Maximally Informative (Underwater) Sensor Placement

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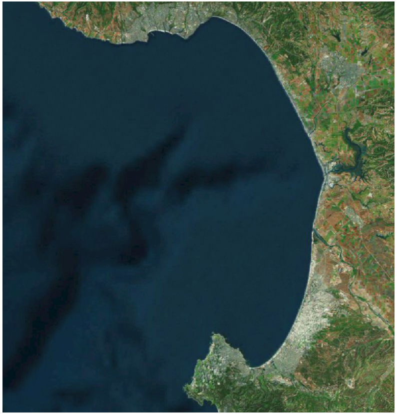
Sensing Correlated Numerical Signals

Example: Acoustic signals in the Monterey Bay.



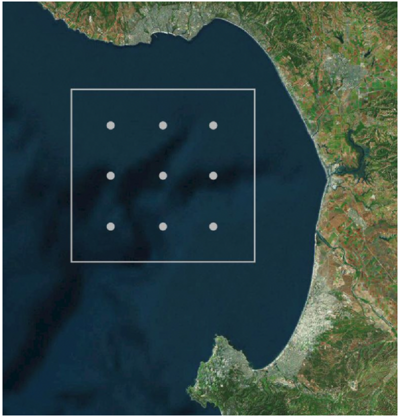
General Problem

Where should data be collected, in order to maximize the amount of "information" gleaned about the entire area of operations (AO)?

- i.e., in order to best infer the numerical signal across the entire AO, **where should sensors be placed?**

Sensor Placement Problem

Example: Placing hydrophones to detect vessels.



Suppose the AO has been discretized into a *finite* number of **locations** (e.g., grid squares). Let:

$$n = \text{number of locations}$$

- The numerical signal X_i at each location i is a random variable.

Only a subset of these locations can be monitored (e.g., with sensors). Let:

$$k = \text{number of locations that can be monitored}$$

Problem

Which k out of the n locations should be **selected**?

Sensor Placement Problem (Continued)

Each selection has an associated **utility**. For each subset $A \subseteq \{1, \ldots, n\} =: [n]$ of the locations, let:

$$u(A) = \text{utility of monitoring the locations in } A$$

Optimization Problem

$$\begin{aligned} \text{maximize} \quad & u(A) \\ \text{subject to} \quad & A \subseteq [n] \\ & |A| = k \end{aligned}$$

What should $u(A)$ be?

Examples: $u(A)$ could measure:

- how well the AO is covered by the sensors; e.g., [Zhao, Yoshida, Cheung & Haws 2013], [Craparo & Karatas 2019]
- the uncertainty (e.g., entropy) associated with the locations in A; e.g., [Ko, Lee & Queyranne 1995], [Chen, Fampa & Lee 2022]
- **how informative the measurements in A are about the remaining locations**; e.g., [Caselton & Zidek 1984], [Krause, Singh & Gusterin 2008]

Utility Function

Use the **mutual information** $u(A) = \mathrm{MI}(A)$ between the sensed and un-sensed locations.

Gaussian Signals

Suppose $X_1, \ldots, X_n$ are jointly Gaussian, where

$$\mathbf{\Sigma}_A = \text{covariance matrix for locations } i \in A \subseteq [n]$$

Fact

$$\text{MI}(A) = \frac{1}{2} \cdot \ln \left(\frac{\det \mathbf{\Sigma}_A \cdot \mathbf{\Sigma}_{[n] \setminus A}}{\det \mathbf{\Sigma}_{[n]}} \right), \quad A \subseteq [n]$$

So, in the Gaussian case, our optimization problem is equivalent to:

$$\begin{aligned} \text{maximize} \quad & \ln(\det \mathbf{\Sigma}_A) + \ln(\det \mathbf{\Sigma}_{[n] \setminus A}) \\ \text{subject to} \quad & A \subseteq [n] \\ & |A| = k \end{aligned} \qquad \text{(P)}$$

Theorem [Bassett, H & Vargas 2022]

The following **semidefinite program** can be viewed as a relaxation of the discrete optimization problem (P):

$$\begin{aligned} \text{maximize} \quad & \ln \det \left(\mathbf{\Sigma}_{[n]} \odot \frac{\mathbf{X} + 1}{2} \right) \\ \text{subject to} \quad & \text{diag}(\mathbf{X}) = 1 \\ & \mathbf{X} \in \text{PSD}_n \end{aligned}$$

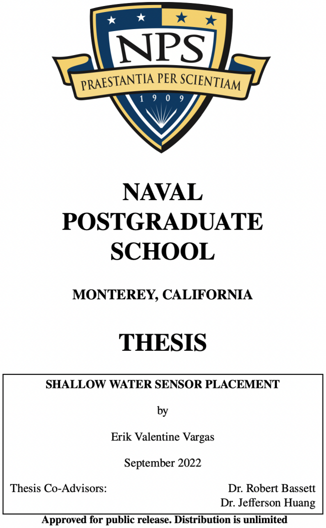
where PSD_n is the set of all symmetric positive semidefinite $n \times n$ matrices.

- Inspired by Goemans-Williamson relaxation of the Max Cut Problem [Goemans & Williamson 1995].

The relaxation can be solved efficiently, and be used in the context of a branch and bound algorithm for (P) [Bassett, H & Vargas 2022].

Summary

- We consider a sensor placement problem formulated as a mutual information maximization problem.
- We propose a new efficiently solvable relaxation of the problem.
- The relaxation can be used in an effective branch & bound algorithm for the original sensor placement problem.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THESIS

SHALLOW WATER SENSOR PLACEMENT

by

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