The Nonhydrostatic Unified Model of the Atmosphere (NUMA): CG Dynamical Core

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http://faculty.nps.edu/projects/NUMA

Collaborators/Acknowledgements

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- Les Carr, Applied Math, NPS
- Emil Constantinescu, Argonne National Laboratory
- James Doyle, NRL Monterey
- Saša Gaberšek, NRL Monterey
- Shiva Gopalakrishnan, Applied Math, NPS
- Jim Kelly, Applied Math, NPS
- Matthias Läuter, Frei University of Berlin /Konrad Zuse High-Performance Computing Center
- Simone Marras, Barcelona Supercomputing Center

Sponsors:

- ONR (Battlespace Environments and Computational Math)
- AFOSR (Computational Math)

Overview of Existing Global Models

Model	Organization	Spatial Discretization	Grid Type	Time-Integrator
Unified Model (NH)	UK Met Office	Finite Difference (2^{nd} order)	Structured (lat-lon)	Semi-Implicit/ Semi-Lagrangian
FIM (Hydrostatic)	ESRL NOAA	Finite Volume (2^{nd} order)	Icosahedral	Explicit
MPAS (NH)	NCAR	Finite Difference (2^{nd} order	Unstructured (hexagons only)	Split-Explicit
NUMA (NH)	NPS/NRL	Spectral Elements/ Disc. Galerkin (arbitrary order)	Unstructured (any grid)	Semi-Implicit: 3D and 1D Vertical

Design Philosophy (2)

Unified Dynamics

- All limited-area models are nonhydrostatic. Resolutions of global models are approaching the nonhydrostatic limit (~10 km).
- Both limited-area and global models utilize the same equations.
- Engineer a common dynamical core for both models, then change grids, force, and boundary conditions.

Unified Numerics

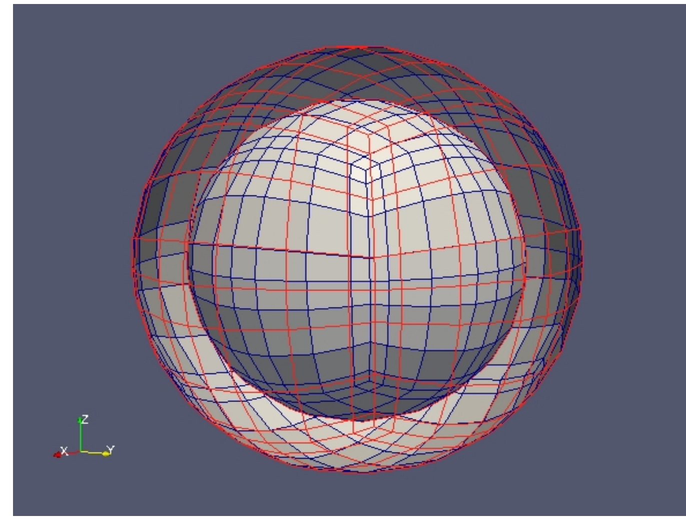
- CG is more efficient for smooth problems at low processor counts.
- DG is more accurate for problems with sharp gradients and more efficient at high processor counts.
- Both EBGs utilize a common mathematical arsenal.
- NUMA allows the user to choose either CG or DG for the problem at hand.

Unified Code

- Code is *modular*, with a common set of data structures.
- New time-integrators, grids, basis functions, physics, etc. may be swapped in and out with ease.
- Code is portable: Successfully installed on Apple, Sun, Linux, and IBM.

Non-Conservative Form (2NC)

- Consider the compressible Euler in Cartesian coordinates (not spherical).
- Mass is conserved and energy can be conserved (up to time-truncation)



$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left(\rho \mathbf{u} \right) = 0$$

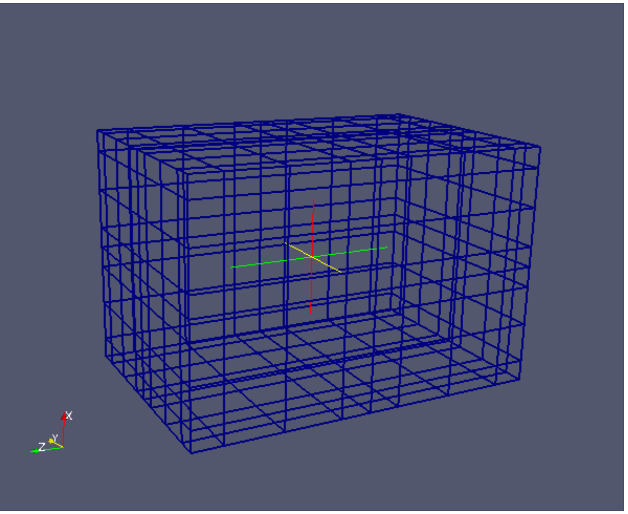
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \bullet \nabla \mathbf{u} + \frac{1}{\rho} \nabla P + g\hat{\mathbf{k}} + \mathbf{f} \times \mathbf{u} = 0$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \bullet \nabla \theta = 0$$

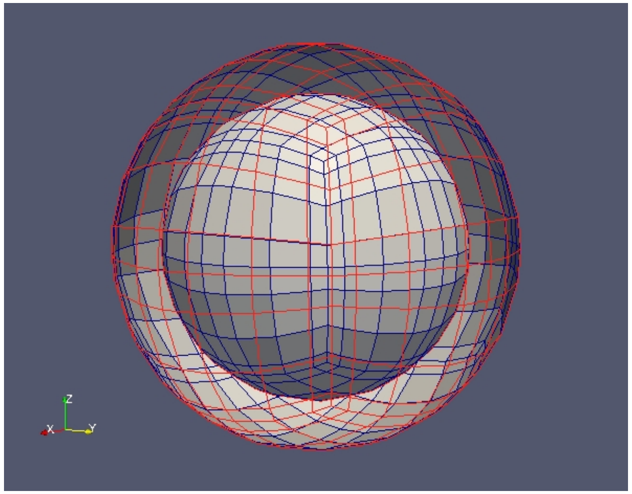
$$P = P_A \left(\frac{\rho R \theta}{P_A} \right)^{\gamma}$$

- The same model may be used for: global, limited-area, and urban-scale modeling (requires different grids, initial conditions, boundary conditions, and physics).

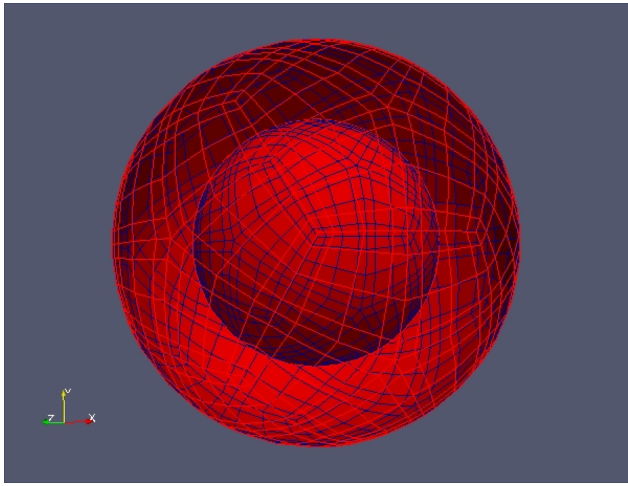
Example of 3D Grids



Mesoscale Modeling Mode



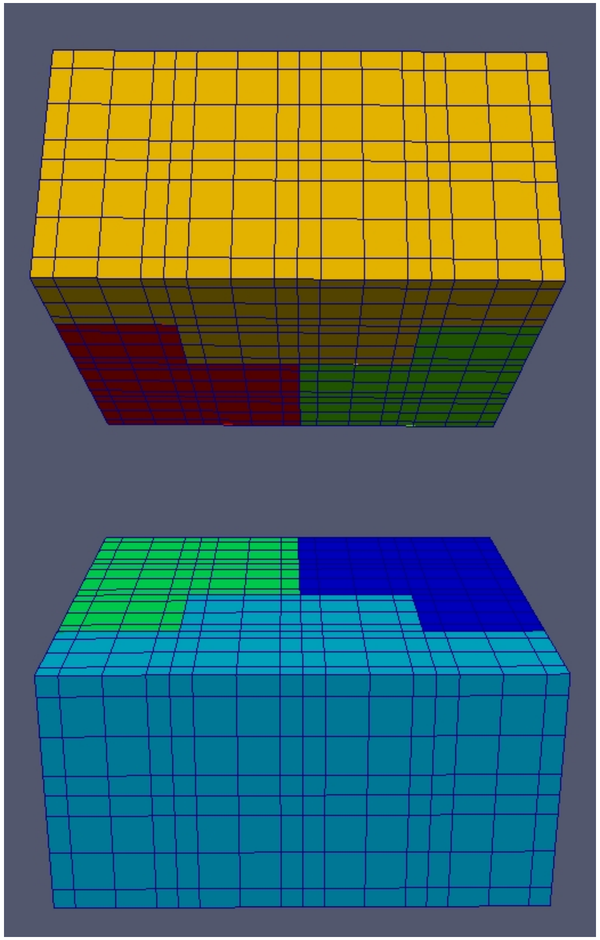
Global Modeling Mode
(Cubed-Sphere)



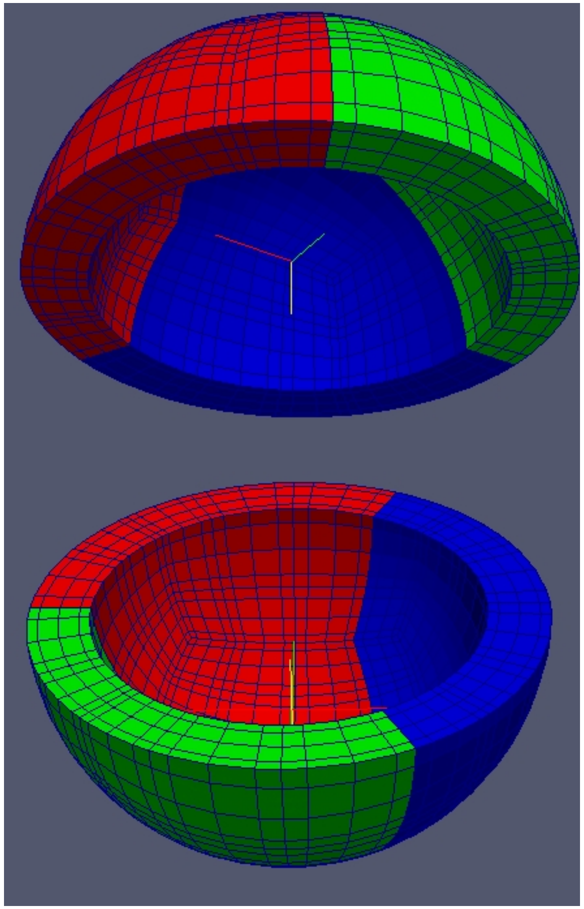
Global Modeling Mode
(Icosahedral)

- NUMA runs in either Mesoscale (Limited-Area) or Global Mode.
- Currently, any (hexahedra-based) grid can be used including completely unstructured grids.
- Parallel Domain Decomposition handled by METIS.

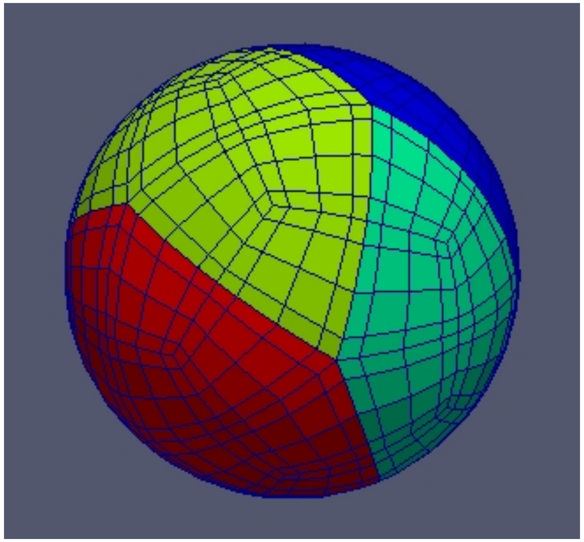
Domain Decomposition via METIS



1: Decomposition of a 3D Cartesian domain using 64 spectral elements (fourth order).

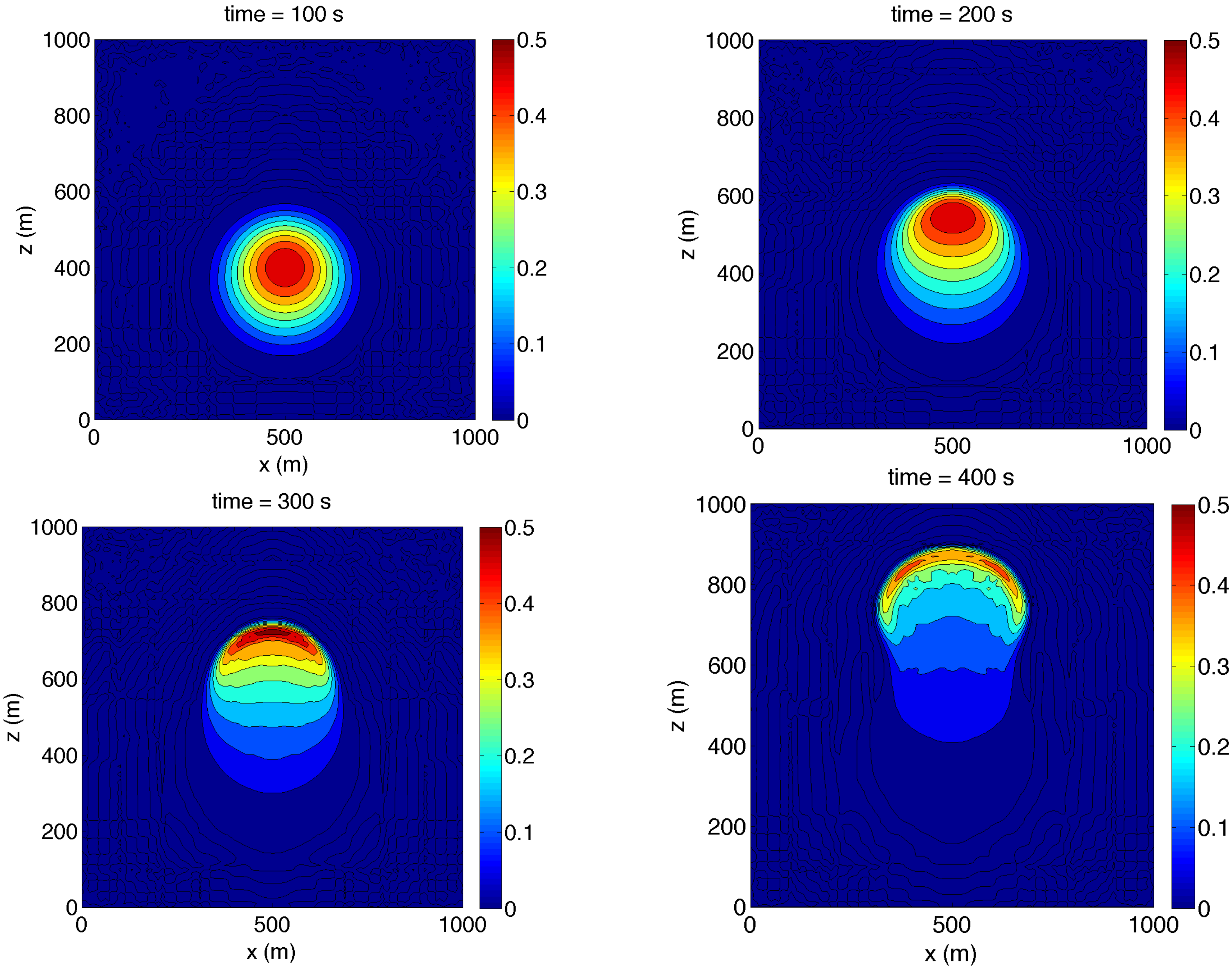


2: Decomposition of a “cubed sphere” using 96 spectral elements (fourth order).



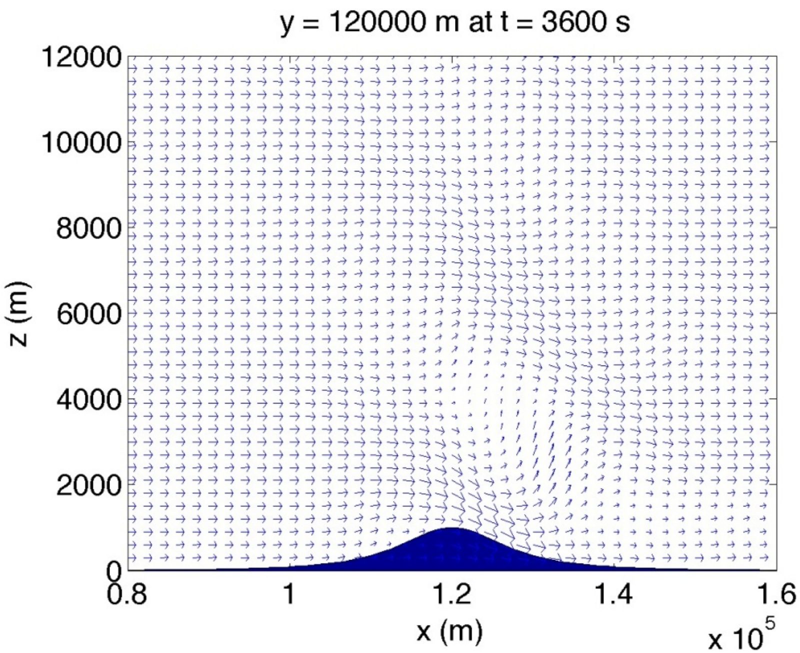
3: Decomposition of an “icosahedral sphere” using 96 elements (fourth order).

Rising Thermal Bubble: 3D

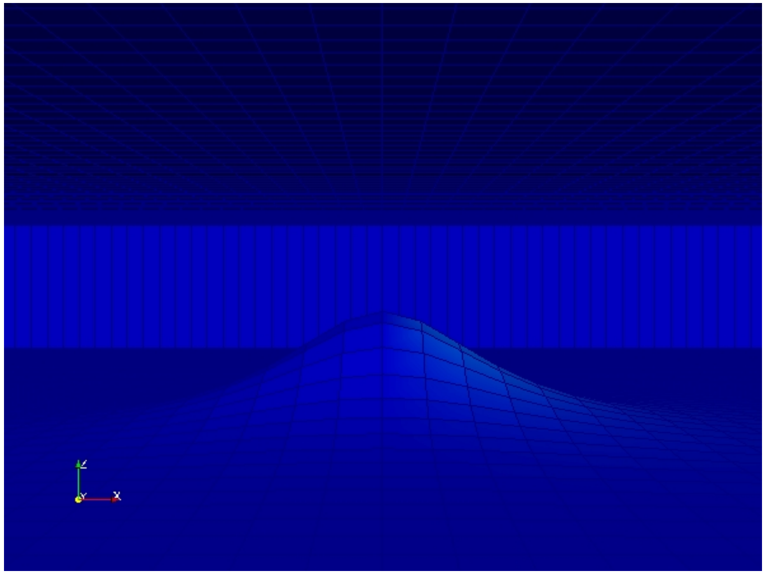


Orographic Flows: Linear Hydrostatic and Nonhydrostatic Mountain (3D)

- Flow of U=20 m/s in an isothermal atmosphere.
- LH Mountain: Solid of revolution of Witch of Agnesi:Mountain height = 1 m with radius 10 km.
- Radiating boundary condition implemented on lateral and top boundaries using an absorbing sponge (Rayleigh friction).

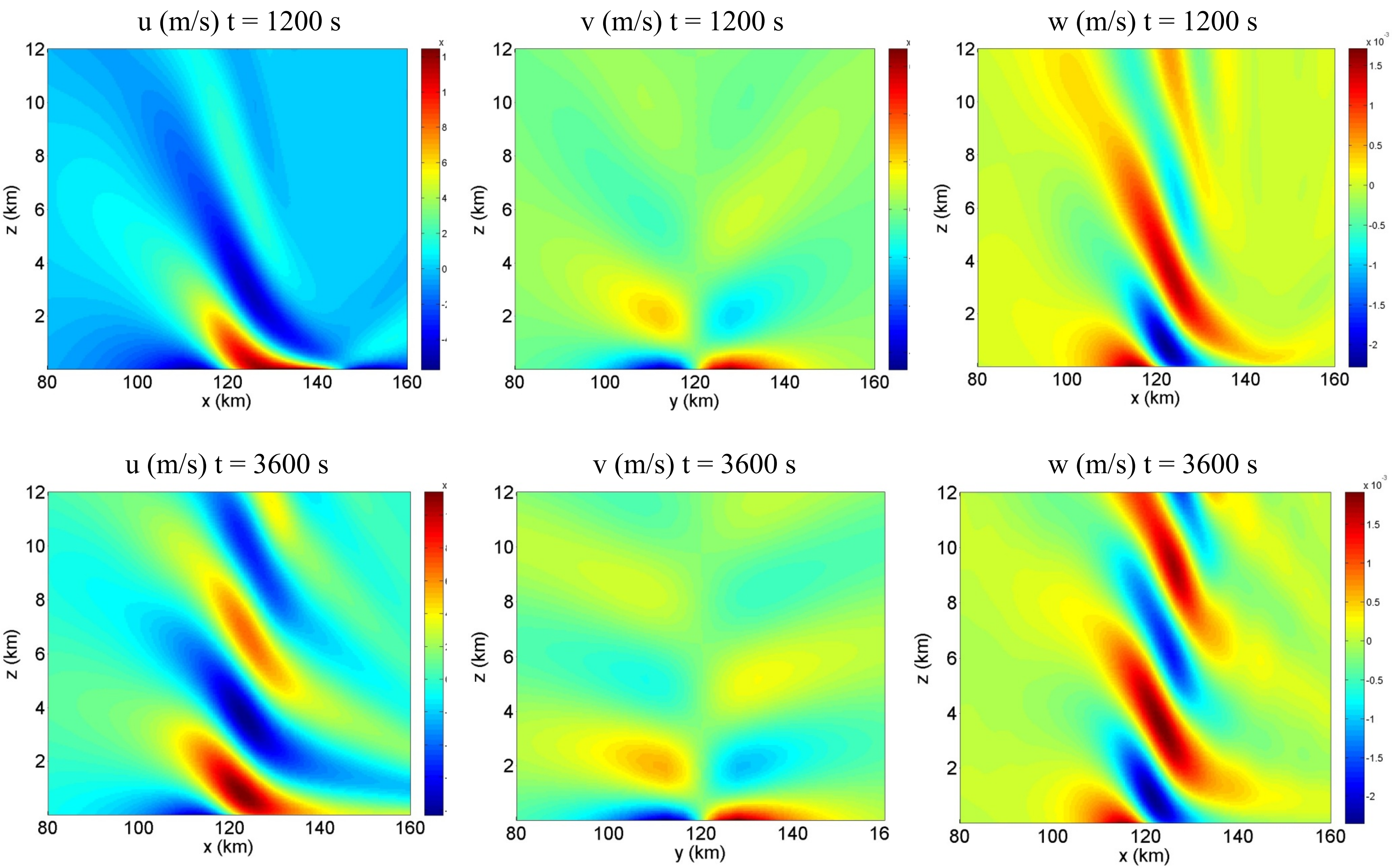


Non-Hydrostatic Mountain

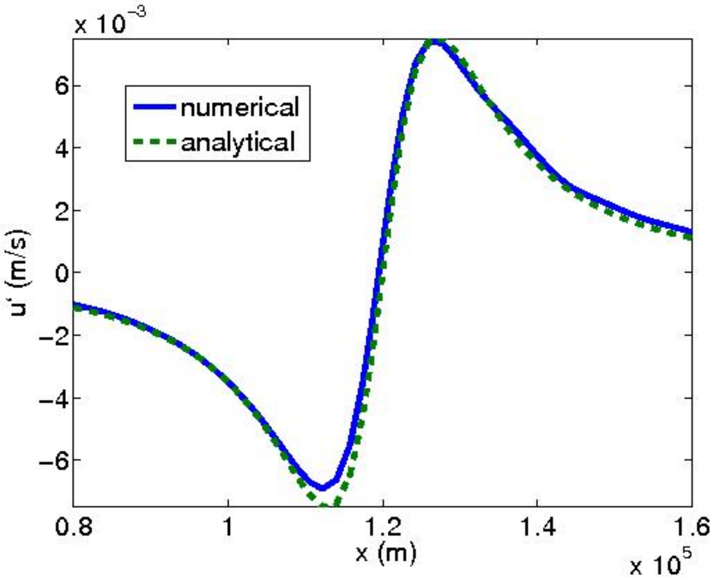


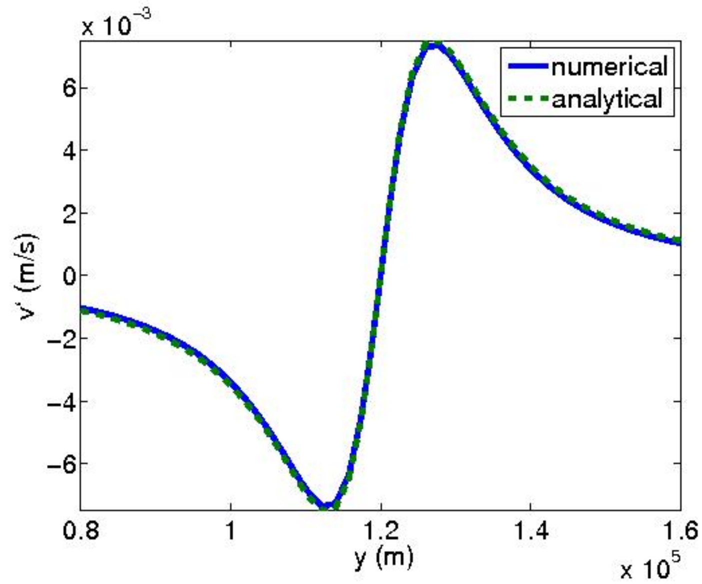
LH Mountain

Linear Hydrostatic Mountain (3D)



3D Linear Hydrostatic Mountain (Verification)





- Mountain case tests orography and sponge BC.
- Analytical approximations for flow over a LHM available in Smith (1980) on mountain surface.
- Decent agreement between theory and numerical model for downstream and cross-stream velocity perturbations.

Results (Global): Pressure Wave

- Proposed by Tomita and Satoh (2004) to test nonhydrostatic global atmospheric models.
- A pressure perturbation is applied to a hydrostatic, isothermal atmosphere

•Top Level z_T=10 km

•Cubed Sphere Grid with 6 x 12 x 12 x 6 = 5184 elements using 4th order polynomials (~210 km horz. Resolution and 0.5 km vertical resolution)

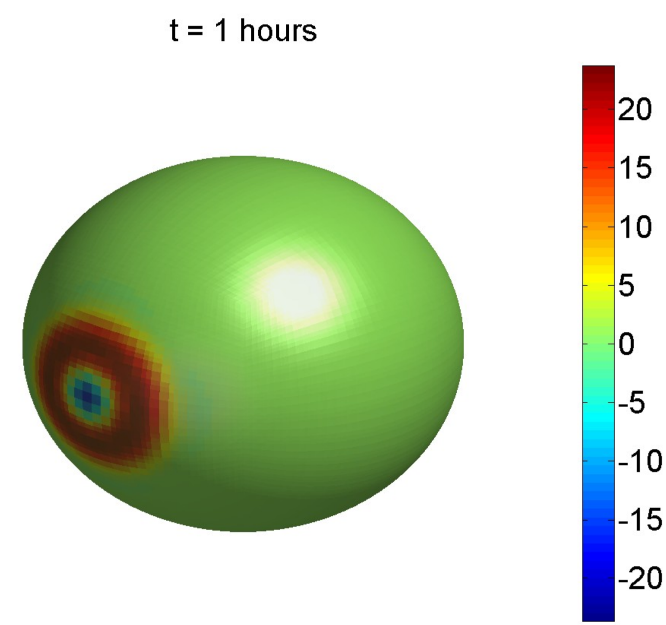
•Model run using Semi-Implicit TI w/ 5 s time-step

$$p' = \Delta p \, f(\lambda,\phi) g(z)$$

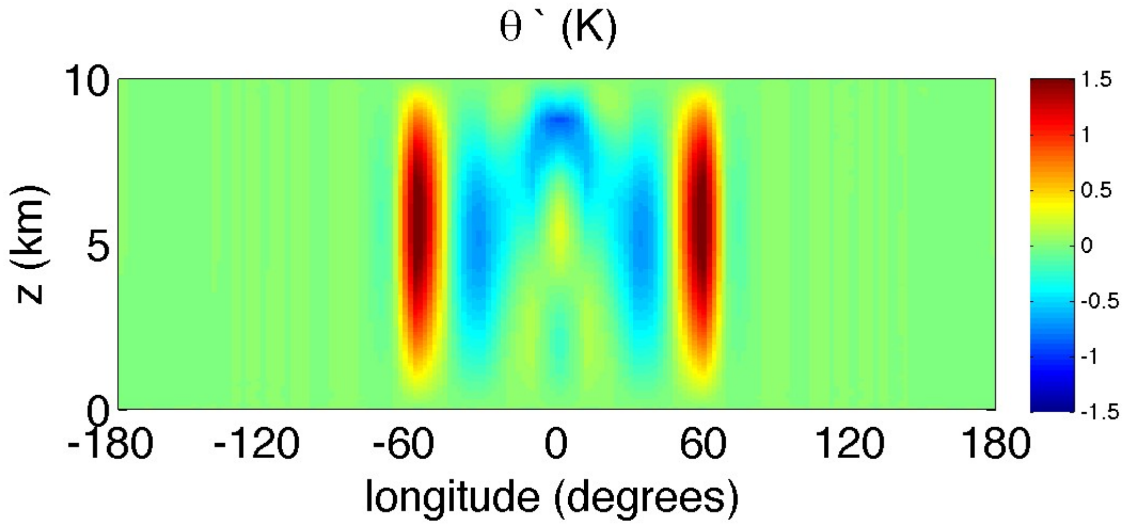
$$f(\lambda,\phi) = \frac{1}{2} H(R-r)\left[1+\cos\left(\frac{\pi r}{R}\right)\right]$$

$$r = a\cos^{-1}\left(\cos\phi\cos\lambda\right)$$

$$g(z) = \sin\left(\frac{\pi z}{z_T}\right)$$

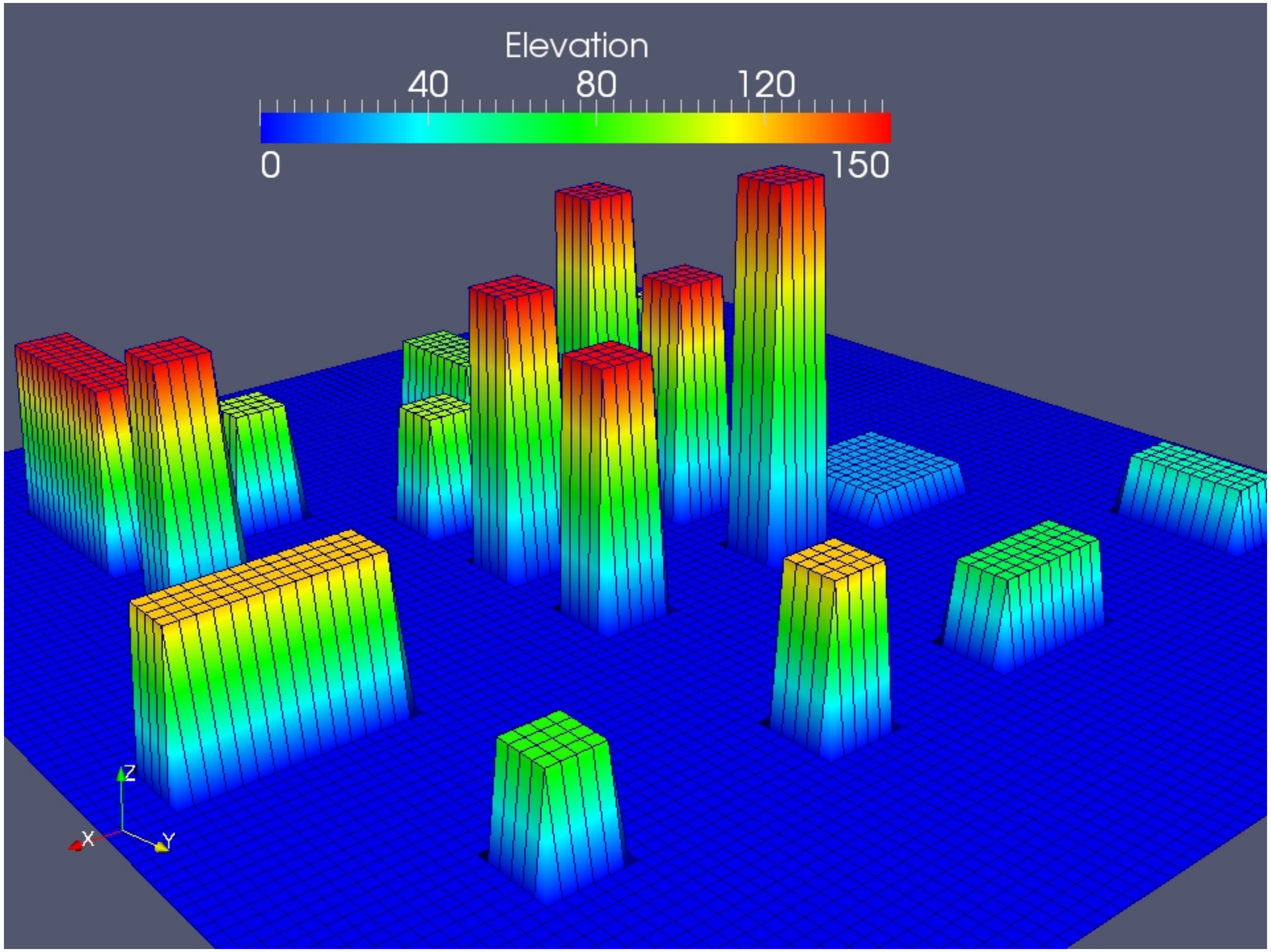


Results (Global): Inertia-Gravity Wave

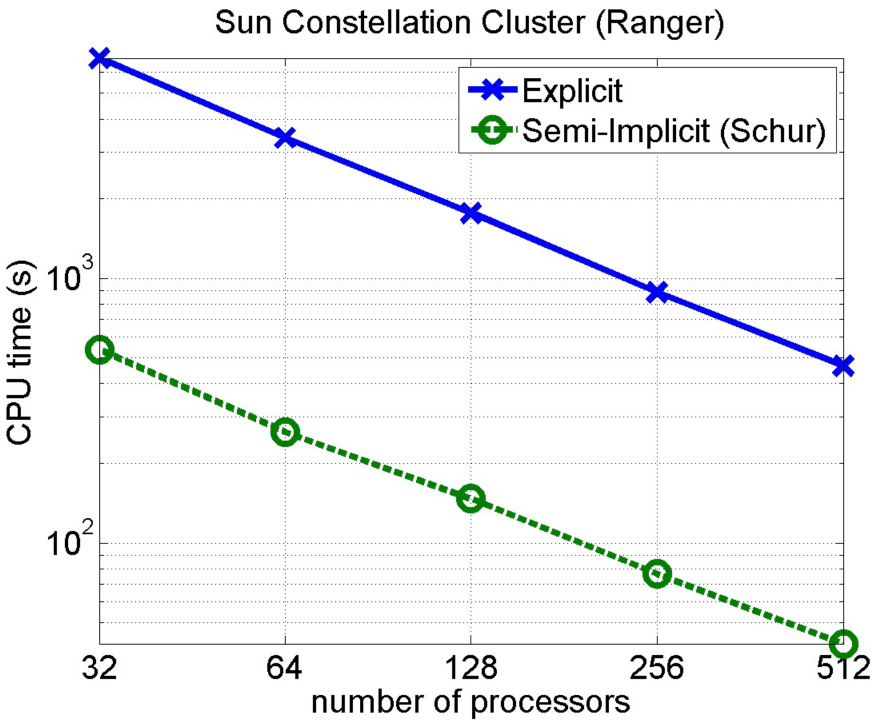


Potential Temperature after 48 hours
with (240x0.5) km resolution

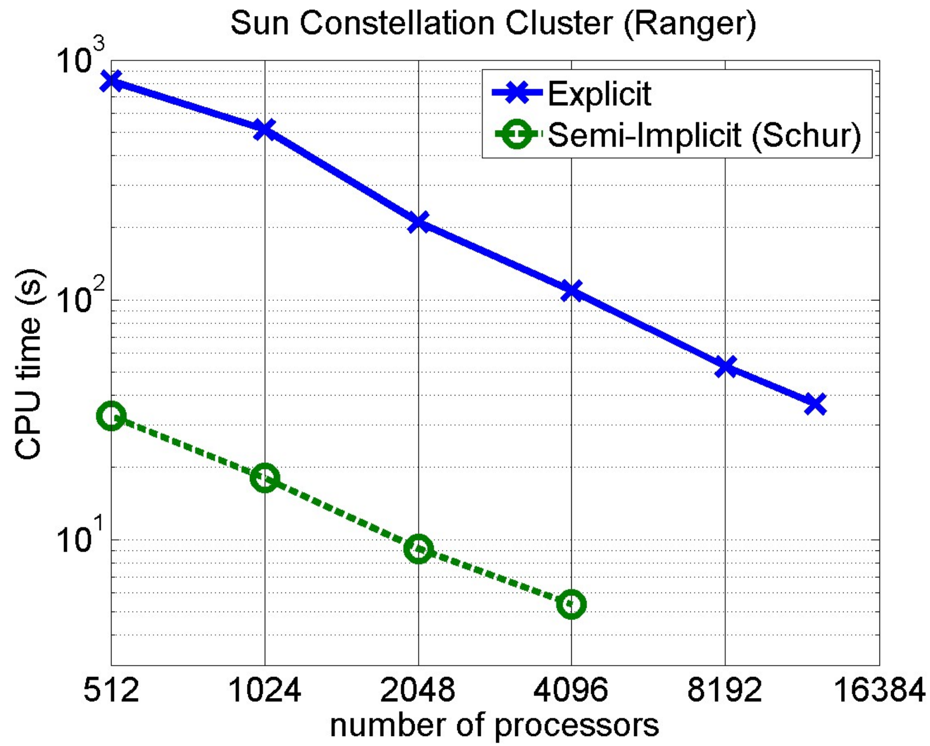
Urban-Scale Modeling



CG: Scalability (Explicit + Semi-Implicit)



Moderate Processor Counts: 110592 elements



Large Processor Counts: 262144 elements

Time-Steps

Explicit RK35: $\Delta t = 0.001$ s

SI BDF2: $\Delta t = 0.01$ s

Summary and Outlook

- NUMA Dynamical Core is quite mature:
  - 3D and MPI .
  - Can use either CG or DG methods.

- We will continue testing the dry dynamics and improve the model to make it as user-friendly as possible.

- Future Projects:
  - We are implementing sub-grid scale parameterizations to NUMA
  - Need more resources added to implement sub-grid scale parameterization, data assimilation, testing, etc.

Backup Slides

Conservative Form (2C)

- Mass is conserved but not energy.
- Conservation (or flux) form is required by DG machinery.

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \mathbf{U} = 0$$

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \bullet \left(\frac{\mathbf{U} \otimes \mathbf{U}}{\rho} + P\mathbf{I} \right) + \rho g \hat{\mathbf{k}} + \mathbf{f} \times \mathbf{U} = 0$$

$$\frac{\partial \Theta}{\partial t} + \nabla \bullet \left(\frac{\Theta \mathbf{U}}{\rho} \right) = 0$$

$$P = P_A \left(\frac{R\Theta}{P_A} \right)^{\gamma}$$

$$\mathbf{U} = \rho \mathbf{u}$$

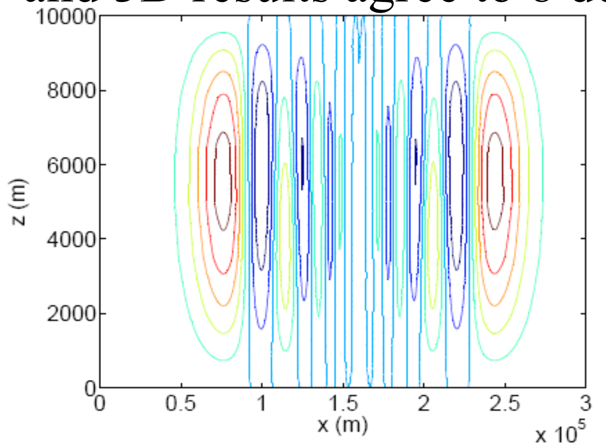
$$\Theta = \rho \theta$$

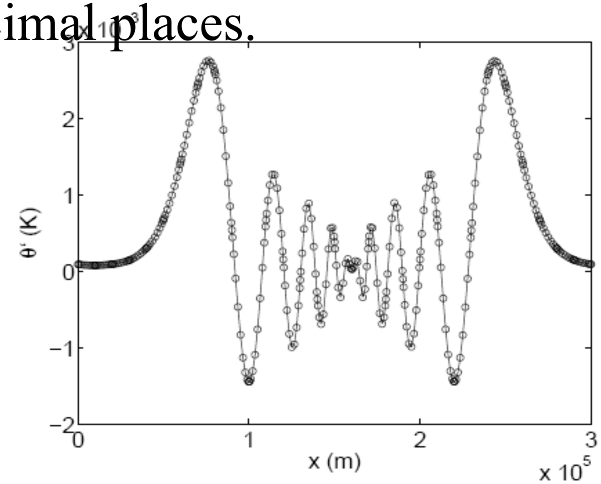
2D Inertia-Gravity Wave

•Standard test suite for 2D mesoscale problems (Skamarock, Doyle, et. al.)

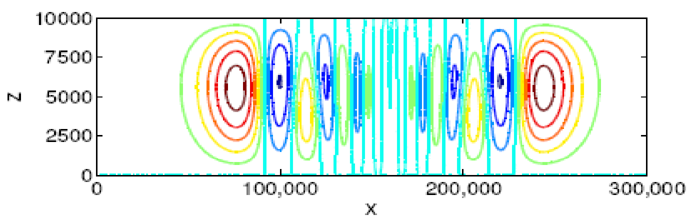
•Potential temperature perturbation after 3000 s for 250 m resolution (120 by 1 by 4 elements) and 10-th order polynomials.

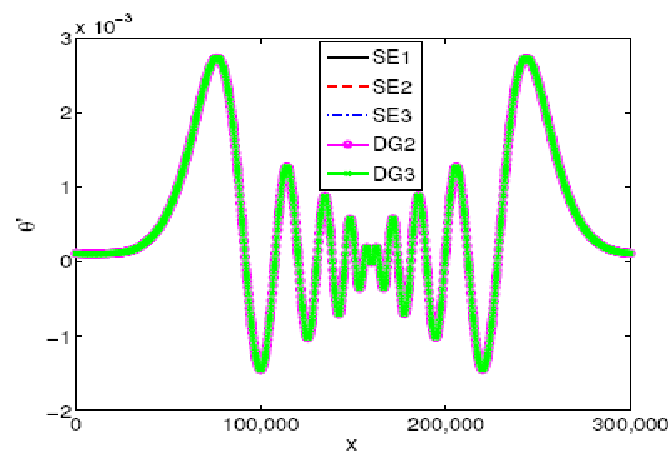
•2D and 3D results agree to 8 decimal places.





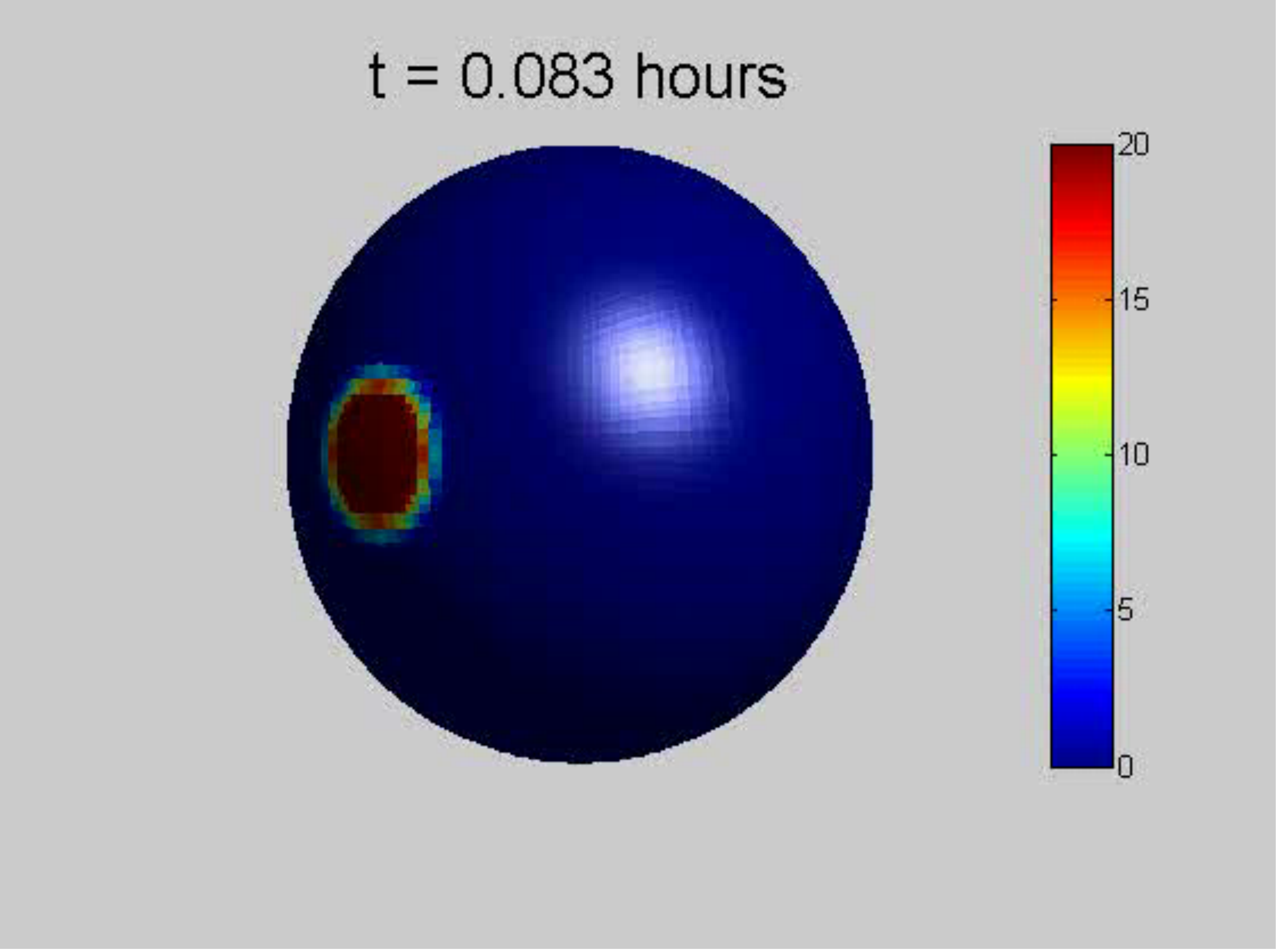
CG-NUMA run with 1 element in y-direction.





2D Density Current results for Giraldo and Restelli (2008).

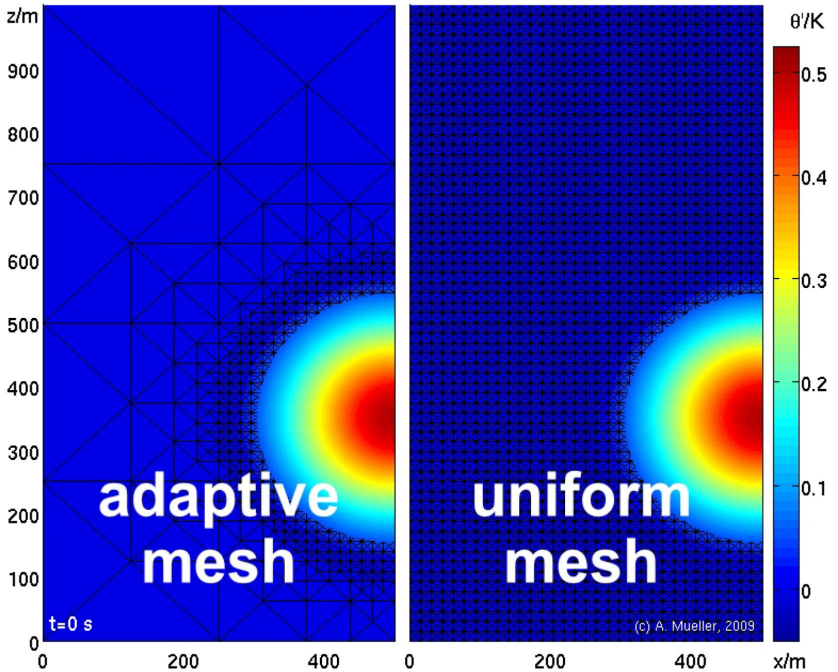
Results (Global): Pressure Wave



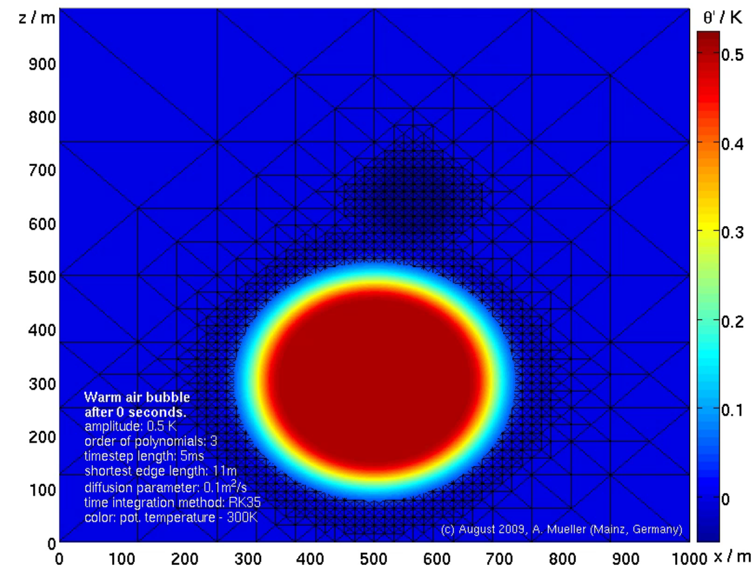
•Acoustic (perturbation) field shown at central vertical level (5 km).

•Wave returns to initial state after about 32 hours, yielding a speed of sound of 348 m/s.

Non-hydrostatic Adaptivity Examples
(Müller, Behrens, Giraldo, Wirth 2010)

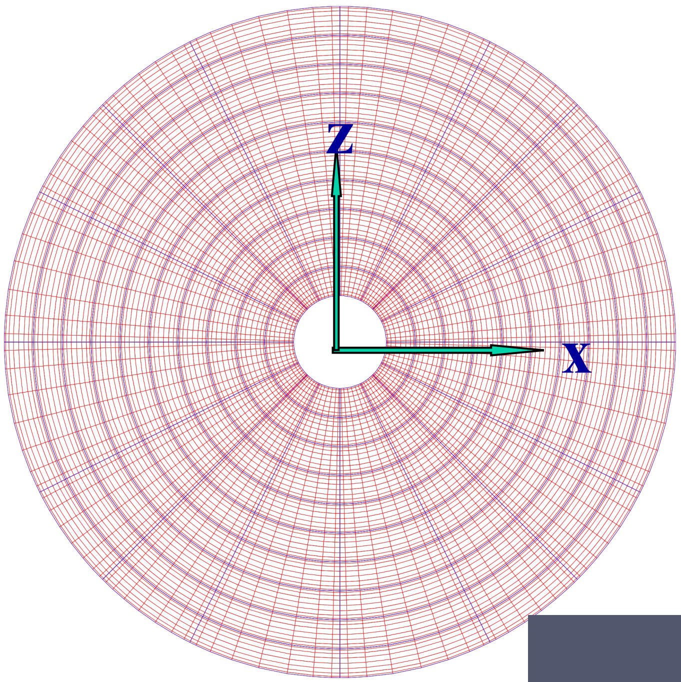


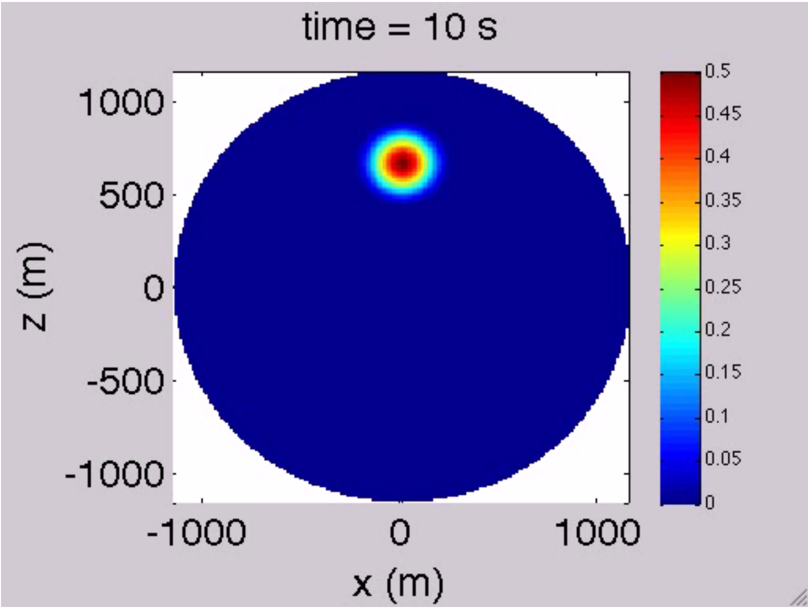
Rising Thermal Bubbles

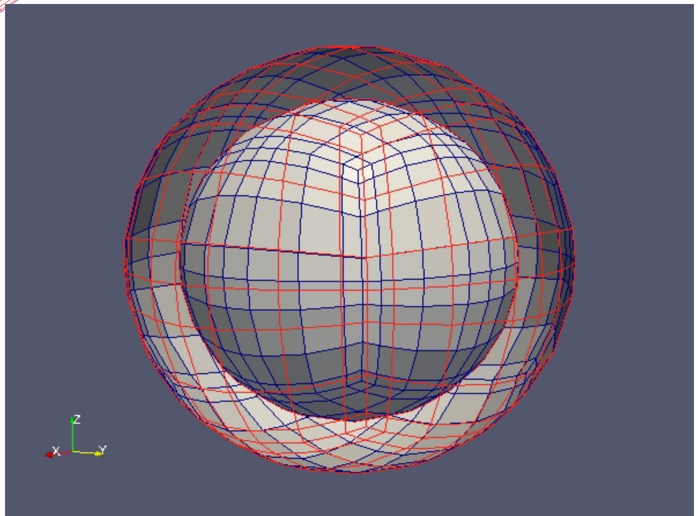


Two (Warm/Cold) Thermal Bubbles

Rising Thermal Bubble
(Global)







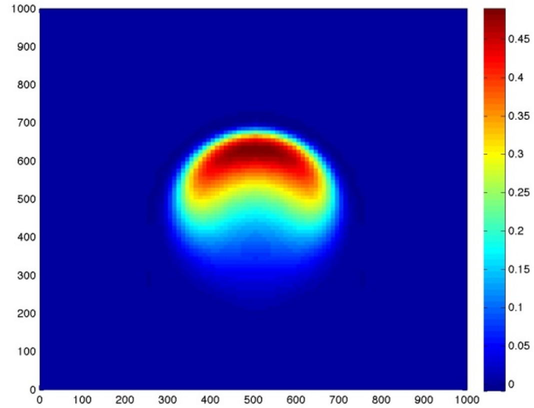
Overview of Existing Limited-Area Models

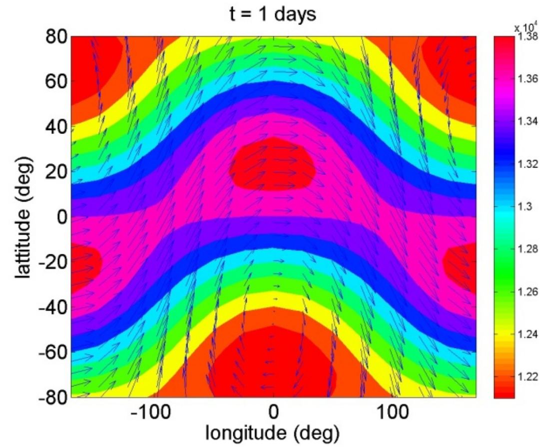
Model	Organization	Spatial Discretization	Grid Type	Time-Integrator
WRF	NCAR	Finite Difference	Structured	Split-Explicit
COAMPS	NRL-Monterey	Finite Difference	Structured	Split-Explicit
Lokal Modell	DWD	Finite Difference	Structured	Split-Explicit/ Semi-Implicit
NUMA	NPS/NRL	Spectral Elements/ Disc. Galerkin	Unstructured (any grid)	Semi-Implicit: 3D and 1D Vertical

Design Philosophy (1)

Numerical Methods

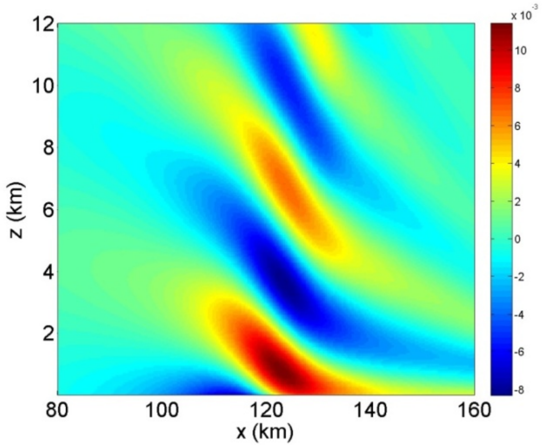
DG (Discontinuous Galerkin)

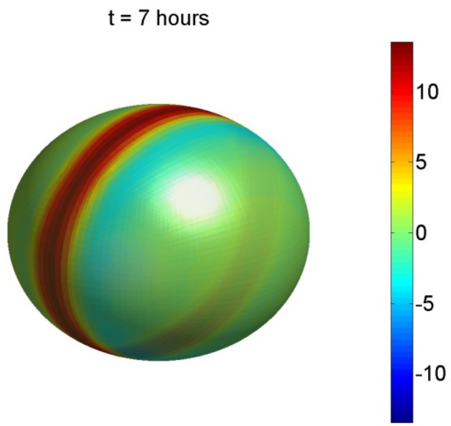




MPI Dry core under development.

CG (Continuous Galerkin)





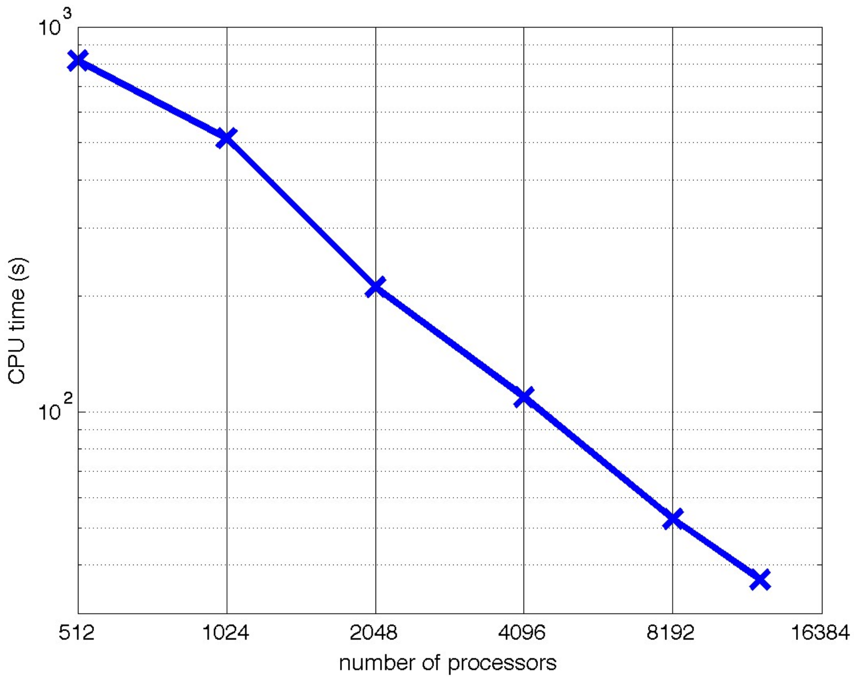
MPI Dry core developed. Physics will be added this year.

Limited-Area (Mesoscale)

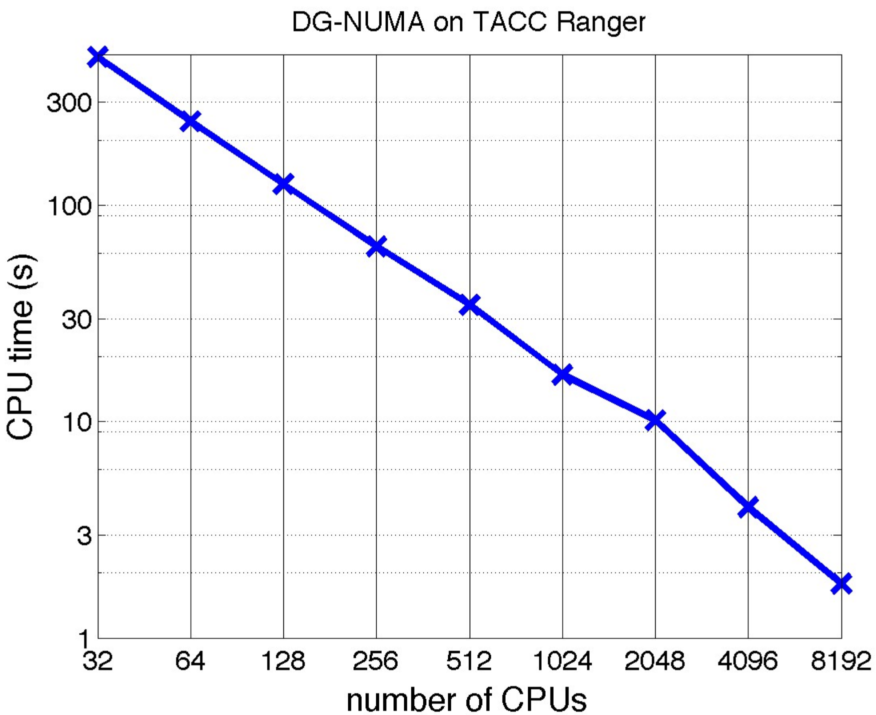
Global

Scale

Performance of the NUMA Model (3D Rising Thermal Bubble)



Continuous Galerkin



Discontinuous Galerkin

16 Million Grid Points