Game-Theoretic Models for Rapid Operational Airlift Network Design in Contested Environments

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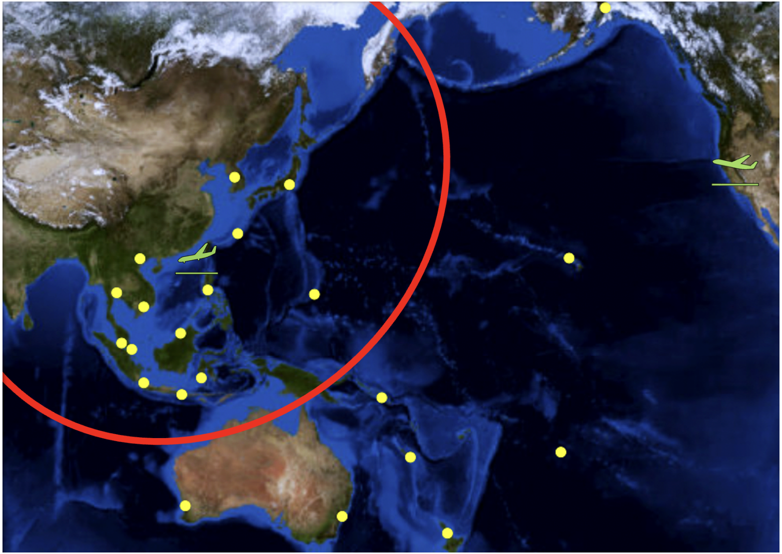
The 91st MORS Symposium
US Military Academy, West Point, NY
14 June, 2023

Question: How should AMC allocate airlift capacity in a contested environment?



Measuring Airlift Capacity

Definition

The US Air Force uses "**Maximum on Ground (MOG)**" as a measure of an airfield's capacity for processing aircraft.

Reference: AFPAM 10-1403

- Indicates, e.g., the amount of support equipment and personnel assigned to an airfield.
- 1 MOG $\approx$ 8 C-17 equivalents per day

A C-17 engine being loaded onto a C-17 at Yokota Air Base, Japan. (Source: Air Mobility Command)

The Problem, and Our Contributions

The Problem: Determine how much MOG should be at each airfield in the airlift network.

Importance: Airlift is an important means of supplying distributed forces over long distances.

Solutions Without Our Help: Recent work at the Naval Postgraduate School [Whitlow 2022] developed an optimization model of the problem, called the **Airlift Optimization Planning Tool (AOPT)**.

- Accounts for "risk to force" via penalties.

Our Contribution

We model the problem as a two-player zero-sum game.

- The attacker selects a subset of the airfields to attack.
- The defender allocates the MOG (similarly to AOPT).

The literature on "attacker-defender" models (see e.g., [Alderson et al. 2011]) can be leveraged to compute prescribed MOG allocations.

The Model: Sketch

$$\min_{Y} \max_{X,A,M} \left\{ X_{ts} - \sum_{i \in \mathcal{N}} q_i Y_i \left[\sum_{j:(i,j) \in \mathcal{A}} X_{ij} + \sum_{j:(j,i) \in \mathcal{A}} X_{ji} \right] \right\}$$

Data:

- $\mathcal{N}$ = set of airfields (i.e., "nodes")
- $\mathcal{A}$ = set of feasible flight legs (i.e., "arcs")
- q_i = penalty for flow through airfield $i \in \mathcal{N}$, if attacked
- Other data related to supply, cargo capacities of aircraft, . . .

Attacker's Decision Variables:

- $Y_i = \begin{cases} 1, & \text{if } i \in \mathcal{N} \text{ is attacked} \\ 0 & \text{otherwise} \end{cases}$

Defender's Decision Variables

- X_{ij} = amount of cargo sent on leg (i, j)
- A_{ij} = number of aircraft sent on leg (i, j)
- M_{ij} = amount of MOG moved from airfield i to j

Constraints: The M_{ij}'s constrain the A_{ij}'s, which constrain the X_{ij}'s.

Full Formulation [Cooper 2023, page 9]

ADAPT Formulation

$$\min_{Y} \max_{X,A,M} \quad X_{ts} - \sum_{i \in \mathcal{N}} q_i Y_i \left[\sum_{j:(i,j)\in\mathcal{A}} X_{ij} + \sum_{j:(j,i)\in\mathcal{A}} X_{ji} \right]$$

$$\text{subject to} \quad \sum_{j:(i,j)\in\mathcal{A}} X_{ij} - \sum_{j:(j,i)\in\mathcal{A}} X_{ji} = \begin{cases} X_{ts}, & \text{if } i = s \\ 0, & \text{if } i \notin \mathcal{N} \setminus \{s,t\} \\ -X_{ts} & \text{if } i = t \end{cases}$$

$$X_{ij} \le \begin{cases} cargocap \cdot A_{ij} & \text{if } (i,j) \in \mathcal{A} \\ supply & \text{if } (i,j) = (t,s) \end{cases}$$

$$\sum_{(i,j)\in\mathcal{A}} M_{ij} \le maxmogmoved$$

$$-\sum_{j:(i,j)\in\mathcal{A}} M_{ij} + \sum_{j:(j,i)\in\mathcal{A}} M_{ji} \le maxmog_i - startmog_i \qquad \forall i \in \mathcal{N}$$

$$\sum_{j:(i,j)\in\mathcal{A}} M_{ij} - \sum_{j:(j,i)\in\mathcal{A}} M_{ji} +$$

$$mogused_i \left(\sum_{j:(i,j)\in\mathcal{A}} A_{ij} + \sum_{j:(j,i)\in\mathcal{A}} A_{ji} \right) \le startmog_i \qquad \forall i \in \mathcal{N}$$

$$\sum_{i \in \mathcal{N}} Y_i \le maxattacks$$

$$X_{ij}, A_{ij}, M_{ij} \ge 0 \qquad \forall (i,j) \in \mathcal{A}$$

$$Y_i \in \{0,1\} \qquad \forall i \in \mathcal{N}$$

NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

GAME-THEORETIC MODELS FOR RAPID OPERATIONAL AIRLIFT NETWORK DESIGN IN CONTESTED ENVIRONMENTS

by

Alexander J. Cooper

March 2023

Thesis Advisor: Jefferson Huang
Second Reader: Peter A. Nesbitt

Approved for public release. Distribution is unlimited.

Determining MOG Allocations

ADAPT Formulation

$$\min_{Y} \max_{X,A,M} \quad X_{ts} - \sum_{i \in \mathcal{N}} q_i Y_i \left[\sum_{j:(i,j) \in \mathcal{A}} X_{ij} + \sum_{j:(j,i) \in \mathcal{A}} X_{ji} \right]$$

$$\text{subject to} \quad \sum_{j:(i,j) \in \mathcal{A}} X_{ij} - \sum_{j:(j,i) \in \mathcal{A}} X_{ji} = \begin{cases} X_{ts}, & \text{if } i = s \\ 0, & \text{if } i \notin \mathcal{N} \setminus \{s,t\} \\ -X_{ts} & \text{if } i = t \end{cases}$$

$$X_{ij} \leq \begin{cases} cargocap \cdot A_{ij} & \text{if } (i,j) \in \mathcal{A} \\ supply & \text{if } (i,j) = (t,s) \end{cases}$$

$$\sum_{(i,j) \in \mathcal{A}} M_{ij} \leq maxmogmoved$$

$$- \sum_{j:(i,j) \in \mathcal{A}} M_{ij} + \sum_{j:(j,i) \in \mathcal{A}} M_{ji} \leq maxmog_i - startmog_i \qquad \forall i \in \mathcal{N}$$

$$\sum_{j:(i,j) \in \mathcal{A}} M_{ij} - \sum_{j:(j,i) \in \mathcal{A}} M_{ji} + mogused_i \left(\sum_{j:(i,j) \in \mathcal{A}} A_{ij} + \sum_{j:(j,i) \in \mathcal{A}} A_{ji} \right) \leq startmog_i \qquad \forall i \in \mathcal{N}$$

$$\sum_{i \in \mathcal{N}} Y_i \leq maxattacks$$

$$X_{ij}, A_{ij}, M_{ij} \geq 0 \qquad \forall (i,j) \in \mathcal{A}$$

$$Y_i \in \{0,1\} \qquad \forall i \in \mathcal{N}$$

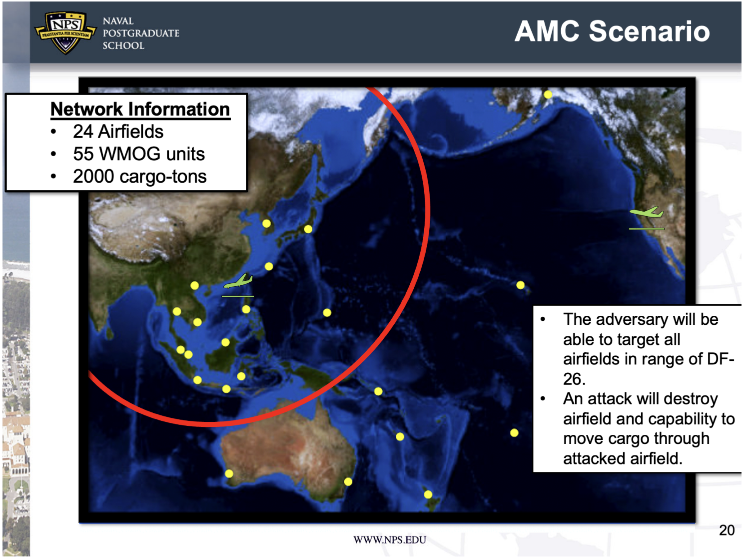
Basic Idea

1. Pick an initial attack Y.
2. Solve the defender's problem for X, A, M, given Y.
3. Try to find a better attack Y, given X, A, M.
4. Repeat ...

More specifically (see [Cooper 2023, Section 3.2]),

- decompose the minimax problem into a "master" problem and a "defender subproblem", and
- iteratively add "Benders cuts" to the master problem.

Application to Notional Scenario



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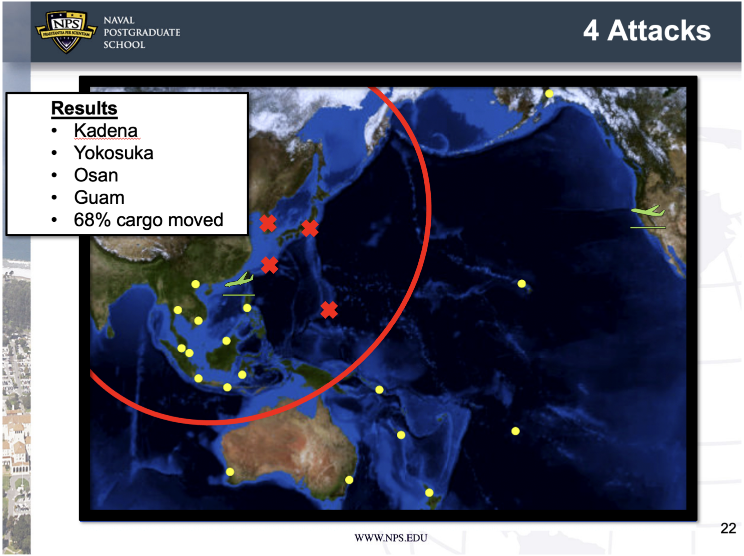

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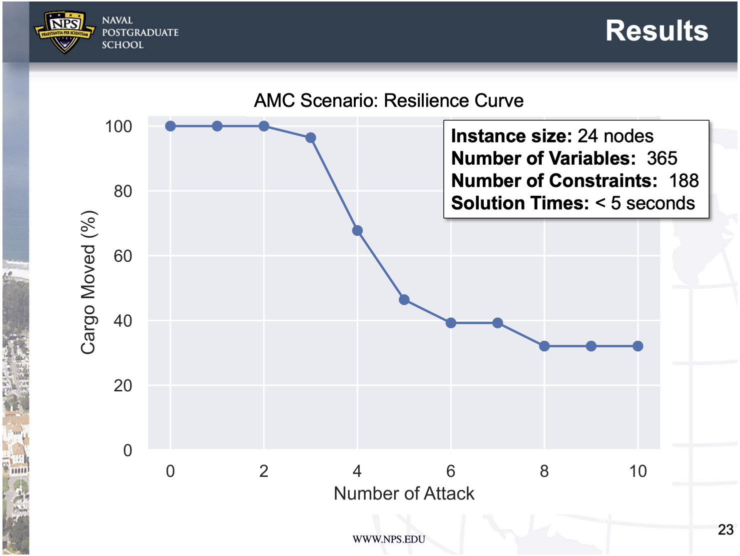

Take-Aways and Ongoing Work

Take-Aways

- Formulated attacker-defender model for allocating airlift capacity in a contested environment.
- The model can be used to find "robust" MOG allocations.

Ongoing Work

- The model has been extended to supplying theater entry points with multiple cargo types, and penalties for unsatisfied demand.
- Part of an ongoing project sponsored by the Air Force Office of Scientific Research.