013_Q2	2013_Q3	2013_Q4
M15	3BHBCT	BCT14	BCT14	BCT53	BCT53	BCT53	BCT24	BCT24	BCT24
M15	3BIBCT	BCT49	BCT22	BCT22	BCT22	BCT22	BCT22	BCT23	BCT23
M15	3BIBCT	BCT48	BCT48	BCT48	BCT48	BCT20	BCT20	BCT20	BCT32
M15	3BIBCT	BCT47	BCT47	BCT47	BCT47	BCT25	BCT25	BCT25	BCT25
M15	3BIBCT	BCT54	BCT54	BCT54	BCT21	BCT21	BCT21	BCT40	BCT40

compare fit and risk levels based on a quantitative and objective foundation. Figure 4 shows an example of BCTAO overall results. In this scenario, the total objective function is −282.34 fit units. (We display z^{BCTAO} for a given solution, even if ABCTAO has been used to adjust the final BCT assignments.) There are 620 BCTs assigned to mission parts. The fit score is 87.66. Given the existing BCT types, if each mission received its most favorable BCT, the maximum fit value would have been 121.13 (i.e., BCTAO is able to find a schedule that meets 72.36 percent of the maximum possible effectiveness level, if every mission always received its best BCT fit). The solution incurs a risk penalty of (negative) 370 fit units, for its 37 unmet mission parts. The next block reports the number of exact fits (i.e., missions assigned to their most favorable BCT). Thus, planners have a combined measure of BCT suitability and BCT sufficiency. In our implementation, we represented the BOG:dwell (mobilized:demobilized for RC units) concept using the BOG level calculation, which divides time a unit is deployed by the total time the unit has spent in the system. Thus, using Table 1, the AC component will have a BOG level between .23 and .42 (.2703 in this case), and the RC component between .2 and .25 (.2347). The last block reports BOG levels by BCT type. Note that because the model is constraining the solution space only by component (AC and RC), there are two RC BCT types ("BSBCT" and "3BHBCT") with BOG levels outside of the range calculated from Table 1.

ANALYSIS OF RESULTS

Overview

The TRAC study team ran more than 300 combinations through the optimization models, varying BCT types, quantities, number of BCTs in the active and reserve components, BOG:dwell

```
Objective value (total fit - risk) = -282.34
   Filled mission-parts: 620 (fit: 87.66 out of a max. of 121.13)
   Unmet mission-parts:  37 (risk penalty:     -370.00)

Fraction of exact fits:
      620 filled mission-parts, of which 342.00 are with exact fit (0.55)
      657 total mission-parts, of which 342.00 are with exact fit (0.52)

BOG levels (overall, by component):
                   dmin      actual    dmax
AC                 0.0000    0.2703    1.0000
RC                 0.0000    0.2347    1.0000

BOG levels (by BCT type and component):
BSBCT              AC        0.3125
3BIBCT             AC        0.2729
3BHBCT             AC        0.2338
BSBCT              RC        0.2857
3BIBCT             RC        0.2381
3BHBCT             RC        0.1837
```

Figure 4. Notional output of BCTAO: Overall results.

constraints (that, for RC units, actually account for time mobilized versus time not mobilized), and BCT type transition dates. The team used this information to determine the optimal mix that provided the lowest risk and highest assignment of optimal BCTs for the mission demands.

All of our scenarios have 28 time periods (seven years divided quarterly). Depending on the case, we consider between 57 and 73 BCTs, and from 19 to 31 missions, for a total of 41 to 58 mission parts. Mission duration is variable (some missions may last two or three periods only, whereas others may last all 28 periods). Thus, a typical problem includes 700 to 900 individual demands for a BCT assignment.

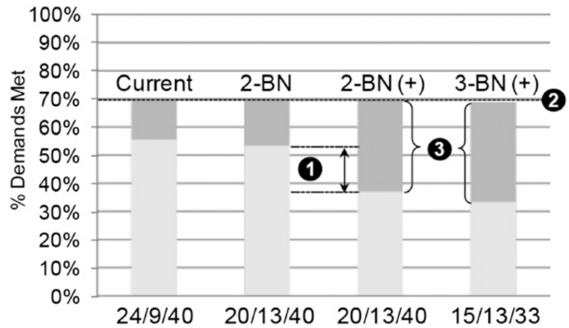

Figure 5. ISC ''A'': one MCO. The graph shows sufficiency (percent of demand met) and effectiveness (preferred BCT choices, upper portion of each bar) for different configurations of two- and three-battalion BTCs. Below each column, the number of BCTs is represented as HBCTs/SBCTs/IBCTs.

Selected Analysis Results

This section summarizes selected results obtained with the BCTAO model, as discussed by Pippin and Morey (2010).

Figures 5–7 compare best-case mixes in each of the three ISCs. The first three bars in each of the three ISC figures are alternatives with a 73 BCT structure, and the fourth bar is an alternative with a 61 BCT structure. The specific number of BCTs in each alternative is represented by the numbers below the bar as HBCTs/SBCTs/IBCTs. The ''+'' for the last two alternatives denotes that all BCTs have engineering battalions (BEBs) and the IBCTs have protected mobility. In the first three alternatives, the HBCTs and IBCTs have two maneuver battalions, whereas in the last alternative all BCTs have three maneuver battalions.

Each figure shows the mission percent demands met by each structure. Sufficiency (measure of demands met) is shown by the height of each bar. With BOG:dwell enforced at 1:2 for AC and mobilized:demobilized enforced at 1:4 for RC (reflecting current policy), approximately 70 percent of the demands are met for ISC ''A,'' 66 percent for ISC ''B,'' and 64 percent for ISC ''C.'' (Sensitivity runs were conducted and it was determined that greater than 96 percent of demands could be met in all ISCs with a BOG:dwell ratio of 1:1.3 and 1:2.2 for AC and RC, respectively.) The upper portion of each bar represents the scenario demands met with preferred forces (effectiveness), and the lower portion is the scenario demands met with less-preferred forces.

The numbered comments below correspond to the numbers in black circles on each of three figures.

1. The addition of the protected mobility and BEB to the two-battalion design resulted in a substantial increase in demands met by preferred forces, especially in a COIN-heavy ISC (specifically, for ISC ''C'').
2. Three-battalion (+) design met about the same number of demands as all two-battalion designs.
3. Equally sufficient, the two-battalion (+) and three-battalion (+) designs offered more effective options to meet strategic demands.

Note that the effectiveness of the three-battalion (+) design is greater in each of the three ISC cases, achieving an almost 10 percent

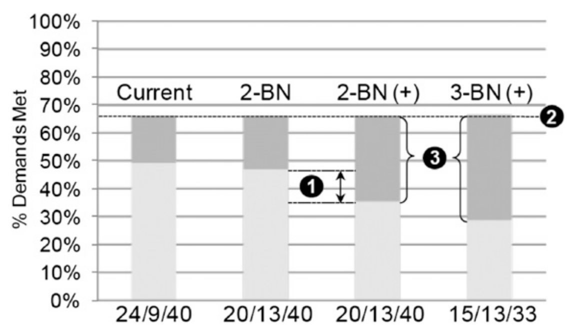

Figure 6. ISC ''B'': Two MCO+.

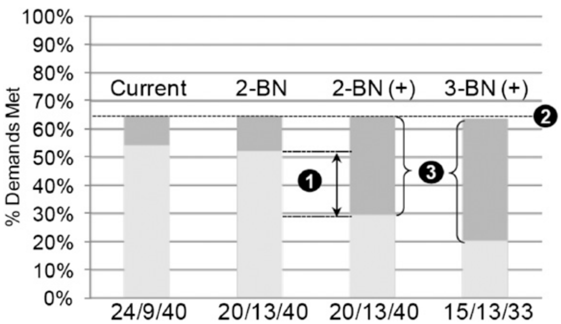

Figure 7. ISC "C": One COIN.

greater effectiveness than the two-battalion (+) design in ISC "C" (COIN). When considering a wide range of options requiring multiple BCTs, about 75 percent of warfighters judged the three-battalion (+) design as the most effective design of those considered.

CONCLUSIONS

We have developed the BCTAO model and described its use supporting the 2010 BCT force-mix analysis conducted by the TRAC. During the course of the study, further improvements that could be made to the model were identified. After the 2010 study described here, further study was required by the Army to help senior Army leaders decide on the best path for Army modernization to take. The body of analysis built over the last eight years has consistently shown that the Army could reorganize all BCTs to a three-battalion design, even in the face of overall troop reductions in the Army, while maintaining a sufficient number of BCTs to meet worldwide demands and, at the same time, increasing operational effectiveness.

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