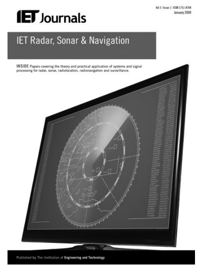
Techset Composition Ltd, Salisbury
Doc: H:/Iee/RSN/Articles/Pagination/RSN20110349.3d

www.ietdl.org

Published in IET Radar, Sonar and Navigation
Received on 25th October 2011
Revised on 4th February 2012
doi: 10.1049/iet-rsn.2011.0349



ISSN 1751-8784

Extending the unambiguous range of polyphase P4 CW radar using the robust symmetrical number system

N. Paepolshiri P.E. Pace D.C. Jenn

Department of Electrical and Computer Engineering, Naval Postgraduate School, Spanagal Hall, Monterey, CA, USA
E-mail: pepace@nps.edu

Abstract: Polyphase continuous waveform radar systems often use polyphase P4 modulation because of the periodic autocorrelation zero sidelobes and good Doppler tolerance. The P4's unambiguous detection range is equal to the number of subcodes within a code period. To extend the unambiguous detection range beyond a single-code period, develops a novel relationship between the polyphase P4 code and the robust symmetrical number system (RSNS) where the P4 phase values within the code period match the symmetrical residues within an RSNS sequence. By transmitting $N \geq 2$ coprime P4 code periods, the unambiguous range is extended by considering the paired values from each sequence. With this new approach, the unambiguous range is extended to the greatest length of combined coprime phase sequences that contain no repeated paired terms. The flexibility of the RSNS allows the unambiguous range to be extended using a fewer number of subcodes within each code period. The combined phase sequences also have an inherent integer Gray code property to control range detection errors. Examples are shown to demonstrate the feasibility of the approach and to compare the unambiguous detection range to that of a single P4 code period.

1 Introduction

Continuous waveform (CW) radar systems have a superior low probability of intercept (LPI) performance over pulse train radar systems because of their low-average power transmitted (e.g. 1 W) and their use of pulse compression techniques. Note that we extend the concept of pulse compression to CW (nonpulsed) waveforms since the techniques are similar and the objectives are the same. Since pure CW waveforms cannot resolve the target's range, periodic modulation techniques are used, such as frequency-modulated CW (FMCW), frequency-shift keying (FSK), noise modulation, phase-shift keying (PSK), as well as hybrids of these techniques [1]. Owing to the advances in high-speed processing and direct digital synthesis modules, the use of PSK techniques in CW radar is highly advantageous. CW radar systems that transmit and receive PSK signals can have small range resolution cells and are ideally suited for many sensor applications for situational awareness, including over-the-horizon-radar systems and multiple-input multiple-output configurations.

Polyphase coding refers to PSK modulation of the CW carrier. Polyphase sequences are of finite length with N_c subcodes, each with a discrete phase value to modulate the CW carrier. The number of subcodes is taken from an alphabet of size $N_c \geq 2$, which is also the processing gain (PG) of the radar excluding any post-detection integration. The polyphase code period is $T = N_c t_b$ where t_b is the size of each subcode which determines both the 3 dB bandwidth of the waveform $B = 1/t_b$ and the range resolution $\Delta R = ct_b/2$. The unambiguous range is limited by the number of subcodes within a code period as $R_u = N_c \Delta R$. Increasing the number of phase subcodes in the sequence results in a greater PG in the receiver or equivalently a larger signal-to-noise ratio (SNR).

With a CW polyphase waveform, the matched filter in the receiver is a coherent, range–Doppler correlation processor that performs a cross correlation between the received N_p returned code periods from the target and N_r reference code periods where each reference code period is the complex conjugate of the transmitted polyphase code [2]. To reduce the periodic ambiguity sidelobe levels the N_r cross correlation output values can be added together. When the polyphase signals are returned and have an impressed Doppler shift, the correlation process used in the receiver compression operation is not perfect, resulting in a certain amount of correlation loss because of the phase shifts across the code period T.

One of the most important PSK codes used in CW radar is the P4 code [3, 4]. The P4 code consists of discrete phases of a linear chirp waveform taken at specific time intervals. It is derived by converting a linear FMCW to baseband using a local oscillator on one end of the frequency sweep and sampling the inphase (I) and quadrature (Q) video at the Nyquist rate. The P4 phase steps exhibit a parabolic distribution that is symmetrical. The P4 is also a Doppler tolerant perfect code in that it exhibits a perfect periodic autocorrelation function – namely zero autocorrelation sidelobes [1].

Although the P4 polyphase modulation can serve as a transmission waveform for a CW radar system, there are several limitations. Most significantly, the unambiguous detection range of the waveform R_u is limited by the number of subcodes within a code period $R_u = N_c \Delta R$. Increasing the number of subcodes to extend the unambiguous range requires a larger range–Doppler correlation processor to provide the code compression. With an increased code length, a larger code compression time is required as well as an increase in the bulk memory requirements in the radar digital processor. Further, a significant amount of correlation loss is incurred if the total number of code periods returned from the target N_p is less than the number of code periods N_r used in the correlation processor, as would be the case because of a limited time-on-target.

Considering the symmetrical, parabolic distribution of the P4 polyphase code, this study develops a novel relationship between the P4 code and the robust symmetrical number system (RSNS) in order to significantly extend the unambiguous range beyond a single code period $T = N_c t_b$. The RSNS is a modular system consisting of $N \geq 2$ integer sequences with each sequence associated with a coprime modulus m_i from the set $m_1, m_2, \ldots, m_N$ [5, 6]. Each integer within an RSNS sequence is called a symmetrical residue and corresponds to a single subcode within the phase code sequence. By using $i = 1, 2, \ldots, N$ ($N \geq 2$) P4 code sequences that correspond to an RSNS coprime modulus, the unambiguous range is extended by considering the paired values from each sequence. The unambiguous range is extended from the number of subcodes within a single-code period to the greatest length of combined coprime phase sequences that contain no repeated paired terms which is called the RSNS dynamic range $\hat{M}_{\mathrm{RSNS}}$. Collectively processing $N \geq 2$ symmetrical residue P4 phase code sequences, the unambiguous target detection range is extended from $R_u = cT/2 = ct_b N_c/2$ for an individual code to $R_{u_{\mathrm{RSNS}}} = ct'_b \hat{M}_{\mathrm{RSNS}}/2$ where t'_b is the subcode period. In addition, the combined RSNS phase sequences have an inherent integer Gray code property that makes it particularly attractive for controlling any possible errors in the detected target's range that might occur. Owing to the flexibility offered by the RSNS, the modular code periods can also have a small number of subcodes.

2 CW phase modulation radar

In polyphase CW radar, the phase-shifting operation is performed in the radar's transmitter, with the timing information generated from the receiver–exciter. The transmitted complex signal can be written as [1]

$$s(t) = A e^{j(2\pi f_c t + \phi_k(t))} \tag{1}$$

where A is the signal amplitude, f_c is the carrier frequency and $\phi_k(t)$ is the time-dependent phase modulation. For a code period T, the CW signal is shifted in phase every subcode with each code period consisting of $k = 1, \ldots, N_c$ subcodes. Each subcode with phase ϕ_k has a duration of t_b. For a polyphase sequence, the code period is

$$T = N_c t_b \tag{2}$$

If cpp is the number of cycles of the carrier frequency per subcode, the bandwidth B of the transmitted signal is

$$B = \frac{f_c}{\mathrm{cpp}} = \frac{1}{t_b} \tag{3}$$

The return signal from the target $x(t)$ can be written as

$$x(t) = \eta A e^{j\{2\pi(f_c+\nu)(t-\tau)+\phi_k(t-\tau)\}} \tag{4}$$

where η is the amplitude attenuation coefficient of the received signal because of the target scattering and the propagation path loss. The roundtrip delay τ for a target at range R is $\tau = 2R/c$, where c is the speed of light in free space. The Doppler frequency ν owing to the target motion is $\nu = 2V_r/\lambda$ where λ is the wavelength and V_r is the target's relative radial velocity.

In Fig. 1, a range–Doppler correlation receiver for zero Doppler offset is illustrated [2]. The received waveform from the target is processed by a filter matched to a subcode duration t_b. A phase detector then detects the phase of each subcode. The detected output signal is then sent through a tapped delay line where each delay D is t_b. Instead of using one reference code period, the return signal from the target with N_p code periods is correlated using a range–Doppler correlator that contains a cascade of N_r copies of the N_c reference coefficients in order to improve the ambiguity sidelobe structure [2]. As the signal is strobed through the tapped delay line, it is multiplied by the reference signal. At each step, the multiplication for each delay is summed separately for each of the N_r reference code periods. To reduce the sidelobe structure further, a weighting function C_i can be added. To compute the entire range–Doppler ambiguity space, the correlation outputs from each subcode delay are multiplied by $q^k = e^{j2\pi k \Delta\nu t_b}$ for $\nu = \Delta\nu$ where k ranges from 0 to $N_r N_c - 1$. For $\nu = 2\Delta\nu$, the output of the previous $\nu = \Delta\nu$ operation is again multiplied by $q^k = e^{j2\pi k \Delta\nu t_b}$ and so on for the entire ambiguity space.

In general, the larger the PG the greater is the SNR improvement in the signal processing. If the radar signal processing input SNR is $\mathrm{SNR}_{R_{\mathrm{in}}}$ and the radar signal processing output SNR is $\mathrm{SNR}_{R_{\mathrm{out}}}$ then

$$\mathrm{PG} = \frac{\mathrm{SNR}_{R_{\mathrm{out}}}}{\mathrm{SNR}_{R_{\mathrm{in}}}} \tag{5}$$

The PG for a polyphase code is equal to the time-bandwidth product

$$\mathrm{PG} = T(1/t_b) = (N_c t_b)/t_b = N_c \tag{6}$$

and is equal to the number of subcodes within a code period [1].

3 P4 Polyphase code

The P4 polyphase code is conceptually derived by converting a linear frequency modulation waveform to baseband by using a synchronous local oscillator that is offset in the I and Q detectors. The sampling of I and Q video at the Nyquist rate yields the polyphase code P4 [4]. The P4 consists of the discrete phases of the linear chirp waveform taken at specific time intervals and exhibits the same range–Doppler coupling associated with the chirp waveform. However, the peak sidelobe levels are lower than those of the unweighted chirp waveform. Various weighting techniques can be applied to reduce the sidelobe

doi: 10.1049/iet-rsn.2011.0349

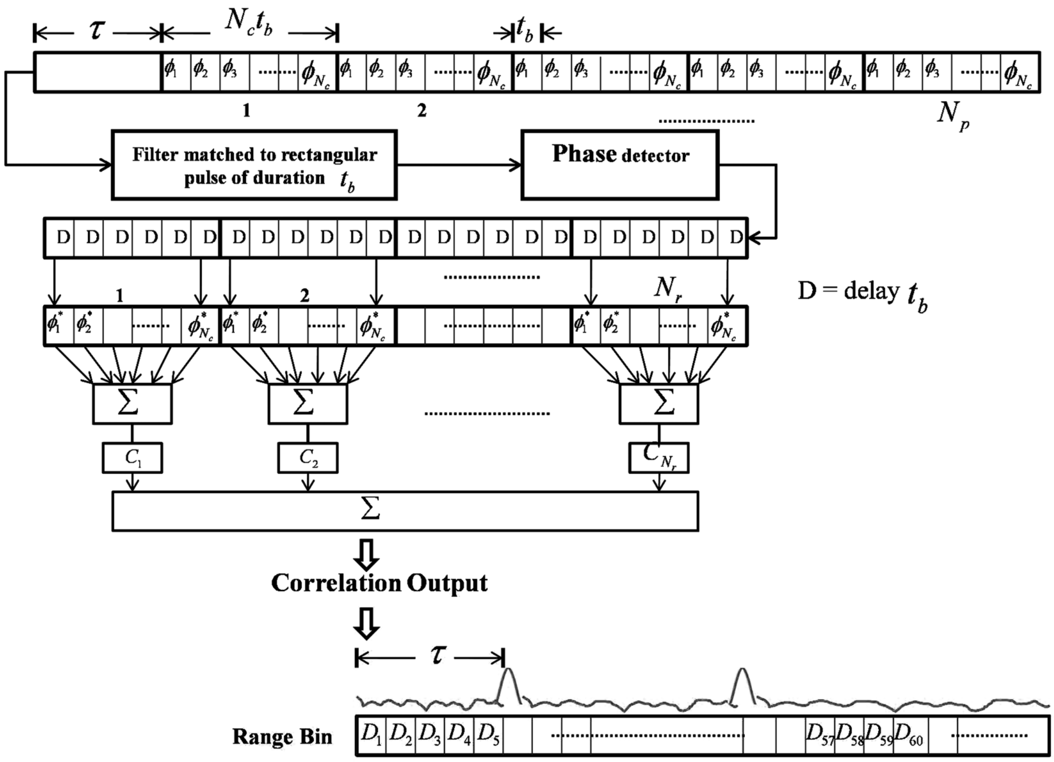


Fig. 1 *Compression receiver matched to N_r periods of a transmitted polyphase code for $\nu = 0$ and the receiver's range bin for detection (adapted from [2])*

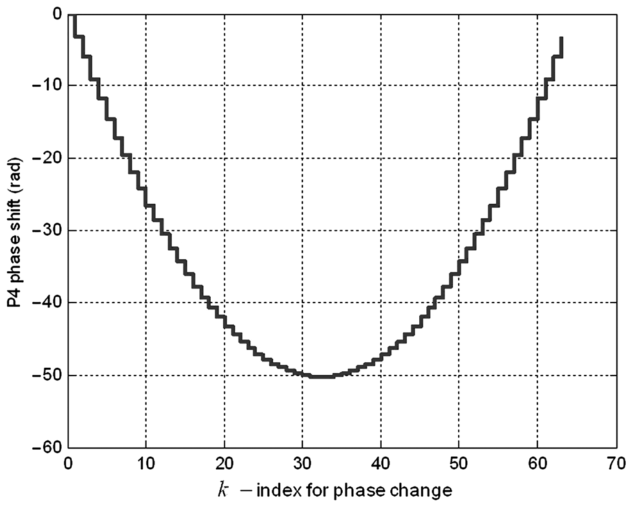


Fig. 2 *Discrete P4 phase values for $N_c = 64$*

levels further. The phase sequence of a P4 signal is given by

$$\phi_k = \frac{\pi}{N_c}(k-1)^2 - \pi(k-1) \tag{7}$$

where $k = 1, \ldots, N_c$ is the subcode index and N_c is the PG. The discrete phase values for the P4 code for $N_c = 64$ are shown in Fig. 2.

4 Robust symmetrical number system

The RSNS is a modular system consisting of $N \geq 2$ integer sequences with each sequence associated with a coprime modulus m_i, $i = 1, \ldots, N$. Owing to the presence of ambiguities, the set of integers within each RSNS sequence do not form a complete residue system [6]. The ambiguities within each modulus sequence are resolved by the use of additional moduli and considering the vector of paired integers from all N sequences. The RSNS is based on the sequence

$$RS'_{m_i} = [0, 1, 2, \ldots, m_i - 1, m_i, m_i - 1, \ldots, 2, 1] \tag{8}$$

To form the N-sequence RSNS, each term in (8) is repeated N times in succession. The integers within one folding period of a sequence are then

$$\begin{aligned} RS_{m_i} = [&0, 0, \ldots, 0, 1, 1, \ldots, 1, \ldots, m_i - 1, \ldots, m_i - 1, \\ &m_i, m_i, \ldots, m_i, m_i - 1, \ldots, m_i - 1, \ldots, 1, \ldots, 1] \end{aligned} \tag{9}$$

This results in a periodic sequence where each code period consists of

$$N_{c_{i_{\mathrm{RSNS}}}} = 2Nm_i \tag{10}$$

subcodes [5], which is also the period of the sequence. Each sequence corresponding to m_i is also right (or left) shifted by $s_i = i - 1$ places. The chosen shift values $\{s_1, s_2, \ldots, s_N\}$ must form a complete residue system modulo N. The shift values s_i for each sequence do not affect the dynamic range $\hat{M}_{\mathrm{RSNS}}$ but do make a difference in the location of the beginning position of $\hat{M}_{\mathrm{RSNS}}$, h, and the ending position, $h + \hat{M}_{\mathrm{RSNS}} - 1$.

After choosing the beginning position h, it is useful to know the symmetrical residues at this point in order to align them for each sequence. If each sequence is extended periodically with period $2Nm_i$ as $RS_{m_i,h+n2Nm_i} = RS_{m_i,h}$

where $n \in 0, \pm 1, \pm 2, \ldots$, then RS_{m_i} is called a symmetrical residue of $(h + n2Nm_i)$ modulo $2Nm_i$. Since each period consists of $2Nm_i$ integers, the symmetrical residues are determined by first subtracting off an integer number of $2Nm_i$ integers as [7]

$$n_i = h - \left\lfloor \frac{h}{2Nm_i} \right\rfloor 2Nm_i \tag{11}$$

This value is then used to find the symmetrical residue RS_{m_i} as

$$RS_{m_i} = \begin{cases} \left\lfloor \dfrac{n_i - s_i}{N} \right\rfloor, & s_i \le n_i \le Nm_i + s_i + 1 \\ \left\lfloor \dfrac{2Nm_i + N - n_i + s_i - 1}{N} \right\rfloor, & Nm_i + s_i + 2 \le n_i \le 2Nm_i + s_i - 1 \end{cases} \tag{12}$$

The construction of the N sequences ensures that the paired integers from the N sequences differ by only one integer, resulting in an integer Gray code property.

The calculation of $\hat{M}_{\mathrm{RSNS}}$ is a function of N and the chosen m_i. A closed-form solution for computing $\hat{M}_{\mathrm{RSNS}}$ for $N = 2$ moduli is reported in [8], and summarised as follows.

For $m_1 \ge 3$ and $m_2 = m_1 + 1$

$$\hat{M}_{\mathrm{RSNS}} = 3(m_1 + m_2) - 6 = 6m_1 - 3 \tag{13}$$

For $5 \le m_1 < m_2$ and $m_2 \le m_1 + 2$

$$\hat{M}_{\mathrm{RSNS}} = 4m_1 + 2m_2 - 5 \tag{14}$$

For $5 \le m_1 < m_2$ and $m_2 \ge m_1 + 3$

$$\hat{M}_{\mathrm{RSNS}} = 4m_1 + 2m_2 - 2 \tag{15}$$

A closed-form solution for $N = 3$ moduli of the form $2^z - 1, 2^z, 2^z + 1$ is

$$\hat{M}_{\mathrm{RSNS}} = \frac{3}{2}m_1^2 + \frac{15}{2}m_1 + 7 \tag{16}$$

where z is any integer and $m_1 \ge 3$ [6]. An efficient algorithm for computing $\hat{M}_{\mathrm{RSNS}}$ and its position for any general set of moduli is reported in [9].

Let $\boldsymbol{X}_h$ be the vector of N paired integers from each sequence within the RSNS at h where h is the beginning position of the vector $\boldsymbol{X}_h$. In Table 1, the symmetrical residues for an $N = 3$ RSNS system with $m_i = [3\ 4\ 5]^{\mathrm{T}}$ and right shift $s_i = [0,\ 1,\ 2]^{\mathrm{T}}$ are shown. For this example, from (16) the RSNS dynamic range is $\hat{M}_{\mathrm{RSNS}} = 43$, and the
Q1 position begins at $h = 61$ with the vector $\boldsymbol{X}_{61} = [2\ 4\ 1]^{\mathrm{T}}$. The set of integers that lie within the dynamic range $\hat{M}_{\mathrm{RSNS}} = 43$ contain no ambiguities. Note in Table 1 the integer Gray code property where any code transition results in just one integer changing value by ± 1. Also shown is the bin index a, which runs from 0 to 42 (total 43) that will be used to refer to the range bin position.

5 Robust symmetrical residue-P4 phase code

Owing to the symmetrical distribution of the P4 phase values, the RSNS symmetrical residues within a modulus can be related to a P4 code sequence. The RSNS symmetrical residues can be folded into the P4 phase sequence (9) by associating the symmetrical residues $RS_{m_i,k'}$ with the P4 subcode phases. The phase index k' runs from $1, \ldots, N_{c_{i_{\mathrm{RSNS}}}}$, where $N_{c_{i_{\mathrm{RSNS}}}}$ is the number of RSNS subcodes within one code period. The symmetrical residue-P4 phase sequence for modulus m_i is given by

$$\phi_{m_i,k'} = \left[\frac{\pi}{2m_i}(RS_{m_i,k'} - m_i)^2\right] - \frac{\pi}{2}m_i \tag{17}$$

and from (10) $N_{c_{i_{\mathrm{RSNS}}}} = 2m_iN$, where N is the number of modular polyphase waveforms transmitted. For example, if $N = 2$ and the first modulus in the sequence is $m_1 = 4$, then $N_{c_{1_{\mathrm{RSNS}}}} = 16$. From (9) the symmetrical residue integers within the modulus are $RS_{m_1=4,k'} = [0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 3, 3, 2, 2, 1, 1]$ which consists of $N_{c_{1_{\mathrm{RSNS}}}} = 16$ integers. From (17) the phase sequence in radians is $\phi_{m_1=4,k'} = [0, 0, -2.75, -2.75, -4.71, -4.71, -5.89, -5.89, -6.28, -6.28, -5.89, -5.89, -4.71, -4.71, -2.75, -2.75]$. We now let N_{c_i} be the number of subcodes or processing gain, within a P4 sequence (7). The number of subcodes in a P4 sequence is related to the number of subcodes within a symmetrical residue-P4 sequence by

$$N_{c_i} = \frac{N_{c_{i_{\mathrm{RSNS}}}}}{N} = 2m_i \tag{18}$$

Consequently, the number of P4 subcodes N_{c_1} for $m_1 = 4$ is

$$N_{c_1} = \frac{N_{c_{1_{\mathrm{RSNS}}}}}{N} = 8 \tag{19}$$

Recall that cpp represents the number of carrier frequency cycles per subcode. The cpp value for a P4 subcode is N times that of a symmetrical residue-P4 subcode. That is, for the P4, the subcode period is

$$t_b = \frac{\mathrm{cpp}}{f_{\mathrm{c}}} \tag{20}$$

and for the symmetrical residue-P4, the subcode period is

$$t_b' = \frac{\mathrm{cpp}}{Nf_{\mathrm{c}}} \tag{21}$$

From (20) and (21), we have

$$t_b = N\,t_b' \tag{22}$$

Note that from (18) the symmetrical residue-P4 code period is

$$T_i = t_b' N_{c_{i_{\mathrm{RSNS}}}} = t_b N_{c_i} N \tag{23}$$

For example, in Fig. 3 we see that the phase sequence for the symmetrical residue-P4 for $m_1 = 16$, $N = 2$ and $N_{c_{1_{\mathrm{RSNS}}}} = 64$ is actually the same as the P4 phase code sequence using $N_{c_1} = 32$.

doi: 10.1049/iet-rsn.2011.0349

Table 1 RSNS symmetrical residues for $m_1 = 3$ ($s_1 = 0$), $m_2 = 4$ ($s_2 = 1$), $m_3 = 5$ ($s_3 = 2$) (after [5])

n	$m_1 = 3$	$m_1 = 4$	$m_3 = 5$	a	Range (m)
61	2	4	1	0	0–50
62	2	4	0	1	50–100
63	3	4	0	2	100–150
64	3	3	0	3	150–200
65	3	3	1	4	200–250
66	2	3	1	5	250–300
67	2	2	1	6	300–350
68	2	2	2	7	350–400
69	1	2	2	8	400–450
70	1	1	2	9	450–500
71	1	1	3	10	500–550
72	0	1	3	11	550–600
73	0	0	3	12	600–650
74	0	0	4	13	650–700
75	1	0	4	14	700–750
76	1	1	4	15	750–800
77	1	1	5	16	800–850
78	2	1	5	17	850–900
79	2	2	5	18	900–950
80	2	2	4	19	950–1000
81	3	2	4	20	1000–1050
82	3	3	4	21	1050–1100
83	3	3	3	22	1100–1150
84	2	3	3	23	1150–1200
85	2	4	3	24	1200–1250
86	2	4	2	25	1250–1300
87	1	4	2	26	1300–1350
88	1	3	2	27	1350–1400
89	1	3	1	28	1400–1450
90	0	3	1	29	1450–1500
91	0	2	1	30	1500–1550
92	0	2	0	31	1550–1600
93	1	2	0	32	1600–1650
94	1	1	0	33	1650–1700
95	1	1	1	34	1700–1750
96	2	1	1	35	1750–1800
97	2	0	1	36	1800–1850
98	2	0	2	37	1850–1900
99	3	0	2	38	1900–1950
103	3	1	2	39	1950–2000
101	3	1	3	40	2000–2050
102	2	1	3	41	2050–2100
103	2	2	3	42	2150–2200

6 Radar application of the robust symmetrical residue-P4 phase code

The block diagram of the CW radar using the symmetrical residue-P4 phase code modulation is shown in Fig. 4. In the transmitter, the first step is to generate and store the sequence values $RS_{m_i,k'}$ for each modulus. Then, a direct digital synthesiser uses each residue sequence $RS_{m_i,k'}$ to generate the $N_{c_{i\text{RSNS}}} = 2Nm_i$ subcode phase values $\phi_{m_i,k'}$ according to (17) for each modulus. Each subcode corresponding to $RS_{m_i,k'}$ has a length of t'_b. Together the $N_{c_{i\text{RSNS}}}$ subcodes have a code period of T_i. To detect the target's range unambiguously, each coprime sequence must have $\hat{M}_{\text{RSNS}}$ subcodes. Each sequence is amplified and transmitted in succession.

Upon reception, the signal is amplified and downconverted to an intermediate frequency compatible with an available analogue-to-digital (ADC) converter technology. The output of the ADC is processed by a moving target indication (elliptic) filter to remove the clutter, and then the phase of each subcode is detected. The output of the phase detector is processed by a phase code compressor that is followed by a non-coherent post-detection integration (PDI) process. The output of the PDI is sent to a constant false alarm rate processor to detect and save the range bin of the target for that modulus sequence. This series of steps is performed for each symmetrical residue-P4 phase code sequence, and the detected target's range bin for each code is sent to an algorithm to resolve the target's true range after all N symmetrical residue sequences are obtained [10].

To illustrate the concept, the previous example is worked using $N = 3$ RSNS moduli with $m_1 = 3$, $m_2 = 4$ and $m_3 = 5$. For these moduli, the dynamic range from (16) is $\hat{M}_{\text{RSNS}} = 43$. The transmitted signal is shown in Fig. 5. Each sequence has a length equal to $\hat{M}_{\text{RSNS}}$ subcodes. For the first modulus, $m_1 = 3$, there are four symmetrical

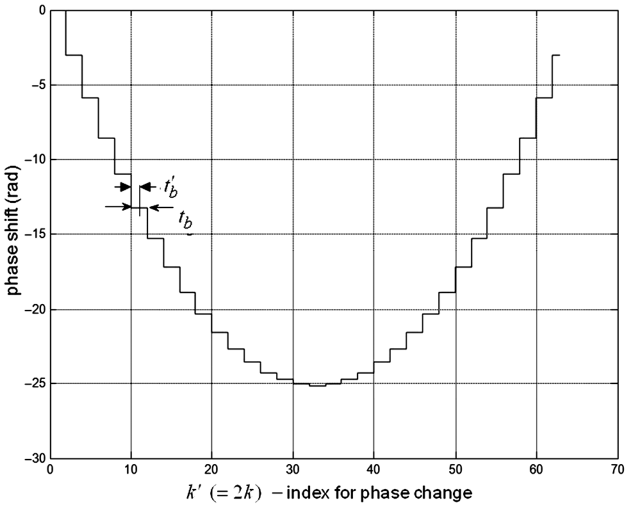


Fig. 3 *Symmetrical residue-P4 subcode phase values within a code period using $m_1 = 16$, $N = 2$ and $N_{c_{1\,RSNS}} = 64$ shown with the P4 subcode phase values using $N_{c_i} = 32$*

residues within the sequence: 0, 1, 2 and 3. The sequence code period corresponding to m_1 needs $N_{c_{1_{\mathrm{RSNS}}}} = 2Nm_1 = 2 \times 3 \times 3 = 18$ subcodes. For $m_2 = 4$, the symmetrical residues are 0, 1, 2, 3 and 4. The required number of subcodes is $2 \times 3 \times 4 = 24$. For $m_3 = 5$, the symmetrical residues are 0, 1, 2, 3, 4 and 5. The number of subcodes is $2 \times 3 \times 5 = 30$. As shown in Fig. 5, the beginning of each sequence for $m_1 = 3$, $m_2 = 4$ and $m_3 = 5$ are $k' = 8$, $k' = 13$ and $k' = 30$, respectively, since, from (11) and (12), $X_{61} = [2\ 4\ 1]^{\mathrm{T}}$.

Each reference code must have a length equal to the code period T_i for each modular waveform in order to correctly calculate the code compression output. Referring again to Fig. 5, the reference signals start from the beginning of the transmitted signal. The number of subcodes within each sequence (multiple code periods) is $\hat{M}_{\mathrm{RSNS}}$. The number of code periods needed to cover $\hat{M}_{\mathrm{RSNS}}$ is

$$\hat{N}_{\mathrm{p},m_i} = \frac{\hat{M}_{\mathrm{RSNS}}}{N_{c_{i_{\mathrm{RSNS}}}}} = \frac{\hat{M}_{\mathrm{RSNS}}}{2m_i N} \tag{24}$$

Sending 43 subcodes for $m_1 = 3$ requires $\hat{N}_{\mathrm{p},3} = 43/18 = 2.4$ code periods. For $m_2 = 4$ and $m_3 = 5$, the number of subcodes required are $N_{c_{2_{\mathrm{RSNS}}}} = 24$ and $N_{c_{3_{\mathrm{RSNS}}}} = 30$ with the number of transmitted code periods $\hat{N}_{\mathrm{p},4} = 1.8$ and $\hat{N}_{\mathrm{p},5} = 1.4$, respectively. The waveform $m_1 = 3$ has more than one complete code period, so we can expect to see two compressed pulses from modular waveform $m_1 = 3$. However, there will be only one compressed pulse for the other two modular waveforms.

In Fig. 6*a*, the symmetrical residues for $m_1 = 3$, $m_2 = 4$ and $m_3 = 5$ with $\hat{M}_{\mathrm{RSNS}} = 43$ are shown. Also, the phase sequences from each modular waveform are plotted in Fig. 6*b*. Since $RS_{m_i,h+n2Nm_i} = RS_{m_i,h}$ where $n \in \{0, \pm 1, \pm 2, \ldots\}$, the phase sequences are periodic with $\phi_{m_i,k'+n2Nm_i} = \phi_{m_i,k'}$.

Using all sequences, the paired terms are mapped to a specific range bin a. Continuing the example for $m_1 = 3$, $m_2 = 4$ and $m_3 = 5$, we show the robust symmetrical residue-paired values and the corresponding ranges in Table 1. The signal bandwidth is 3 MHz and the subcode period is $t_b' = t_b/N = 0.33\ \mu$s. For the range resolution shown the new unambiguous range using the symmetrical residue-P4 modulation is

$$R_{u_{\mathrm{RSNS}}} = \frac{c\hat{M}_{\mathrm{RSNS}} t_b'}{2} \tag{25}$$

For this example, the maximum unambiguous range is $R_{u_{\mathrm{RSNS}}} = 2150$ m and the range resolution is $\Delta R = ct_b'/2 = 50$ m.

An example of the target detection for $m_1 = 3$, $m_2 = 4$ and $m_3 = 5$ is illustrated in Fig. 7. The target is in range bin $a = 5$. From Table 1, the corresponding range of the detected target is between 250 and 300 m. First, the phase code signal for $m_1 = 3$ (top) is compressed giving the symmetrical residue $\mathrm{RS}_3 = 2$. The process is the same for the modular waveforms $m_2 = 4$ and $m_3 = 5$ and the symmetrical residues containing the target are $\mathrm{RS}_4 = 3$ and $\mathrm{RS}_5 = 1$. The three robust symmetrical residues are paired as [2 3 1] and must be converted to a more convenient, for example, binary number and can be computed with a look up table or using the algorithm as described in [10].

7 Practical considerations

To examine the practical advantage of the RSNS P4 approach, we note that for a single P4 code period, the unambiguous range for a single modulus m_i is limited to

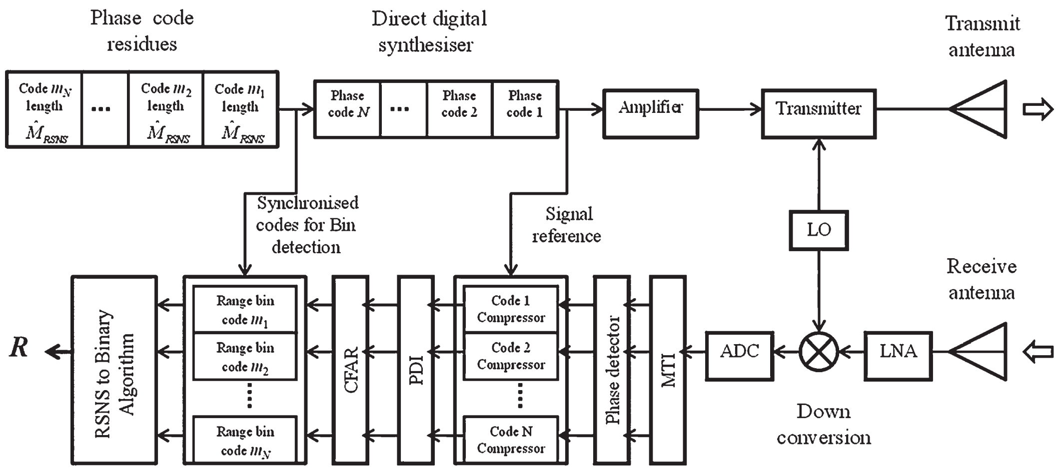


Fig. 4 *Block diagram for the radar using N residue-P4 phase code sequences*

doi: 10.1049/iet-rsn.2011.0349

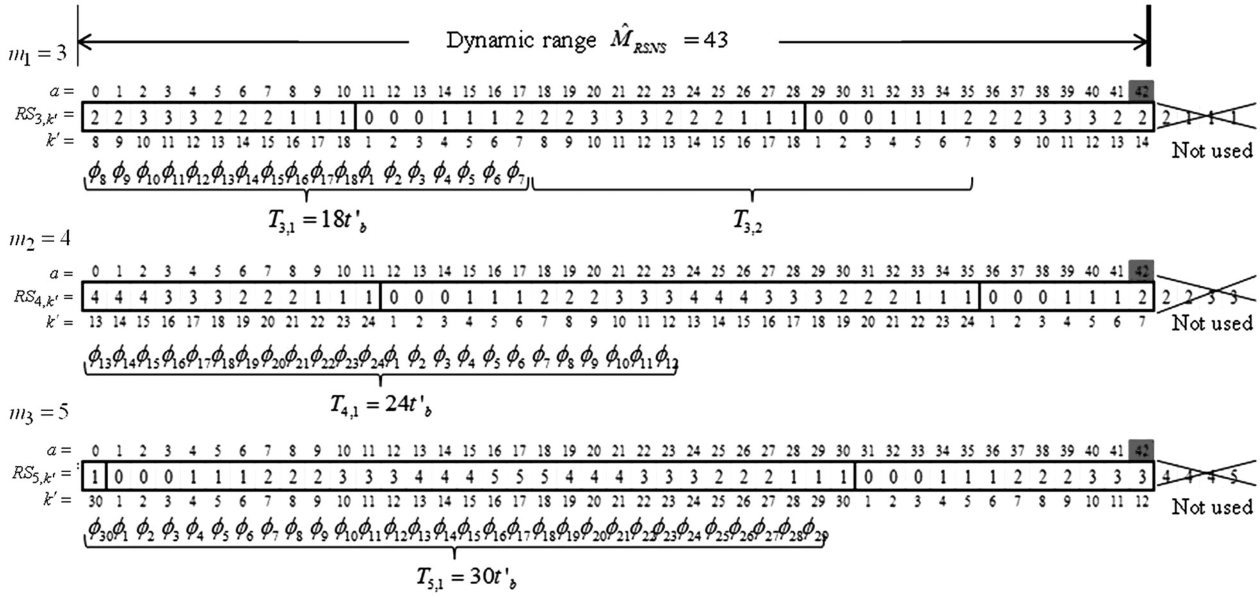


Fig. 5 *Illustration of transmitted signal using N = 3 robust symmetrical residue-P4 phase code sequences for $m_1 = 3$, $m_2 = 4$ and $m_3 = 5$*

$R_{u_i} = ct_b N_{c_i}/2$. With the RSNS-P4 waveform concept the range can be extended to $R_{u_{\mathrm{RSNS}}} = ct'_b \hat{M}_{\mathrm{RSNS}}/2$. To compare the detection range of the RSNS-P4 waveform with the corresponding single P4 code sequences, we note that the average signal power P_{CW} and the maximum detection range $R_{\max}$ are related as [1]

$$R_{\max} = \left[\frac{P_{\mathrm{CW}} G_{\mathrm{t}} G_{\mathrm{r}} \lambda^2 \sigma}{(4\pi)^3 \, k_{\mathrm{B}} T_0 F_{\mathrm{R}} B_{\mathrm{Ri}} (\mathrm{SNR}_{R_{\mathrm{out}}}/\mathrm{PG}) L} \right]^{(1/4)} \tag{26}$$

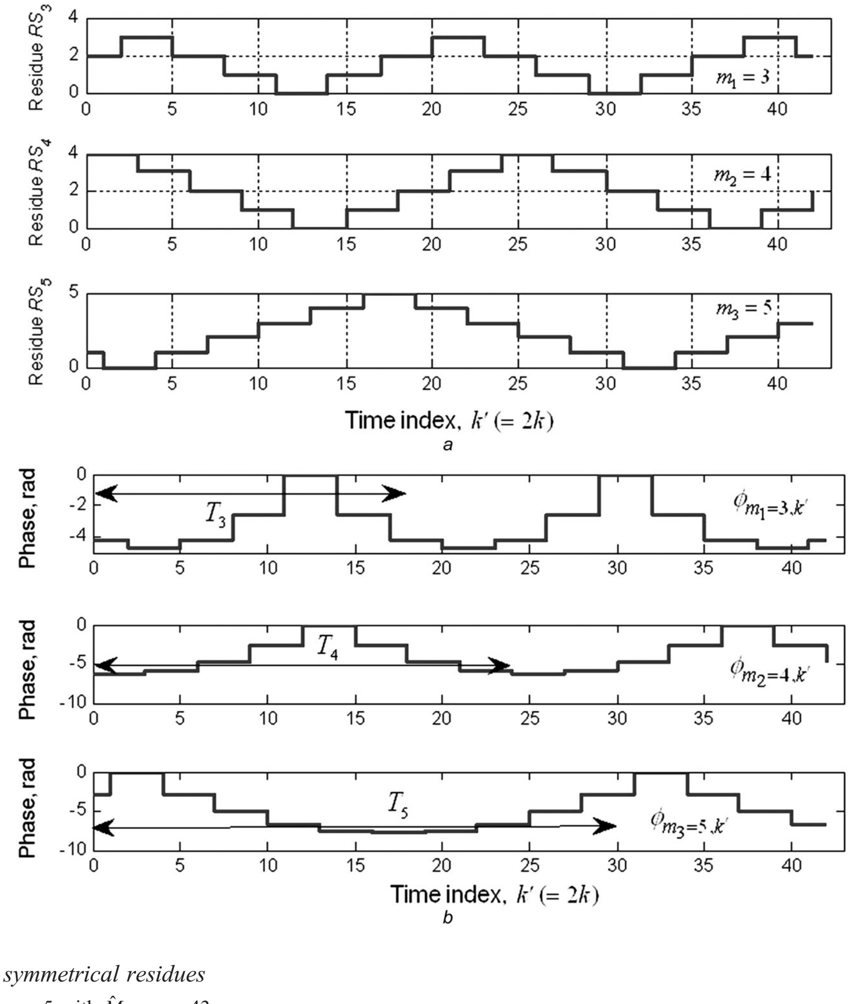


Fig. 6 *Plot of the RSNS symmetrical residues*

a With $m_1 = 3$, $m_2 = 4$and $m_3 = 5$ with $\hat{M}_{\mathrm{RSNS}} = 43$

b Their phase values

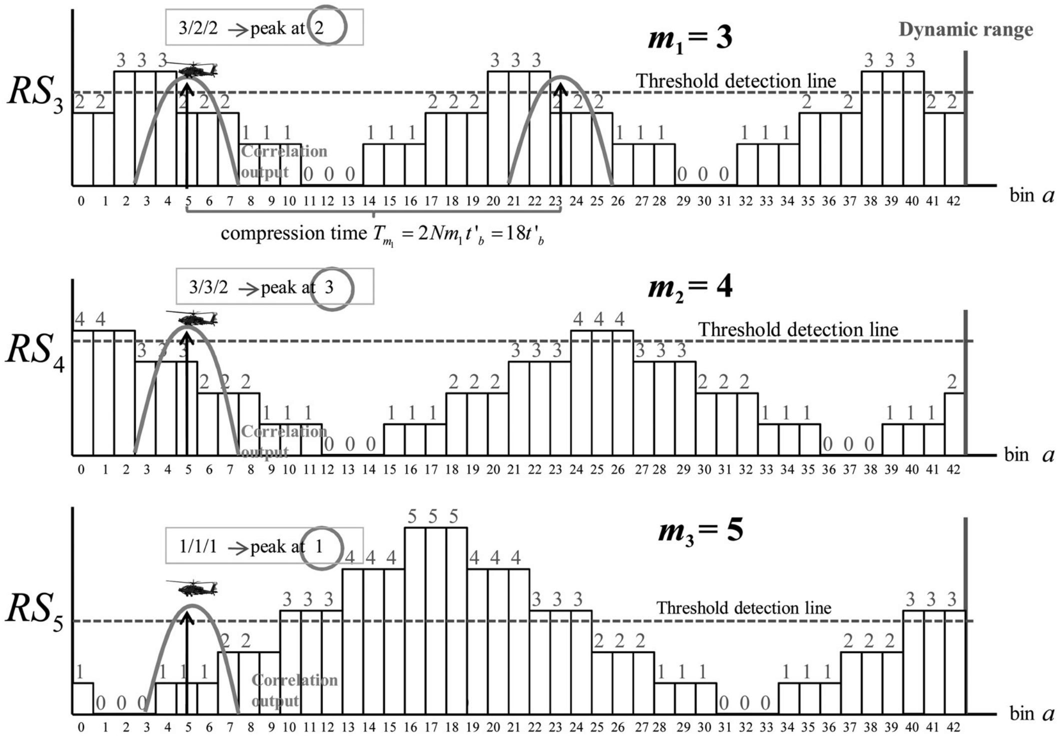


Fig. 7 *Illustration of target detection by using the range bin matrix for RSNS-P4 code with $m_1 = 3$, $m_2 = 4$ and $m_3 = 5$*

where $k_B = 1.38 \times 10^{-23}$ J/K (Boltzman's constant), $T_0 = 290$ K is the ambient noise temperature, F_R is the receiver noise factor, B_{Ri} is the radar receiver's input bandwidth in Hz, G_t and G_r are the transmitting and receiving antenna gains, $L(L \geq 1)$ a factor to account for losses and σ is the target's radar cross section in square metre. Propagation and loss factors have been omitted, since they do not directly affect the comparison.

The maximum resolvable detection range is limited by the unambiguous range. That is, the radar may be able to detect targets beyond R_u of the waveform, but there are ambiguities that must be resolved, commonly by using coincidence ranging [11, 12]. In Fig. 8, the maximum detection range is plotted as a function of the required CW power using $N = 3$ for $m_1 = 3$, $m_2 = 4$, and $m_3 = 5$ with a constant $\mathrm{SNR}_{R_{out}} = 13$ dB, $B_{Ri} = 3$ MHz, ($t'_b = 0.333$ μs), $G_t = G_r = 33$ dB, $f_c = 3$ GHz, $\lambda = 0.1$ m, $\sigma = 100$ m^2 and $T_0 = 290$ K, a loss factor of $L = 4/3$ and $F_R = 5$ dB. For comparison, the unambiguous range R_{ui} is shown for each P4 code used individually with the corresponding N_{ci} subcodes. For the P4 code, from (18), $N_{c_1} = 18/3 = 6$, $N_{c_2} = 8$, and $N_{c_3} = 10$ for $m_1 = 3$, $m_2 = 4$ and $m_3 = 5$, respectively. The individual P4 unambiguous ranges are $R_{u_1} = 3 \times 10^8 \times 10^{-6} \times 6/2 = 900$ m, $R_{u_2} = 1200$ m and $R_{u_3} = 1500$ m, respectively; all are less than what is achieved using the $N = 3$ RSNS-P4 waveform, which is $R_{u_{\mathrm{RSNS}}} = c\hat{M}_{\mathrm{RSNS}}\, t'_b/2 = 2150$ m. Note that to achieve this additional range, an increase in CW power is required as expected.

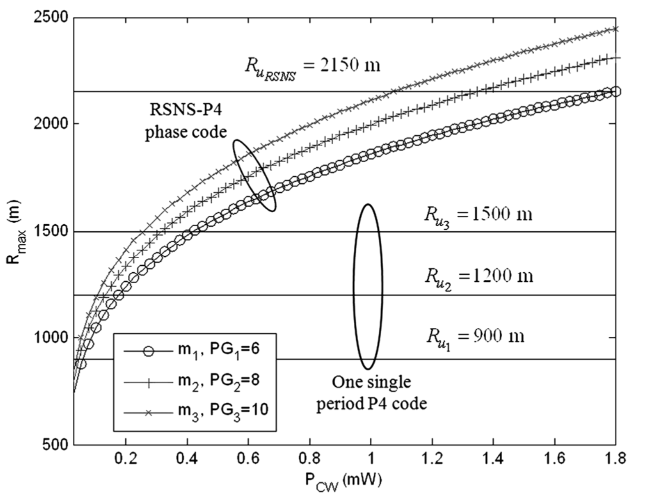


Fig. 8 *Comparison of the maximum unambiguous range of CW radar system for $\mathrm{SNR}_{R_{out}} = 13$ dB using the RSNS-P4 code for $m_1 = 3$, $m_2 = 4$ and $m_3 = 5$ against using each P4 code individually for $N_{c_i} = N_{c_{iRSNS}}/N$*

Detecting the target's correct range without error is a significant advantage of the RSNS encoding of the P4 waveform. Examination of Table 1 shows that a range bin error of ± 1 in any one coprime sequence (e.g. because of noise), may cause the resolved target's range to be off by only one range bin. The target's velocity also presents a limitation on the transmitted waveform. In order that the target does not transition a subcode or range bin until the $N\hat{M}_{\mathrm{RSNS}}$ subcodes are reflected from the target, the target's maximum allowable relative radial velocity must satisfy from (20) and (21)

$$\Delta R > (t_b \hat{M}_{\mathrm{RSNS}})\, V_r \tag{27}$$

or

$$N\hat{M}_{\mathrm{RSNS}} < \frac{c}{2V_r} \tag{28}$$

For the example shown, if the velocity is Mach 2 (680 m/s), then the product $N\hat{M}_{\mathrm{RSNS}} < 22.06 \times 10^4$. Note that (28) provides considerable flexibility in the waveform design,

doi: 10.1049/iet-rsn.2011.0349

allowing a large unambiguous range using only a few coprime sequences. This adds considerably to the LPI nature of the waveform.

8 Summary and conclusions

It was demonstrated that the RSNS can be used in the construction of P4 polyphase waveforms. Using multiple RSNS moduli significantly extends the unambiguous range of the P4 modulation beyond a single code period. In addition, all of the desirable properties of the P4 code are maintained such as Doppler tolerance and periodic ambiguity sidelobe performance [1]. Also, when using the RSNS mapping, the number and choice of moduli is flexible. Consequently, the number of subcodes within a code period can be made significantly smaller resulting in a smaller digital correlation processor. In addition, the combined RSNS phase sequences have an inherent integer Gray code property that makes it particularly attractive for controlling any possible errors in the detected target's range that might occur. The limitations on the product $N\hat{M}$ show that a considerable number of symmetrical residue-P4 waveforms can be designed and implemented. The RSNS mapping allows control of the maximum unambiguous range and range resolution over a wide parameter space, limited only by the capability of the hardware. With components such as direct digital synthesisers and digital receivers, the waveform can be reconfigured for different code lengths adding to the LPI characteristics of the waveform.

We note that the unambiguous range of a CW Frank code and the P2 (not a perfect code) can also be extended in a similar manner using a residue number system (RNS) mapping because of the saw-tooth waveform characteristic of the phase distribution within a code period [13]. The residue-Frank phase code however, does not have an integer Gray code property and, as such, if the detected target's range bin in one of the transmitted sequences is incorrect, a very large range error will occur. For the P1 and P3 polyphase (perfect) codes, a number theoretic transform is not obvious. We also note that the Doppler responses of the P3 and P4 codes have much lower peak sidelobes (secondary maxima) than the Frank, P1 and P2 codes and have comparable image lobes that are because of the polyphase codes being derived from a Nyquist rate sampling of the linear chirp phase characteristics [14].

9 Acknowledgment

This work was supported by the Naval Postgraduate School Center for Joint Services Electronic Warfare.

10 References

1 Pace, P.E.: 'Detection and classifying low probability of intercept radar' (Artech House, Norwood, MA, 2009, 2nd edn.)
2 Levanon, N.: 'CW alternatives to the coherent pulse train-signals and processors', *IEEE Trans. Aerosp. Electron. Syst.*, 1992, **28**, (2), pp. 387–395
3 Lewis, L.B.: 'Linear frequency modulation derived polyphase pulse compression codes', *IEEE Trans. Aerosp. Electron. Syst.*, 1982, **AES-18**, (5), pp. 637–641
4 Lewis, B.L., Kretschmer, F.F.: 'A new class of polyphase pulse compression codes and techniques', *IEEE Trans. Aerosp. Electron. Syst.*, 1981, **AES-17**, (3), pp. 364–372
5 Pace, P.E., Styer, D., Akin, I.A.: 'A folding ADC preprocessing architecture employing a robust symmetrical number system with Gray-code properties', *IEEE Trans. Circ. Syst.*, 2000, **47**, (5), pp. 462–467
6 Luke, B.L., Pace, P.E.: 'N-sequence RSNS ambiguity analysis', *IEEE Trans. Inf. Theory*, 2007, **53**, (5), pp. 1759–1766
7 Arvizo, M.R., Calusdian, J., Hollinger, K.B., Pace, P.E.: 'Robust symmetrical number system preprocessing for minimizing encoding errors in photonic analog-to-digital converters', *Opt. Eng.*, 2011, **50**, (80) Q2
8 Styer, D., Pace, P.E.: 'Two-channel RSNS dynamic range', *IEEE Trans. Circ. Syst.*, 2002, **49**, (3), pp. 395–397
9 Luke, B.L., Pace, P.E.: 'Computation of the robust symmetrical number system dynamic range'. IEEE Information Theory Workshop, ITW, Dublin, 2010
10 Luke, B.L., Pace, P.E.: 'RSNS-to-binary conversion', *Trans. Circ. Syst. I, Regul. Pap.*, 2007, **54**, (9), pp. 2030–2043
11 Long, W.H., Harringer, K.A.: 'Medium PRF for the AN/APG-66 radar', *Proc. IEEE*, 1985, **73**, (2), pp. 301–311
12 Simpson, J.: 'PRF set selection for pulse Doppler radars'. IEEE Region 5 Conf., 1988: 'Spanning the Peaks of Electrotechnology', 1988, pp. 38–44
13 Paepolshiri, N.: 'Extending the unambiguous range of CW polyphase radar systems using number theoretic transforms'Naval Postgraduate School, Master's thesis, September 2011
14 Kretschmer, F., Lewis, B.L.: 'Doppler properties of polyphase coded pulse compression waveforms', *IEEE Trans. Aerosp. Electron. Syst.*, 1983, **AES-19**, (4), pp. 521–531

www.ietdl.org

RSN20110349

Author Queries

N. Paepolshiri, P.E. Pace, D.C. Jenn

Q1 IEE style for matrices and vectors is to use bold italics. Please check that we have identified all instances.

Q2 Please provide the page numbers in reference [7].