TWO-DIMENSIONAL ARRAY ANTENNA PATTERNS

Introduction

According to the principle of pattern multiplication, the radiation pattern of an array of identical elements (i.e., identical element patterns) can be written as the product (phasor quantities)

$$|F(\theta,\phi)| = |AF(\theta,\phi)||EF(\theta,\phi)\hat{e}|$$

where $|AF(\theta,\phi)|$ is the array factor and $|EF(\theta,\phi)|$ is the element factor. The element factor is actually a vector quantity where the unit vector $\hat{e}$ denotes the polarization of the element. The array factor depends only on the geometrical arrangement of the elements and their excitation conditions. The element factor depends only on the type of element.

The coordinate system is shown in Figure 1. If the *x*-*y* plane corresponds to the earth's surface and the *z* axis the zenith, the angles α and γ are the azimuth and elevation angles, respectively. The array is in the *x*-*y* plane.

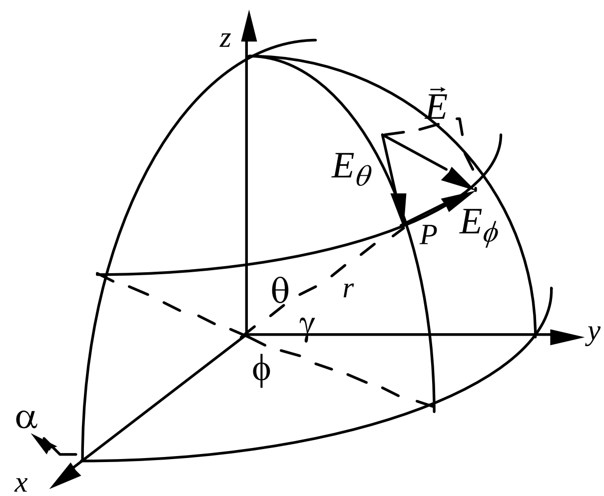


Figure 1: Coordinate system

For a periodic array of elements that are laid out on a rectangular lattice in the *x*-*y* plane, the array factor can be expressed as the sum

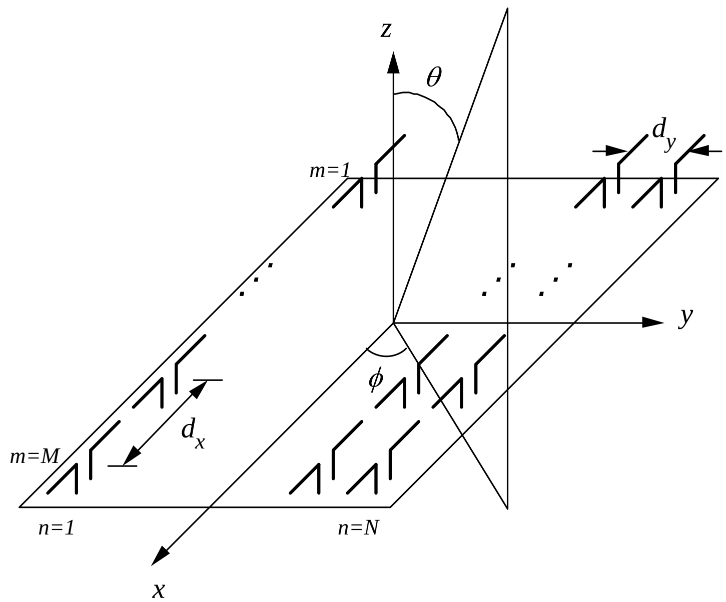
$$|AF(\theta,\phi)| = \left| \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} e^{j\Phi_{mn}} \exp\left(jk(d_x u(m-1) + d_y v(n-1))\right) \right|$$

where d_x and d_y are the spacings along the *x* and *y* axes, $k = 2\pi/\lambda$, and the *x* and *y* direction cosines are

$$u = \sin\theta\cos\phi,\ v = \sin\theta\sin\phi,\ \text{and}\ w = \cos\theta\,.$$

The amplitude and phase excitation coefficients for element (m,n) are A_{mn} and Φ_{mn}, which are controlled by the method of feeding the element. An example of a two-dimensional array of dipoles is shown in Figure 2. Note that the above formula is written for element m=n=1 located at the origin, x=y=0. However, the diagram shows the center of the array at the origin. The shifting of the reference merely adds a phase factor (exponential factor) to the array factor, which does not affect the magnitude of the array factor. Also note that if the number of elements is large, then the array dimensions are approximately

$$L_x \approx M\, d_x \text{ and } L_y \approx N\, d_y.$$



For uniform amplitudes, the two dimensional array factor for scanning the main beam to the direction (θ_s, ϕ_s) can be written as

$$|AF(\theta,\phi)| = \underbrace{\left|\frac{\sin(M(\psi_x - \psi_{sx})/2)}{M\sin((\psi_x - \psi_{sx})/2)}\right| \left|\frac{\sin(N(\psi_y - \psi_{sy})/2)}{N\sin((\psi_y - \psi_{sy})/2)}\right|}_{\text{NORMALIZED ARRAY FACTOR}}$$

where

$$\psi_x = kd_x \sin\theta\cos\phi, \quad \psi_{sx} = kd_x \sin\theta_s \cos\phi_s$$
$$\psi_y = kd_y \sin\theta\sin\phi, \quad \psi_{sy} = kd_y \sin\theta_s \sin\phi_s$$

Two-dimensional Array Patterns: *array2d.m*

The array factor for a two-dimensional array is computed by *array2d.m*. The user can select from several types of amplitude distributions. Also, phase shifter roundoff algorithms can be selected to observe the effects of digital phase shifters on the beam position, gain and sidelobe level. *array2d.m* may call several functions, depending upon the calculations requested by the user. They include `taylor`, `bayliss`, `triangular` and `cosine` which compute aperture distributions, and `tuncate` and `rro` which execute phase shifter roundoff algorithms.

The user is asked to input the range of angles for the pattern calculation. If the start and stop values for ϕ are identical, then a pattern cut for that value of ϕ is generated. Similarly, if the start and stop values for θ are identical, then a pattern cut for that value of θ is generated. If a range of both θ and ϕ are given, then a two-dimensional contour, mesh, or both is plotted in direction cosine space. The range of u and v will correspond to the given range of θ and ϕ.

Amplitude Tapering

The amplitude and phase coefficients are used to scan the beam, control the sidelobe level and, in some cases, shape the radiation pattern. Some common amplitude distributions for controlling sidelobes are given in Table 1.

Table 1: Amplitude tapers

Distribution	First sidelobe or parameters
Uniform (Reference)	-13.2 dB (large array)
Cosine on a pedestal	$a, b: \ a + (1-a)\cos^{b}(\|\pi x / L_x\|)$
Taylor (sum beam)	SLL, $\bar{n}$
Bayliss (difference beam)	SLL, $\bar{n}$
Triangular	-26 dB (large array)

The user is asked to select the amplitude distribution in the two principal planes. The distribution is assumed to be separable in x and y; that is, $A_{mn} = A_{x_m} A_{y_n}$ and $\Phi_{mn} = \Phi_{x_m} \Phi_{y_n}$. The amplitudes are set in the function *getamplitudes.m*.

The aperture efficiency is computed from the formula:

$$\rho_a = \frac{\left|\sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn}\right|^2}{MN \sum_{m=1}^{M} \sum_{n=1}^{N} \left|A_{mn}\right|^2}$$

and the directivity of an array of isotropic elements is given by the formula

$$D = D_u \rho_a = \frac{4\pi A}{\lambda^2} \rho_a$$

where

D_u = directivity of a uniform array of the same size

A = aperture area $\approx L_x L_y$

The directivity is the maximum value of the directive gain, which is identical to the gain if the antenna is lossless. In general, losses other than aperture tapering will reduce the directivity of the antenna. When losses are included, the gain is

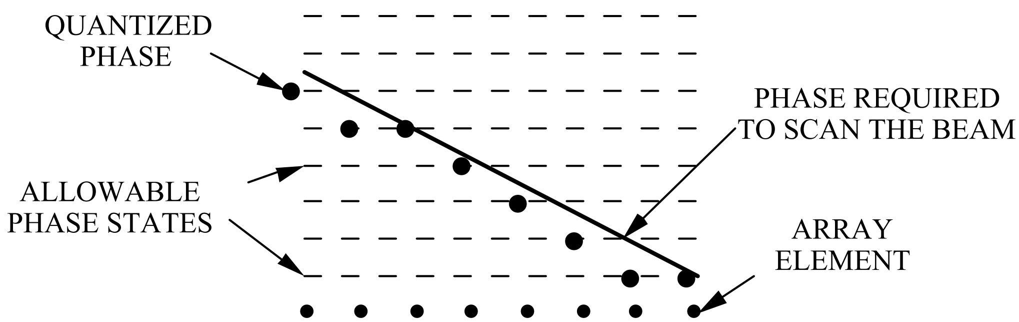
$$G = D\rho$$

where ρ is the efficiency factor.

Phase Shifter Roundoff

Phase shifters are used to control the location and shape of the antenna beam. Generally a linear phase shift is desired to point a focussed beam in space. The phase shift can be specified in degrees per element or total degrees across the length of the array.

Most phase shifters are digital devices, or at least are digitally controlled. Therefore only discrete values of phase shift are allowed, and they may not be the precise values required a particular element as depicted below:



In practice a roundoff method must be prescribed. Simply truncating or rounding to the closest value will yield a periodic quantization error that causes large sidelobes to occur. Furthermore the beam pointing can deviate significantly from the case of no quantization error. With regard to beam pointing, it is usually best to employ some type of randomization when rounding off. A common method is weighted random roundoff, which is a type of fuzzy logic. Randomization is most effective when the amplitudes of the elements are all roughly the same (i.e., roundoff errors will sum to zero). If not, then large roundoff errors at elements with large amplitudes A_{mn} can dominate.

Directivity calculation

When selected, the directivity of the array is calculated by integration of the pattern as follows:

$$D = \frac{4\pi}{\Omega_A}$$

where

$$\Omega_A = \iint \left|F(\theta,\phi)\right|^2 \sin\theta \, d\theta \, d\phi$$

is the beam solid angle and the normalized pattern function is

$$\left|F(\theta,\phi)\right| = \left|AF(\theta,\phi)\right|\left|EF(\theta,\phi)\right|\left|GF(\theta,\phi)\right|.$$

AF is the normalized array factor, *EF* is the normalized element factor, and *GF* is the normalized ground plane factor. The limits of integration are $0 < \theta < \pi$, $0 < \phi < 2\pi$.

The calculation is done in the script *ArrayDirectivity.m*. Gaussian quadrature is used with 20 points per interval. The number of intervals can be varied: *ndivt* for the θ integration and *ndivp* for the ϕ integration. More intervals are required for larger arrays that have greater pattern variations. **Caution: this calculation can be time consuming for large arrays.**

Convergence can be tested by increasing the number of intervals to see of the result changes significantly.

Ground plane

An infinite ground in the z =0 plane can be selected. The method of images is used to compute the pattern in the upper hemisphere (z >0). If the directivity of an array with a ground is being calculated the limits of integration are automatically changed $0 < \theta < \pi/2$ and *ndivt* is halved. The ground plane factor is included in the pattern and directivity calculations.

Array Elements

Several array elements are selectable including isotropic and dipoles aligned in *x, y* or *z*. The half wave dipole formulas are used. For a half wave dipole along the *z* axis:

$$E_\theta = \frac{j\eta_o I_m \, e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$E_\phi = 0$$

For a half wave dipole along the *y* axis:

$$E_\theta = \frac{j\eta_o I_m \, e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\sin\theta\sin\phi\right)}{1-\sin^2\theta\sin^2\phi}\right]\cos\theta\sin\phi$$

$$E_\phi = \frac{j\eta_o I_m \, e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\sin\theta\sin\phi\right)}{1-\sin^2\theta\sin^2\phi}\right]\cos\phi$$

For a half wave dipole along the *x* axis:

$$E_\theta = \frac{j\eta_o I_m \, e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\sin\theta\cos\phi\right)}{1-\sin^2\theta\cos^2\phi}\right]\cos\theta\cos\phi$$

$$E_\phi = -\frac{j\eta_o I_m \, e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\sin\theta\cos\phi\right)}{1-\sin^2\theta\cos^2\phi}\right]\sin\phi$$

When dipole elements are used the components E_θ and E_ϕ are plotted. Note that the pattern function F normally includes both components (often called the composite field):

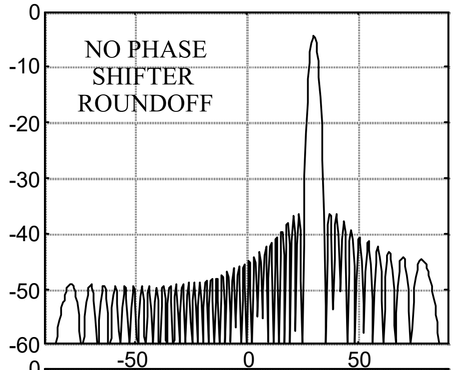
$$|F(\theta,\phi)| = \sqrt{|E_\theta(\theta,\phi)|^2 + |E_\phi(\theta,\phi)|^2}\ .$$

References:

[1] C. A. Balanas, *Antenna Theory*, Wiley.
[2] J. D. Kraus, *Antennas*, McGraw-Hill.

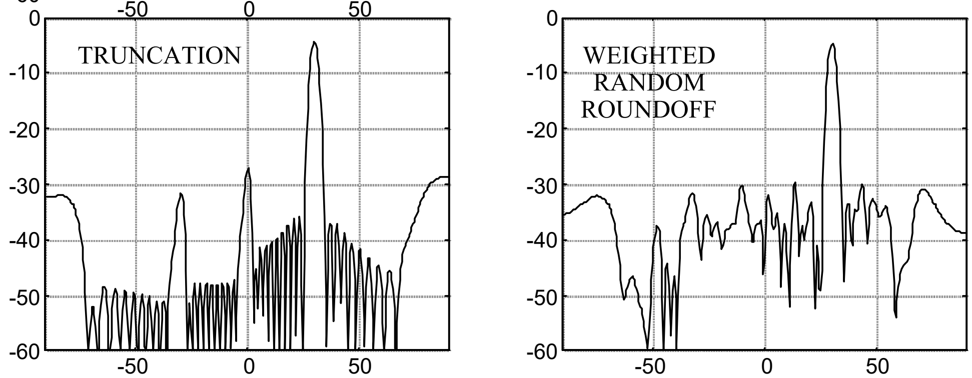
Examples

Example 1: Comparison of roundoff methods for a linear array of 60 elements (30 dB Taylor distribution). If the number of phase shifter bits is B, then the phase shift per element can be as small as $360^\circ / 2^B$. For example, a 4-bit phase shifter has 22.5 degree phase steps.

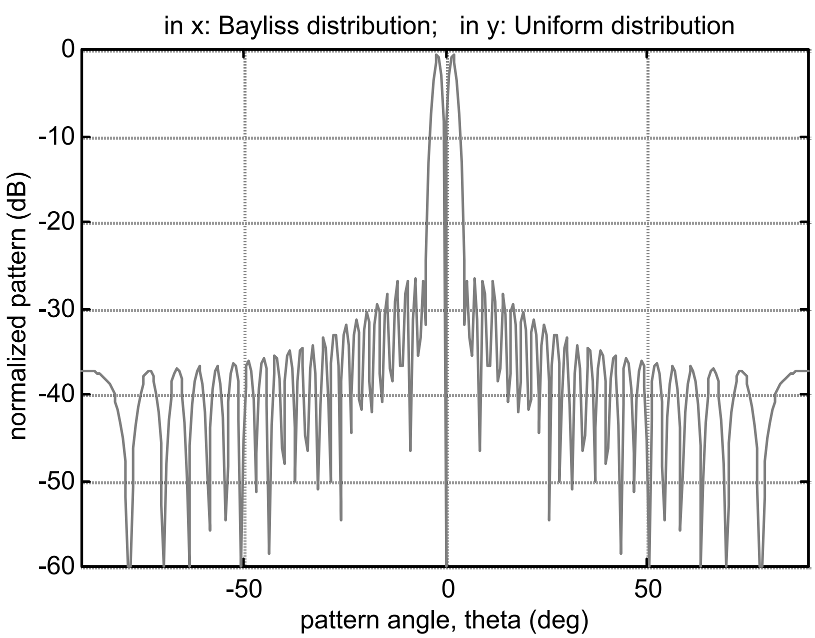


Truncation causes beam pointing errors. Random roundoff methods destroy the periodicity of the quantization errors. The resultant rms error is smaller than the maximum error using truncation.

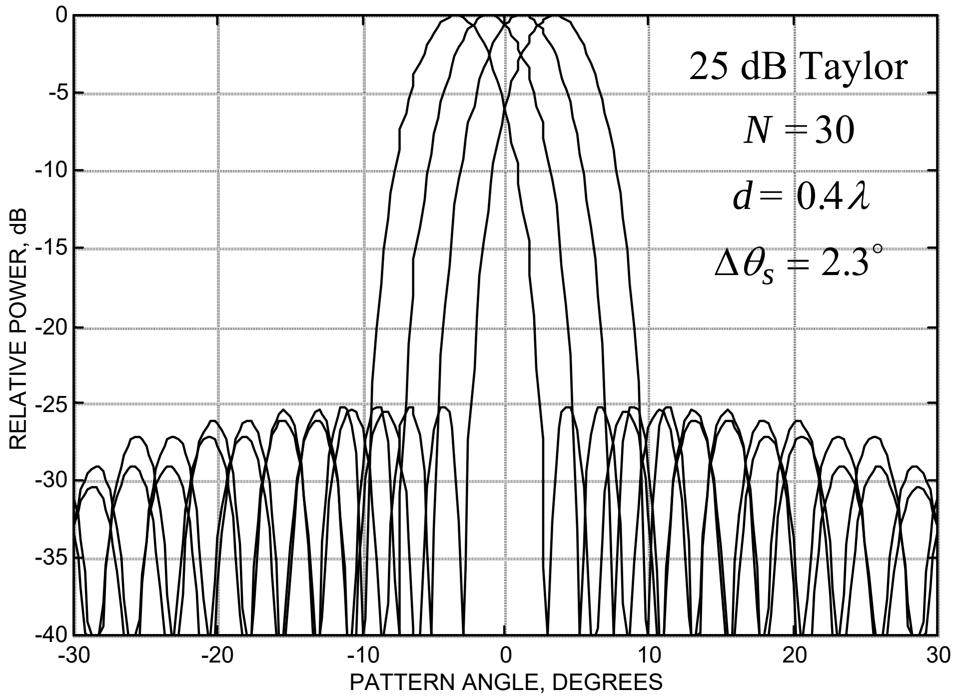
Linear array, 60 elements, $d = 0.4\lambda$
4 bit phase shifters



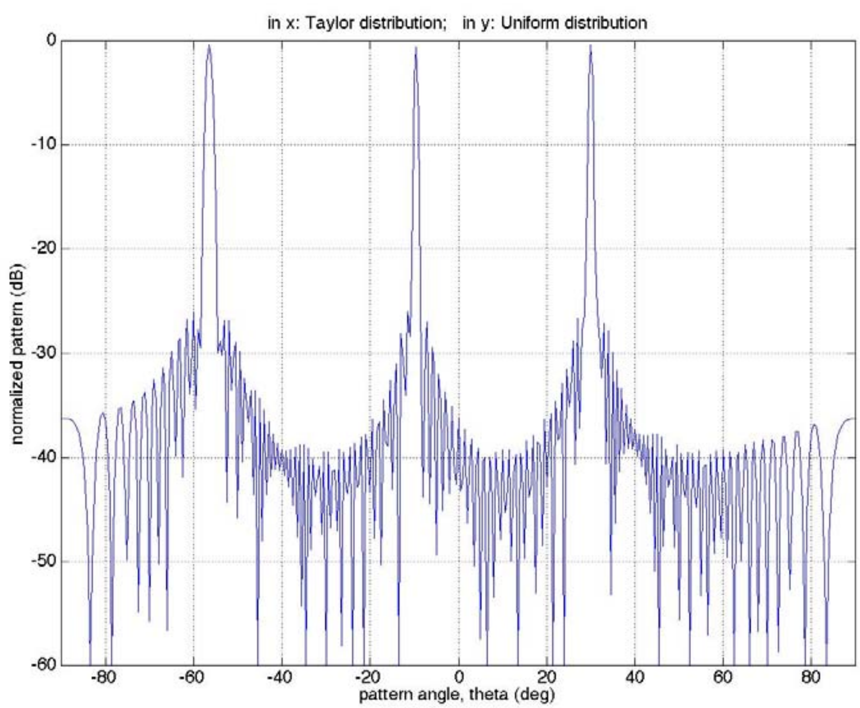
Example 2: Difference beam: 60 elements, 25 dB Bayliss distribution, $d_x = 0.4\lambda$



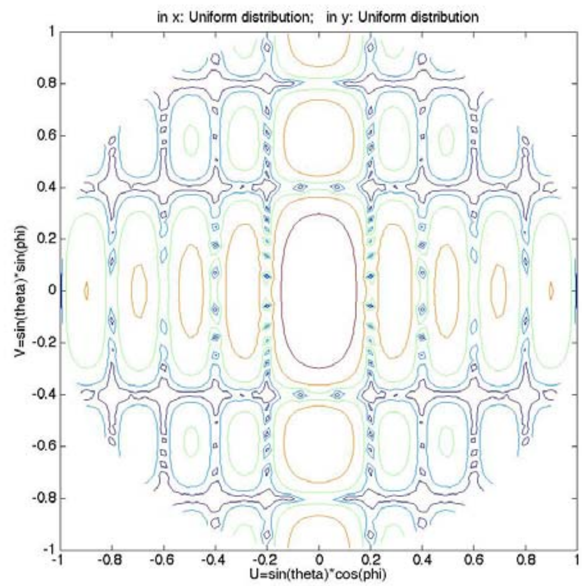
Example 3: Multiple beam antenna, 4 beams squinted at 2.3 degree increments. (In order to plot multiple patterns the user must go into the program an insert “hold on” statements.)

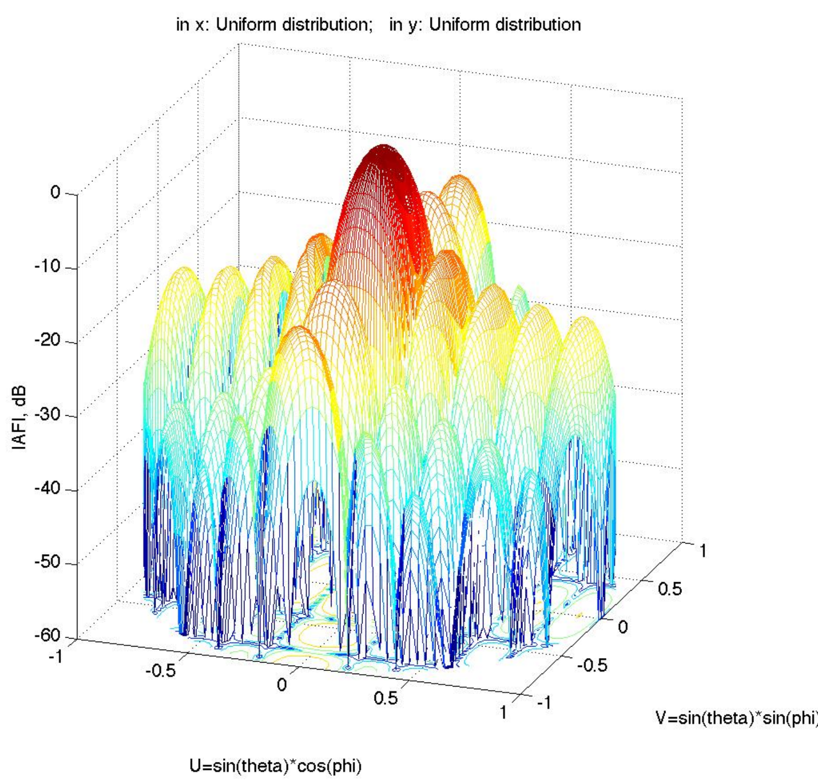


Example 4: Grating lobes; 50 elements, 1.5 wavelength spacing,30 degree scan angle, 25 dB Taylor distribution. Grating lobes are located at about -10° and -56°.



Example 5: Planar array of 10 by 5 elements; uniform distribution in both planes; half wavelength spacing in both planes; no beam scan. The range of values computed are $\theta \leq 90^\circ$ and $0^\circ \leq \phi \leq 360^\circ$ in 1 degree increments. Mesh and contour plots are shown.





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