Maximally Informative Underwater Sensor Placement

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Joint work with Robert Bassett (NPS) and LT Erik Vargas (USN)

About Me

Recent Academic History:

- Applied Math PhD from Stony Brook University (2016)
- Postdoc in Cornell Operations Research (OR) Department (2016-2018)
- Assistant Prof. in NPS OR Department (2018-Present)

Main Research Interests:

- Markov Decision Processes (**MDPs**)
- Dynamic Resource Allocation Problems
- Defense Logistics

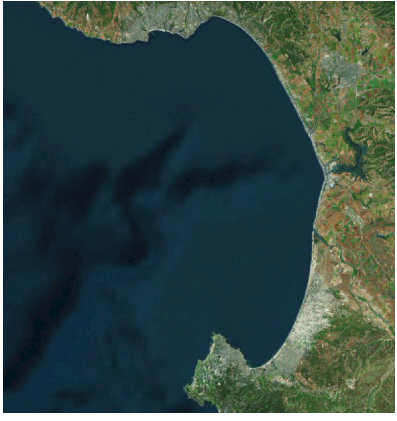
Some Recent Thesis Topics:

- Optimizing Supply Blocks for Expeditionary Units (Capt N.C. Anthony, USMC, June 2021)
- Application of Generative Adversarial Networks to Predicting and Manipulating an Adversary's Behaviors for Mobile Networked Control Systems (LT K. E. Plunkett, USN, March 2023; co-advising with Rudy Yoshida, NPS)
- Approximate Dynamic Programming Methods for the Dynamic Airlift Routing Problem (LCDR A.J. Cooper, USN, March 2023)
- Fleet-Informed Workload Forecasting for the DDNV Material Processing Center (LT A. P. Davidson, USN, September 2023)

For more, see https://faculty.nps.edu/jefferson.huang/

Sensing Correlated Numerical Signals

Example: Acoustic signals in the Monterey Bay.



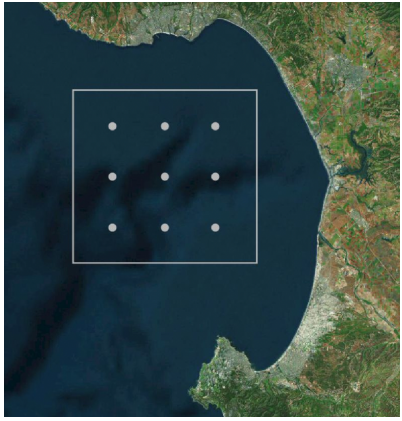
General Problem

Where should data be collected, in order to maximize the amount of "information" gleaned about the entire area of operations (AO)?

- i.e., in order to best infer the numerical signal across the entire AO, **where should sensors be placed?**

Sensor Placement Problem

Example: Placing hydrophones to detect vessels.



Suppose the AO has been discretized into a *finite* number of **locations** (e.g., grid squares). Let:

$$n = \text{number of locations}$$

- The numerical signal X_i at each location i is a random variable.

Only a subset of these locations can be monitored (e.g., with sensors). Let:

$$k = \text{number of locations that can be monitored}$$

Problem

Which k out of the n locations should be **selected**?

Sensor Placement Problem (Continued)

Each selection has an associated **utility**. For each subset $A \subseteq \{1, \ldots, n\} =: [n]$ of the locations, let:

$u(A) =$ utility of monitoring the locations in A

Optimization Problem

$$\begin{aligned} \text{maximize} \quad & u(A) \\ \text{subject to} \quad & A \subseteq [n] \\ & |A| = k \end{aligned}$$

What should $u(A)$ be?

Examples: $u(A)$ could measure:

- how well the AO is covered by the sensors; e.g., [Zhao, Yoshida, Cheung & Haws 2013], [Craparo & Karatas 2019]
- the uncertainty (e.g., entropy) associated with the locations in A; e.g., [Ko, Lee & Queyranne 1995], [Chen, Fampa & Lee 2022]
- **how informative the measurements in A are about the remaining locations**; e.g., [Caselton & Zidek 1984], [Krause, Singh & Gusterin 2008]

Utility Function

Use the **mutual information** $u(A) = \mathrm{MI}(A)$ between the sensed and un-sensed locations.

Gaussian Signals

Suppose $X_1, \ldots, X_n$ are jointly Gaussian, where

$$\mathbf{\Sigma}_A = \text{covariance matrix for locations } i \in A \subseteq [n]$$

Fact

$$\mathrm{MI}(A) = \frac{1}{2} \cdot \ln\left(\frac{\det \mathbf{\Sigma}_A \cdot \mathbf{\Sigma}_{[n]\setminus A}}{\det \mathbf{\Sigma}_{[n]}}\right), \quad A \subseteq [n]$$

So, in the Gaussian case, our optimization problem is equivalent to:

$$\begin{aligned} \text{maximize} \quad & \ln(\det \mathbf{\Sigma}_A) + \ln(\det \mathbf{\Sigma}_{[n]\setminus A}) \\ \text{subject to} \quad & A \subseteq [n] \\ & |A| = k \end{aligned} \tag{P}$$

Theorem [Bassett, H & Vargas 2022]

The following **semidefinite program** can be viewed as a relaxation of the discrete optimization problem (P):

$$\begin{aligned} \text{maximize} \quad & \ln\det\left(\mathbf{\Sigma}_{[n]} \odot \frac{\mathbf{X}+1}{2}\right) \\ \text{subject to} \quad & \mathrm{diag}(\mathbf{X}) = 1 \\ & \mathbf{X} \in \mathrm{PSD}_n \end{aligned}$$

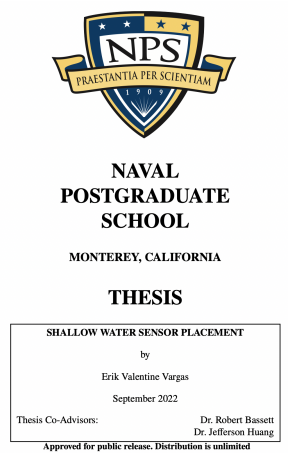
where PSD_n is the set of all symmetric positive semidefinite $n \times n$ matrices.

- Inspired by Goemans-Williamson relaxation of the Max Cut Problem [Goemans & Williamson 1995].

The relaxation can be solved efficiently, and be used in the context of a branch and bound algorithm for (P) [Bassett, H & Vargas 2022].

Summary: Maximally Informative Underwater Sensor Placement

- We consider a sensor placement problem formulated as a mutual information maximization problem.
- We propose a new efficiently solvable relaxation of the problem.
- The relaxation can be used in an effective branch & bound algorithm for the original sensor placement problem.



NAVAL
POSTGRADUATE
SCHOOL

MONTEREY, CALIFORNIA

THESIS

SHALLOW WATER SENSOR PLACEMENT

by

Erik Valentine Vargas

September 2022

Thesis Co-Advisors: Dr. Robert Bassett
Dr. Jefferson Huang

Approved for public release. Distribution is unlimited

Broader Research Interests:

Markov Decision Processes

MDPs Model (Stochastic) Sequential Decision Problems

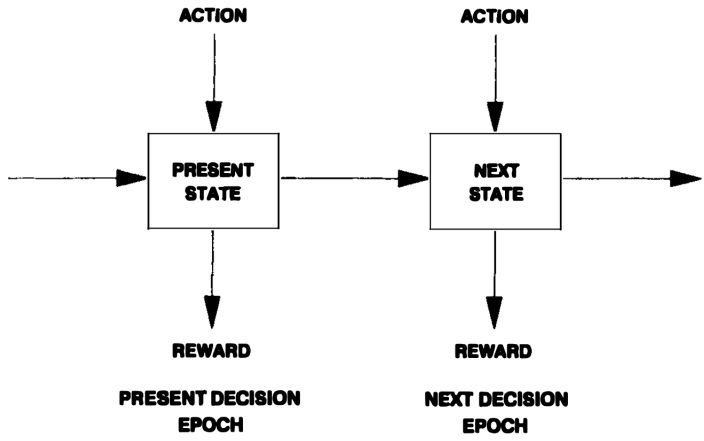


Figure 1.1.1 Symbolic representation of a sequential decision problem.

Source: M. L. Puterman, *Markov Decision Processes: Discrete Stochastic Dynamic Programming*, 2005.

Potential defense-related applications abound...

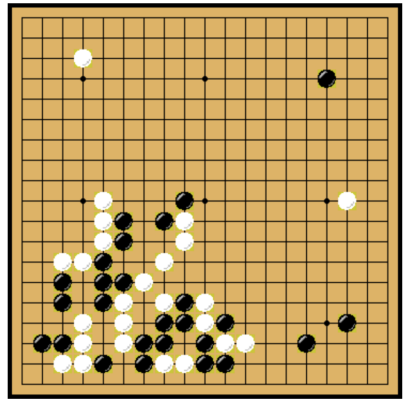
Recent Survey: M. Rempel & J. Cai, A review of approximate dynamic programming applications within military operations research, *Operations Research Perspectives* 8, 2021.

...but computing good solutions at scale is notoriously difficult.

It suffers from what Bellman called "the **curse of dimensionality**,*" meaning that its computational requirements grow exponentially with the number of state variables . . .*

. . . but it is still far more efficient and more widely applicable than any other general method.

Sutton & Barto, *Reinforcement Learning: An Introduction*, 2018 (p. 14)

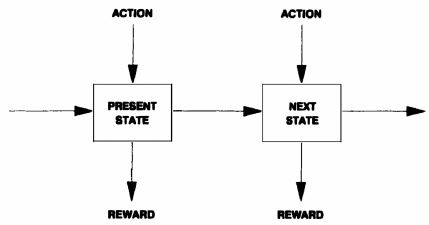


http://norvig.com/atoms.html

The MDP Model

A Markov decision process (MDP) is defined by

- a **state set** $\mathbb{X}$,
- sets of feasible **actions** $A(x)$ for each state $x \in \mathbb{X}$,
- **one-step rewards** $r(x, a)$ for each state $x \in \mathbb{X}$ and action $a \in A(x)$, and
- **transition probabilities** $p(y|x, a)$ for $x, y \in \mathbb{X}$ and $a \in A(x)$.



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Objective: Find an *optimal policy* ϕ that, for each state $x \in \mathbb{X}$, prescribes an action $\phi(x) \in A(x)$ to take.

Finding Optimal Policies

- It suffices to solve some associated **optimality equations** (aka. "Bellman equations")
- Solving the optimality equations provides the **value function**, which indicates which states are more valuable to be in than others.
- The value function can then be used to derive an **optimal policy**.

Example: When the objective is to maximize the expected total reward that is earned, the optimality equations can be viewed as a functional equation

$$v = T(v), \qquad v : \mathbb{X} \to \mathbb{R}$$

where T is a non-linear "optimality operator". Given a solution v to the optimality equation, an optimal policy is

$$\Phi^*(x) = \arg\max_{a \in A(x)} \left[r(x,a) + \sum_{y \in \mathbb{X}} v(y) p(y|x,a) \right]$$

Algorithms for Computing Optimal Policies

There are two main algorithmic paradigms:

Value Iteration: Iteratively approximate the value function $v : \mathbb{X} \to \mathbb{R}$.

Example: Start with an initial guess v_0, and iterate the optimality operator T:

$$v_k = T(v_{k-1}), \qquad k = 1, 2, \ldots$$

Policy Iteration: Iteratively approximate the optimal policy ϕ^*.

Example: Start with an initial policy ϕ_0, and iteratively improve it by identifying actions to switch to.

Note: There is a close connection between policy iteration and applying the simplex method to an associated linear program. Policy iteration can also be viewed as applying Newton's method to finding a root of $T(v) - v$. (See e.g., Puterman (2005) for details.)

Dealing with Computational Intractability

There are two main algorithmic paradigms:

Value Function Approximation: Search within a structured classes of value functions.

Examples:

- Piecewise-constant functions (e.g., via state aggregation)
- Parameterized functions (e.g., linear in hand-selected features, neural networks, . . .)

Policy Function Approximation: Search within structured classes of policies.

Examples:

- Piecewise-constant policies (e.g., via state aggregation)
- Parameterized policies (e.g., linear in hand-selected features, neural networks, . . .)

Reinforcement Learning: Dealing with Unknown/Complex System Dynamics

Idea: Use experience from interacting with an environment (or a simulation model of it) to learn good value or policy function approximations.

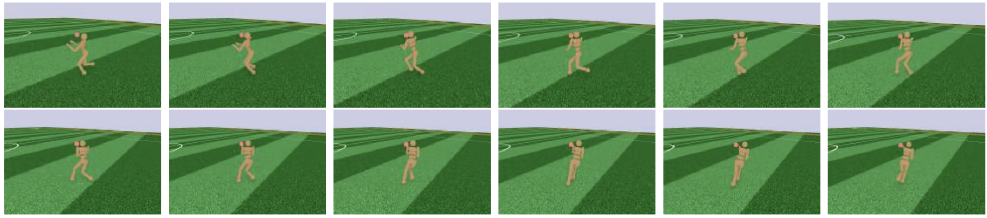

Figure 5: Still frames of the policy learned from RoboschoolHumanoidFlagrun. In the first six frames, the robot runs towards a target. Then the position is randomly changed, and the robot turns and runs toward the new target.

Source: Schulman et al., Proximal policy optimization algorithms, `arXiv:1707.06347v2`, 2017

Reinforcement Learning: Dealing with Unknown/Complex System Dynamics

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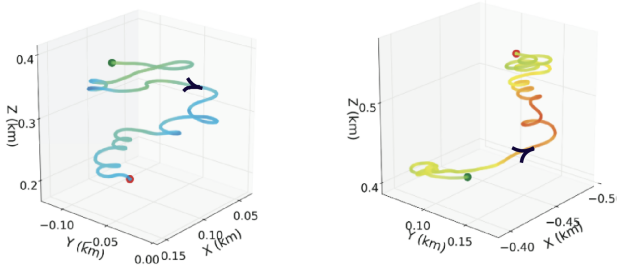

Figure 16.10: Sample thermal soaring trajectories, with arrows showing the direction of flight from the same starting point (note that the altitude scales are shifted). Left: before learning: the agent selects actions randomly and the glider descends. Right: after learning: the glider gains altitude by following a spiral trajectory. Adapted with permission from PNAS vol. 113(22), p. E4879, 2016, Reddy, Celani, Sejnowski, and Vergassola, Learning to Soar in Turbulent Environments.

Source:, Sutton & Barto, 2018 (p. 456)

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