

Bandit Models of Cyber Intrusion

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Network Intrusion Scenario

Task: Collect useful **information** from computers on a **network**.



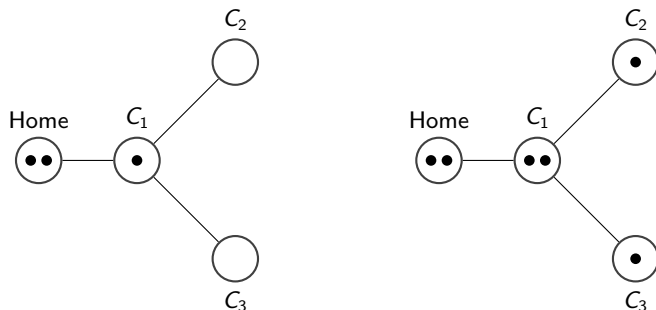
Initially have access to a “**home node**”, from which you can:

- ▶ try to collect information from that node, or
- ▶ try to infiltrate other computers connected to that node.

Question: How should **information collection** be balanced with **infiltration**?

Network Intrusion Scenario

Example:

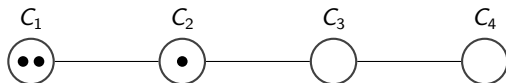


- ▶ 2 dots = **infiltrated** computer (can try to collect info.)
- ▶ 1 dot = **accessible** computer (can try to infiltrate)
- ▶ No dot = **inaccessible** computer.

Linear Network

The home node is initially at one end of a line of K computers.

Example:



- ▶ Trying to collect info from C_k yields a **unit of info** with probability μ_k .
- ▶ Each infiltration attempt is successful with probability s .

Objective: Maximize the **average number of info units** collected over a **discrete** and **finite** number T of **decision epochs**.

Linear Network: “Full” Knowledge

“Full” Knowledge Assumption: The values μ_1, \dots, μ_K and s are **known**.

- ▶ An optimal collect/infiltrate policy can be computed via **dynamic programming**.

Definition

Let $\mu^*(k) := \max_{i=1, \dots, k} \mu_i$, and define the operators $\mathcal{T}_C, \mathcal{T}_I$ on functions $f : \{1, \dots, K\} \rightarrow \mathbb{R}$ by

$$\mathcal{T}_C f(k) := \mu^*(k) + f(k)$$

and

$$\mathcal{T}_I f(x) := sf(k+1) + (1-s)f(k).$$

Optimality Equations

Letting $V_0 \equiv 0$, for $t = 1, \dots, T$ the value function V_t satisfies

$$V_t(k) = \begin{cases} \max\{\mathcal{T}_C V_{t-1}(k), \mathcal{T}_I V_{t-1}(k)\}, & k = 1, \dots, K-1 \\ \mathcal{T}_C V_{t-1}(K), & k = K. \end{cases}$$

Linear Network: “Full” Knowledge

Definition

A collect/infiltrate policy is a **threshold policy** if

collect info at epoch $t \implies$ collect info at epoch $t + 1$.

Theorem

If $T = 3$, then there is an optimal threshold policy.

Conjecture

For any horizon T , there is an optimal threshold policy.

- ▶ To prove the conjecture, it suffices to show that for $k = 1, \dots, K - 1$,

$$V_t(k + 1) - V_t(k)$$

is non-decreasing in t .

Linear Network: “Partial” Knowledge

“Partial” Knowledge Assumption: The chance of successful infiltration s is known, but the values μ_1, \dots, μ_K are unknown.

Idea

Consider policies consisting of **two phases**:

1. Devote the first T_I epochs to attempting to infiltrate new computers.
2. During the remaining epochs, collect info from the infiltrated computers using a bandit algorithm (e.g., UCB).

Theorem (Dor Krutzilber (MAJ, IDF), Master's Thesis, NPS, 2017)

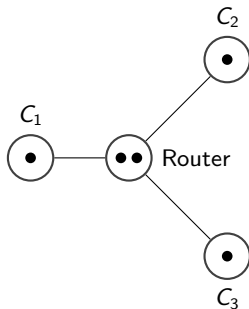
To achieve a regret of

$$O(\sqrt{T \log(T)}),$$

it suffices to let $T_I = O(\sqrt{T/\log(T)})$ and to use UCB.

Using Routers Under “Partial” Knowledge

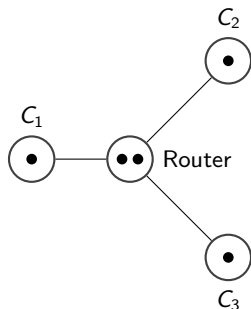
Having access to a **router** node enables you to simultaneously get **filtered intelligence** (e.g., “snippets”) from all computers connected to it.



Question: Suppose you have access to a router that is connected to K infiltrated computers. How should you **optimally extract information** from those K computers over T decision epochs?

- ▶ lots of **filtered** information **vs.** **targeted un-filtered** information

Using Routers Under “Partial” Knowledge



Filtered Information: Suppose that whenever the router is used, the following occurs for each connected infiltrated computer C_k :

- ▶ with probability η_k , C_k responds as if you had tried to collect info from it;
- ▶ with probability $1 - \eta_k$, C_k responds as if you had not tried to collect info from it.

Using Routers Under “Partial” Knowledge

Idea

Consider policies consisting of **two phases**:

1. Use the router during the first T_R decision epochs to **select a subset** of the connected infiltrated computers.
2. Collect info from the selected subset of computers using a bandit algorithm (e.g., UCB).

Subset selection can be done based on **confidence intervals** for the μ_k 's

- ▶ Lykouris, T., E. Tardos, and D. Wali. “Graph regret bounds for Thompson sampling and UCB.” [arXiv](#), May 23, 2019.

Theorem

Suppose the number K of connected infiltrated computers is fixed. To achieve a regret of

$$O\left(\frac{\log(T)}{\min_k \eta_k} + \sqrt{T \log(T)}\right),$$

it suffices to let $T_R = O(\log(T)/\min_k \eta_k)$ and to use UCB.

Summary and Extensions

Summary:

- ▶ **Sequential** network intrusion model, from **attacker's** point of view.
- ▶ Results for **linear network**.
- ▶ Results on using **routers** that provide **filtered** batch observations.



Extensions: network topologies, fatal detections, multiple “players”, ...