NAVAL POSTGRADUATE SCHOOL

Development of an Unified Atmospheric Model (NUMA)

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Motivation for this Work

We are interested in constructing numerical methods for constructing non-hydrostatic mesoscale and global atmospheric models (for NWP applications); this is a unified model. The reason for this is economics - one (production) model is cheaper to support.

Currently, in the U.S. there is a movement to construct one NWP model (NWS, Navy, and Air Force). This National Board (NUOPC=National Unified Operational Prediction Capability) aims to develop a new model that is:

1. Highly scalable on current and future computer architectures
2. Global model that is valid at the meso-scale (i.e., non-hydrostatic)
3. Applicable to medium-range NWP
4. Applicable to decadal time-scales

The following talk outlines a model development effort to meet these needs…

Talk Summary

- Governing Equations
- Spatial Discretization
- Preliminary (Validation) Results
- Parallel Implementation
- Closing Remarks

Governing Equations
(compressible Euler equations)

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{u}) = 0$$ (Mass)

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \bullet \nabla \mathbf{u} + \frac{1}{\rho} \nabla P = -2\mathbf{\Omega} \times \mathbf{u} - \nabla \phi_A$$ (Momentum)

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \bullet \nabla \theta = 0$$ (Energy)

$$P = P_A \left(\frac{\rho R \theta}{P_A} \right)^{\gamma}$$ (Equation of State)

$$\theta = \frac{T}{\pi} \quad and \quad \pi = \left(\frac{P}{P_A} \right)^{R/c_p}$$

$$\mathbf{u} = (u, v, w)^T,$$

$$\mathbf{x} = (x, y, z)^T,$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)^T$$

Spatial Discretization

- Primitive Equations: $$\frac{\partial q}{\partial t} + \nabla \cdot \mathbf{F} = S(q)$$
- Approximate the solution as: $$q_N = \sum_{i=1}^{M_N} \psi_i q_i \qquad \mathbf{F}_N = \mathbf{F}\left(q_N\right) \qquad S_N = S\left(q_N\right)$$
  - **Interpolation O(N)**
- Write Primitive Equations as: $$R(q_N) \equiv \frac{\partial q_N}{\partial t} + \nabla \cdot \mathbf{F}_N - S_N = \varepsilon$$
- Weak Problem Statement: Find
  $$q_N \in \Sigma(\Omega) \; \forall \; \psi \in \Sigma \quad \begin{cases} \Sigma = \left\{\psi \in H^1(\Omega) : \psi \in P_N(\Omega_e) \forall \Omega_e\right\} & \text{(CG)} \\ \Sigma = \left\{\psi \in L^2(\Omega) : \psi \in P_N(\Omega_e) \forall \Omega_e\right\} & \text{(DG)} \end{cases}$$
  - such that
    - **Integration O(2N)**

$$\int_{\mathbf{\Omega}/\mathbf{\Omega_e}} \psi R\left(q_N\right) d\Omega \; = 0$$

Spatial Discretization
(Comparison of CG/DG Methods)

Continuous Galerkin Methods

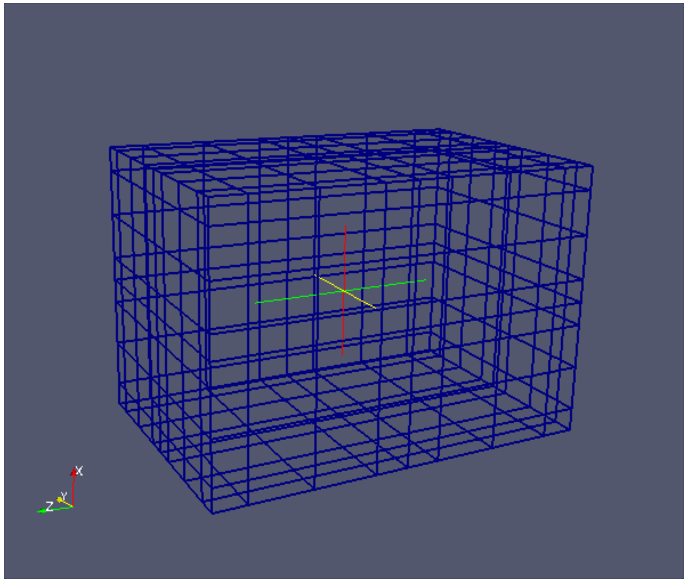
- High order accurate yet local construction (via DSS)
- Simple to construct efficient semi-implicit time-integrators
- In high-order mode, primarily used with quads and inexact integration (e.g., using Lobatto points avoids non-diagonal mass matrix with slight error since integration is O(2N-1))
- No analog of Lobatto points exist on the triangle so costly to use
- Excellent scalability on MPP

Discontinuous Galerkin Methods

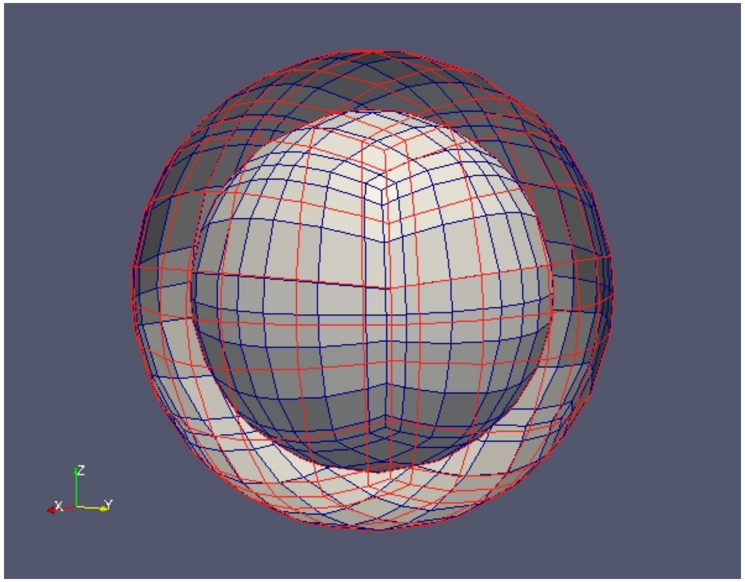
- High order accurate and completely local in nature (no DSS required as in CG)
- High order generalization of the FV (but with compact support)
- Upwinding and BCs implemented naturally (via Riemann solvers)
- Not so easy to construct efficient semi-implicit time-integrators, due to the difficulty in extracting the Schur complement
- Since matrices are all local, using quads or triangles is straightforward and one need not worry as much about exact vs. inexact integration
- Excellent scalability on MPP

Preliminary Results (Model Description)

- Basis functions: 3D tensor products of Lobatto-Gauss-Legendre (LGL) points. Elements are hexahedra (Triangular prisms coming soon).
- Time-Integrators are: explicit SSP-RK, IMEX-BDF2 (Schur and No Schur), Fully-Implicit BDF2 (JFNK), IMEX-RK (currently, No Schur only)
- Mesoscale (limited area) and Global (spherical domain) options



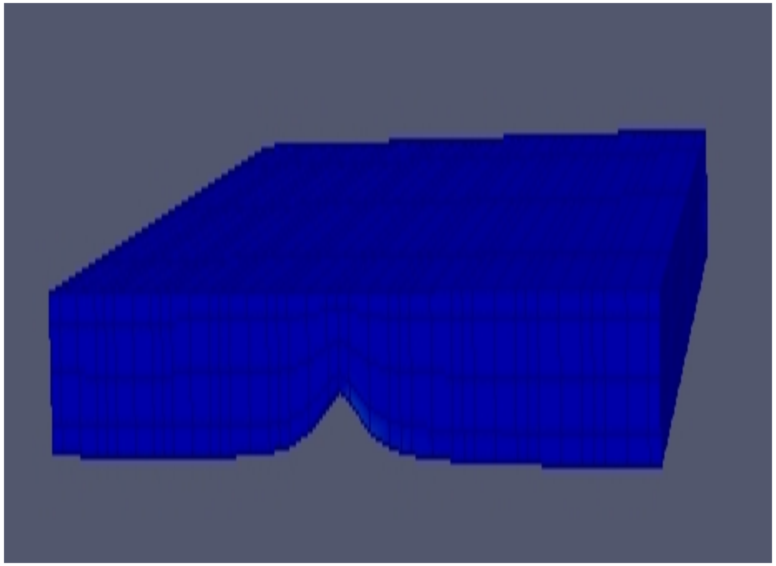
Mesoscale



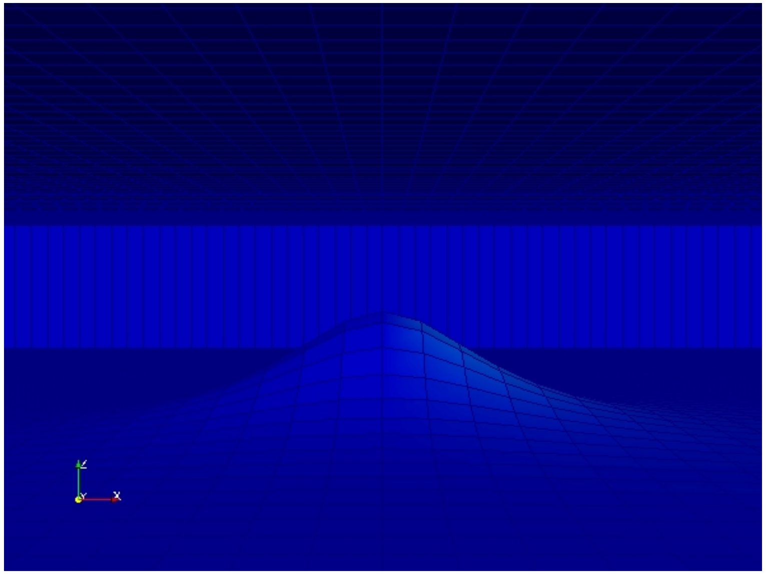
Global

Preliminary Results (Linear Hydrostatic Ridge and Mountain)

- Flow of U=20 m/s in an isothermal atmosphere.
- LH Ridge: Witch of Agnesi ridge: Mountain height = 1 m with radius 10 km.
- LH Mountain: Solid of revolution of Witch of Agnesi: Mountain height = 1 m with radius 10 km.
- Absorbing (sponge) boundary condition implemented on lateral and top boundaries.

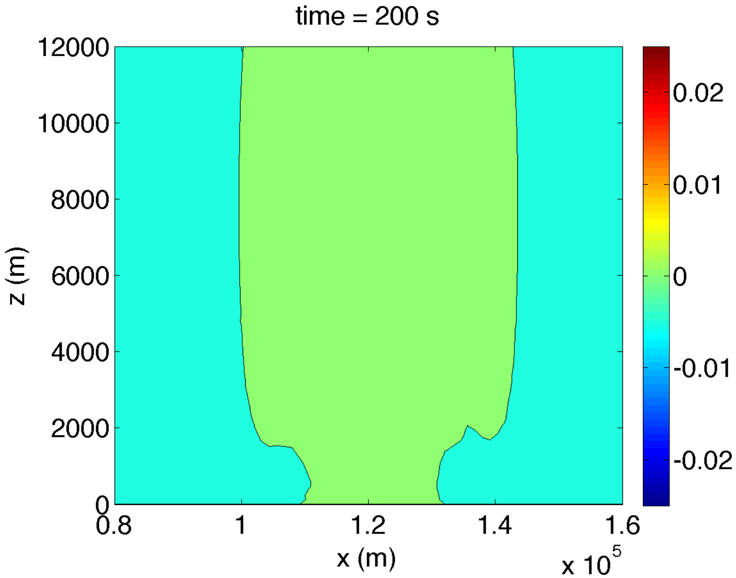


LH Ridge



LH Isolated Mountain

Linear Hydrostatic Ridge



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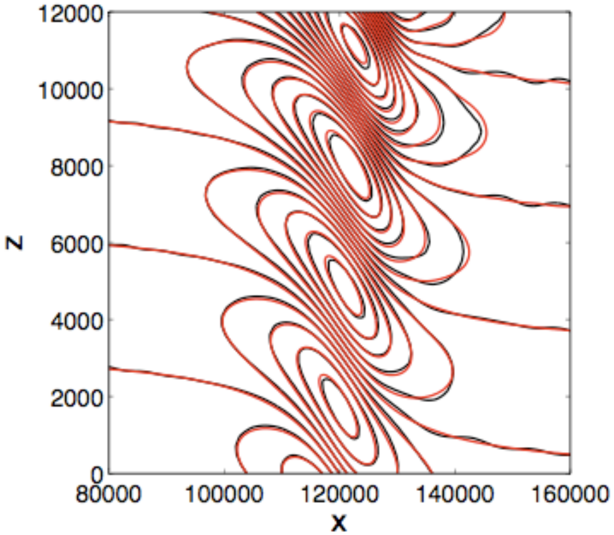

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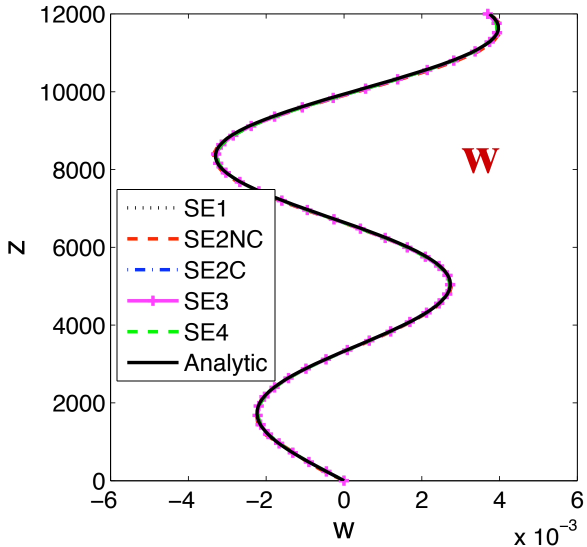

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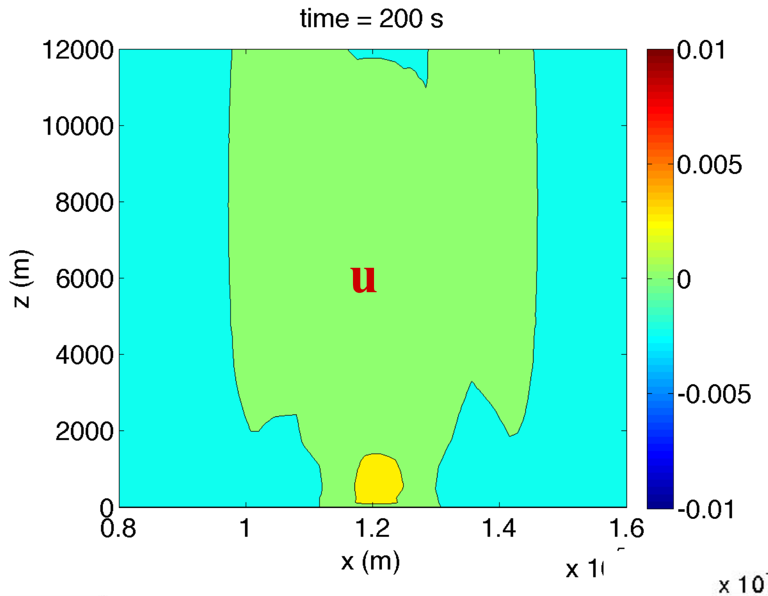
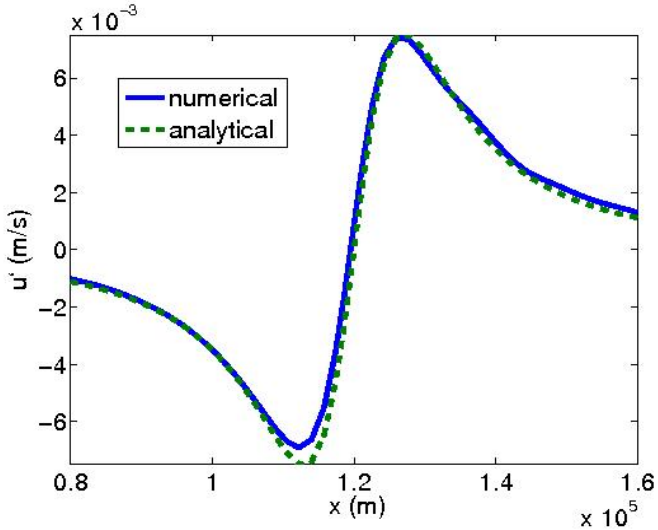
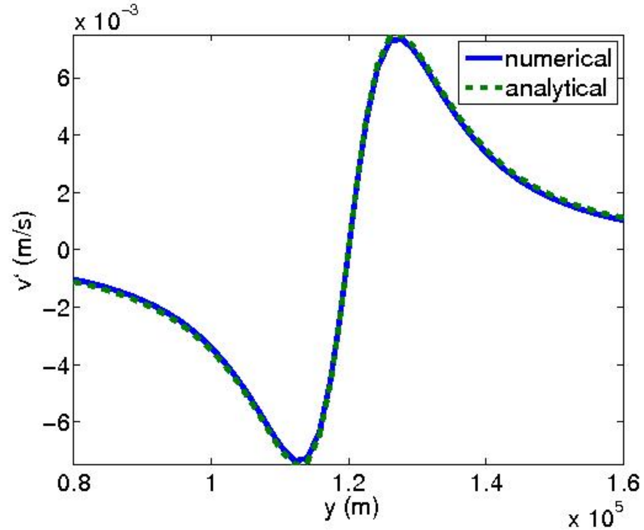
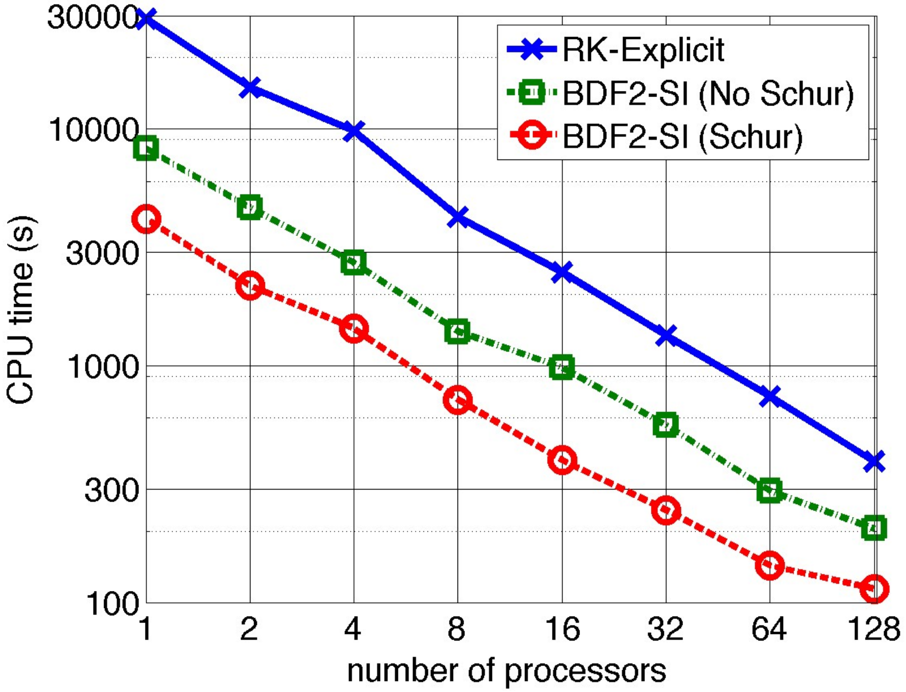
Linear Hydrostatic Isolated Mountain
(Grid Resolution: 2400 x 480 meters)



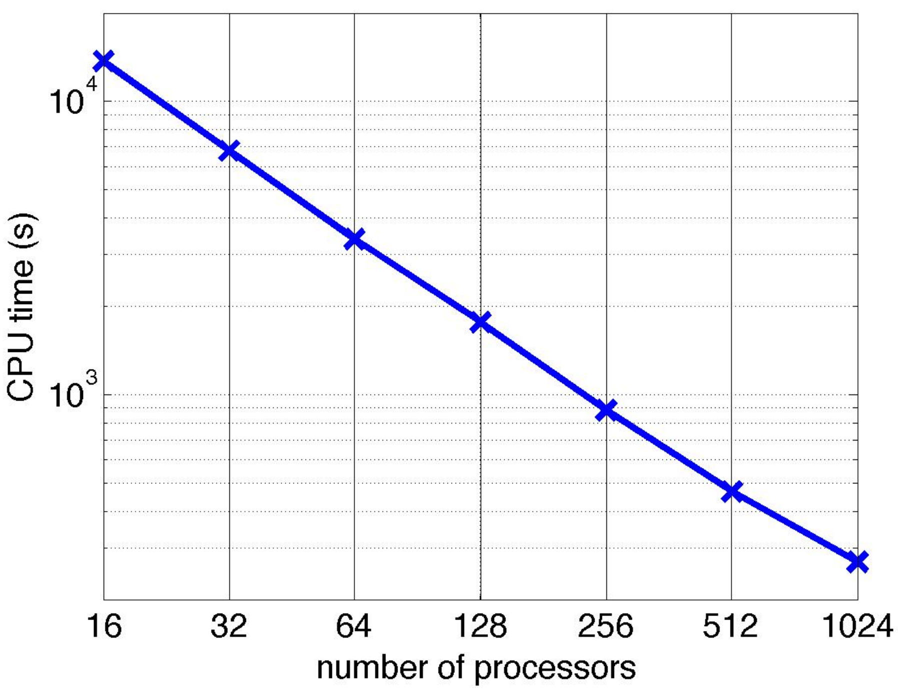




Preliminary Scaling Experiments (Performed on Ranger TACC)



32x32x32 elements with 4th Order Polynomials
(2 Million Grid Points)



48x48x48 elements with 4th Order Polynomials
(7 Million Grid Points)

A Multitude of Challenges Remain

- Further dry physics validation is necessary (e.g., Baroclinic Instability problems).
- Simple moisture has been tested in 2D (manuscript almost finished) and now implementing it in 3D.
- Full sub-grid scale parameterization needs to be included (can compare against older hydrostatic version called NSEAM).
- Interesting question is: how will the NH and H models compare in terms of both solution quality and cost?
- Adaptivity will, eventually, be included (as in A. Müller) but I envision only using triangular prisms.
- Explicit scalability is great but must improve on Semi-Implicit performance (different time-integrators and new approaches for DG such as in M. Restelli's talk on hybridized DG).