Operations Research in Optimization at the Naval Postgraduate School

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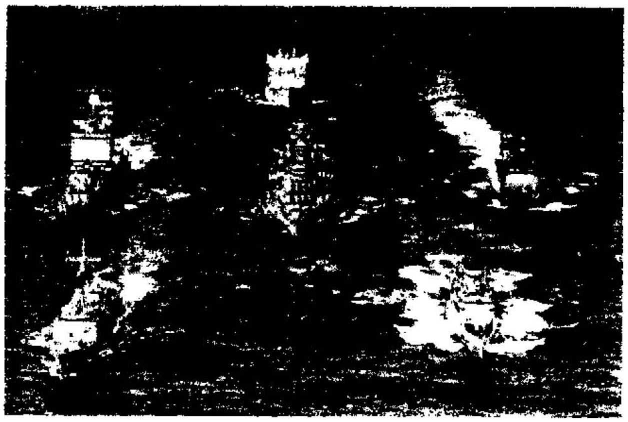

This is a brief description of a long-term research program in optimization theory and practice conducted within the Operations Research Department of the Naval Postgraduate School (NPS). Our program is unique in that our theoretical research is predominantly motivated by pressing real-world problems for which we are allowed very little time to organize and produce viable solutions. The principal sources of these problems are our research sponsors and graduate students with inflexible due dates. We show how beneficial the interaction of research and instruction at NPS can be by use of a typical thesis project. Over the years, our optimization research program has embraced many fields of application and theoretical development out of immediate necessity to support our demanding clients. We have also served as an important conduit between our civilian and military collaborators at other institutions who have assisted us in our research. Projects in the program have been supported by the Office of Naval Research, the National Science Foundation, Department of Energy, Defense Logistics Agency and Joint Chiefs of Staff.

Background

The *Operations Research* (OR) Program at NPS is unique. Operations Research, defined by Webster as the application of scientific and especially mathematical methods to the study and analysis of complex overall problems, was born of military necessity during World War II. Thus, it is not surprising that the OR program at NPS is the oldest and one of the largest such graduate engineering programs in the U.S. Students are usually commissioned officers from uniformed services of the U.S., and from Australia, Canada, Egypt, Germany, Greece, Israel, Korea, Mexico, Turkey and many other allied countries. Typical students have an undergraduate engineering degree, five to ten years of successful professional experience, and the aptitude and diligence to make them the envy of any university. Most of our students pursue the Master's Degree in a program lasting eight to nine quarters. Their first year is occupied by core coursework. The second year begins with a half-quarter experience tour, during which each student is transferred to an operations analysis activity somewhere (e.g., Washington, D.C., Hawaii, Iceland, etc.) and given actual work to do in his or her primary professional specialty. Returning from this taste of reality, advanced coursework, electives, and a thesis complete the program. Although a handful of doctoral students contribute to our research programs, it is principally the Master's thesis requirement which occupies us.

Theses frequently focus on real problems which the student will face immediately following graduation. A thesis topic may be theoretical, or quite practical, but it must be accepted by an advisor and a second reader; these faculty members often act as collaborators with the thesis student, providing an enriched environment for all.

According to Webster, *optimize* is making as perfect, effective, or functional as possible. Optimization in Operations Research evokes a scientific and especially a mathematical connotation for this definition. Computation provides the experimental evidence with which mathematical theory is scientifically tested. Optimization is one of the fundamental areas of study in Operations Research at NPS.

A Case Study

To illustrate the singular nature and pace of our research program in optimization, consider a typical thesis project. Navy LT Clarke Goodman completed his core studies and spent his experience tour in Norfolk, VA. While working on his assigned project, LT Goodman became fascinated with an unrelated activity occupying a signficant number of people and directly affecting the entire U.S. Atlantic Fleet. LT Goodman witnessed the preparation of an *Annual Employment Schedule.* The employment schedule directs the activities of all air, marine, submarine, and surface untis for the following quarter, and informs such units of planned activities for a year in advance. Having been the object of such employment schedules for years, LT Goodman was naturally interested in the reasoning underlying their formulation and in possibly improving their effectiveness and fairness. His questions were unofficially entertained by members of the planning staff.

At NPS, LT Goodman convinced Professor Kevin Wood that the employment schedule problem was a worthy topic, and enlisted my support as a second reader. LT Goodman and Wood worked together to formalize the reasoning for and render a tractable model of the employment scheduling process. They decided to restrict their attention to major *surface combatants* (e.g., frigates, destroyers, cruisers, carriers, etc.) and to use an actual unclassified annual schedule as a mechanism to test their model. LT Goodman returned to Norfolk to formally extract additional data and to receive criticism and additional guidance on his proposed approach.

LT Goodman then devised:

1. A data base for all surface combatants in the U.S. Atlantic Fleet (over 100 ships), including the weapons systems and capabilities of each ship as well as restrictions for fixed commitments (e.g., yard periods, required exercises, or even administrative fiats).
2. A data base for all *major events* in the following year (e.g., fleet exercises, deployment, etc.), including the complementary requirements of each activity (e.g., numbers and types of ships, weapons systems, substitutability of units, mission effectiveness, etc.)
3. A schedule generator which efficiently produces large numbers of alternate employment schedules for each ship, recognizing the unique attributes of each ship and honoring its fixed commitments. A crucial feature of this generator is a mechanism to estimate the contribution of each ship to the mission at hand, given its other commitments as well as its crew and weapons systems endowment.
4. An optimization model to select from the large number of candidate ship schedules a particular set of schedules which satisfies event requirements with maximal effectiveness.
5. A report writer to produce an annual employment schedule in its published format, as well as analyses of the criteria by which such schedules are evaluated (e.g., at-sea time, time between deployments, etc.)

The optimization model can be illustrated by the simplified example in Figure 1.

Figure 1

Set Partition Coefficient Matrix

	Col: / Row	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		Row Sum
Ship	1	1	1	1	1	1											=	1
	2						1	1	1	1							=	1
	3										1	1	1	1	1	1	=	1
Req't/ Event	4								1						1	1	=	1
	5		1		1			1				1			1		=	1
	6			1						1				1			=	1
	7		1			1			1				1		1		=	1

The object is to select from this matrix a set of columns which collectively has exactly one 1 in each row. This deceptively simple model is known as a *set partition problem.* LT Goodman used each column to represent an alternate annual ship schedule, and each row to represent mutual exclusion among all schedules for each ship (ship rows), or appropriate assignment of proper ships and weapons to each event (requirement/event rows). Furthermore, each of his columns has a value (not shown in Figure 1) which contributes additively to the value of the fleet employment schedule and this total value must be maximized.

As the number of rows and columns in the matrix grows, the resulting set partition optimization problem becomes an integer linear program of great combinatoric difficulty. To appreciate the task at hand, consider an artificial case in which only one hundred randomly generated columns need be evaluated. There are 2 to the power 100 (as a decimal number, this is more that 1 followed by 31 zeroes) different candidate solutions to try. Because LT Goodman expected to generate set partitions with tens of thousands of rows plus columns, we contacted Prof. Glenn Graves at UCLA whom we knew to be solving similar problems in crew scheduling for United Airlines. Graves agreed to help us cope with whatever Goodman produced.

Over the next *few weeks,* the data structures and logic of the schedule generator were refined technically to efficiently produce columns restricted to those most likely to be used in an optimal set partition (fleet employment schedule). Considerable modelling effort was devoted to the evaluation of each candidate ship schedule so that the multiple criteria of combat effectiveness, crew fatigue, substitutability of assets, equitable distribution of workload among ships, etc., could all be effectively addressed. In concert, we developed a set-partition solver based on existing general-purpose experimental optimization software (called the X-system by Brown and Graves) and some special-purpose procedures contributed by a prior Master's thesis by Marine CAPT Dan Bausch. The enriched set of (about 20 thousand) columns yielded by the generator was attacked in many experiments aimed at discovering and exploiting special structure in the set-partition matrices and thus reducing solution efforts.

LT Goodman and the professors enlisted to support him assembled in less than two months a prototypic system which accepts a user-friendly input script with ship and event descriptions, and manual preassignments of ships to events, and automatically produces a complete annual employment schedule in less than a minute of time-shared computer time. Objective comparison of the quality of the schedules produced automatically and those produced manually and subsequently published for actual use reveal significant improvements in all criteria. The sheer speed of response of the system suggests that schedulers could employ it as an interactive tool to incorporate non-quantifiable criteria or frequent changes in actual schedules. Technical accomplishments of this work have been presented in international research society meetings and will appear in a research paper in the open literature.

Optimization Research Program

Our objective for the overall optimization research program at NPS is to qualitatively improve the efficiency with which we can solve real-life problems using optimization. Our methods include theoretical as well as extensive empirical investigation of problems presented to us, and of problems posed as wholly artificial test cases.

Many of the classes of problems with which we must deal (e.g., integer programs) are believed to be intractable in the sense that there is strong theoretical evidence (but as yet no proof) that there can never be an algorithm to efficiently compute optimal solutions for all problem instances. Despite the daunting theoretical computational complexity of these problems, the gloomy prospects for success are rarely borne out. Real-life problems seem to exhibit special properties which render them solvable by an approach which identifies and exploits the specializations. This has led us to a guiding ***Principle of Optimization:*** **careful modelling of real-life systems that are shaped by strong economic, physical, social, and/or rational forces yields problems that are dramatically easier to solve than others.** Following this principle, we seek means to effectively identify, extract, express, and exploit in our models and solution methods problem features reflecting these underlying forces.

Solution methods can be characterized as *direct,* in which the problem is solved outright, or *indirect,* in which related sub-problems are solved and their solutions used to assemble a solution to the original problem.

Direct methods benefit from *factorization,* in which some subset of the problem's rows or columns are identified as exhibiting special structure and treated separately by the solution method. Networks are an excellent example of such structure, and we have found that networks are contained within most real-life models.

Indirect methods break up the problem into more manageable pieces, solve the pieces and assess the solutions achieved to see if they solve the original problem. If not, the pieces can be altered, and resolved, perhaps many times, to obtain solutions closer to that required by the original problem. Frequently, solutions achieved by indirect means are approximate and our goal is to achieve minimal error at reasonable cost. The classical indirect approach is *decomposition*, or its currently fashionable heuristic simplification called *Lagrangean relaxation*. We have used both approaches, especially when the *manageable pieces* thus produced are networks. In addition to repeatedly solving the manageable pieces, decomposition usually requires repeated solution of a restricted master problem to which constraints are successively added to ensure convergence. Lagrangean relaxation can be arranged to omit this difficult step, but our experience is that convergence is not so easy to prove theoretically or achieve in practice.

Both direct and indirect methods are improved, especially with integer models, if we modify them to allow violation of each constraint at some cost. Thus, the constraints become aspirations, or goals, and the choice of whether, or by how much to violate constrains is made by the model. We refer to models with linear violation penalties as *elastic* models, and in our experience there is much to recommend them. At first glance, elastic models appear to cheat by violating constraints. However, solutions which offer enormous improvement via some slight constraint violation can be very attractive in real-world problems. Our research has produced new solution methods for network models, such as:

☐ Assignment (e.g., men to jobs, weapons to targets, etc.)
☐ Transportation (e.g., rail shipment from depots to units)
☐ Capacitated Transshipment (e.g., capacitated physical distribution systems)
☐ Generalized Networks (e.g., amplification and/or attenuation in transit)
☐ Integer Generalized Networks (e.g., flows restricted to one of two values)

as well as for other models, such as:

☐ Linear Programming
☐ Mixed Integer Linear Programming
☐ Nonlinear Programming
☐ Quadratic Assignment (e.g., travelling salesman problems)

In cases for which our computer programs do not require extensive field support (which we are not able to provide), we have made the programs available to other academic researchers. Thus, problems formerly at the threshold of the state-of-the-art have been reduced to routine features within even more ambitious models.

The optimization group at NPS has also agreed to apply, when possible, our more complex experimental computer systems to problems supplied to us with a publication release for our research findings.

A partial topical list of applications of optimization which we have pursued includes:

☐ Physical distribution system design
☐ Energy production, distribution and consumption
☐ Ship scheduling and fleet employment
☐ Munitions procurement, storage and shipment
☐ Production scheduling
☐ Search, detection and surveillance
☐ Vehicle dispatching and routing
☐ Target assignment
☐ Mobilization
☐ Manpower
☐ Network reliability
☐ Engineering design
☐ Real-time process optimization
☐ Capital budgeting

An important role for NPS and its research programs is to serve as a medium for communication among academic, military, and civilian researchers. The list above gives ample evidence of the general usefulness of optimization. We have scrupulously shown how to schedule supertankers with the same techniques used for Naval surface combatants, how to bake crackers and cookies or produce ammunition, how to dispatch trucks or allocate sorties to military aircraft, how to build capital portfolios or plan flight tests of new fighter aircraft, and so forth for many application areas.

We are concerned that we can often solve models much more efficiently than we can implement, validate, support or understand them. Accordingly, in concert with our modelling research, we have developed several prototypic user-friendly model-building and solution-interpretation systems. (Necessity leads to invention: a misspelling-tolerant system for naming model entities was developed following a comical late-night terminal session in which students futilely sought transportation data for *Albuquerque.*)

Even more ambitiously, the entire process of formulation, model statement, data specification, and model validation and interpretation has been formally addressed by Professor Gordon Bradley, Ph.D. student Army MAJ Bob Clemence, and Professor Art Geoffrion (UCLA). Geoffrion has specified a formal approach called *structured modelling*, and Bradley and MAJ Clemence have extended the concept and implemented a system called LEXICON to illustrate the efficacy of the approach. They have extended this work by developing the concept of typing and type validation that allows automatic verification that the model correctly expresses the modeler's intentions.

All these research activities have contributed to a rather large optimization research software suite which enables quick response to new modelling challenges. Yet, there are always demands for solutions to larger, more difficult models. But such is the nature of research, each breakthrough leads to more challenges for our program to pursue.

Biography

Dr. Gerald G. Brown is Professor of Operations Research at the Naval Postgraduate School. He chairs the Doctoral Program in Operations Research, and serves as the National Research Council Postdoctoral Research Advisor in Optimization. His contributions to computational techniques in mathematical modelling have received wide recognition. He has had extensive consulting experience with large U.S. corporations and the federal government in the application of state-of-the-art mathematical programming techniques to a wide variety of problems.

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