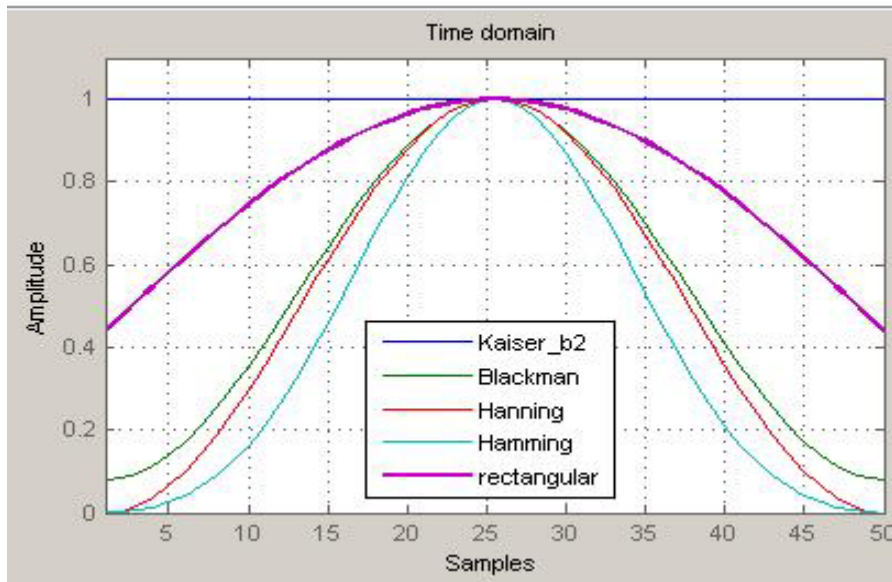


## II. Spectrograms

- [p. 2] Definition of window/frame
- [p. 4] Various windows and their properties
- [p. 7] Resolution issues
- [p. 8] Length effects for the Fourier Transform FT)
- [p. 13] Sliding window and the FT: short-time FT (STFT); spectrogram
- [p. 18] References

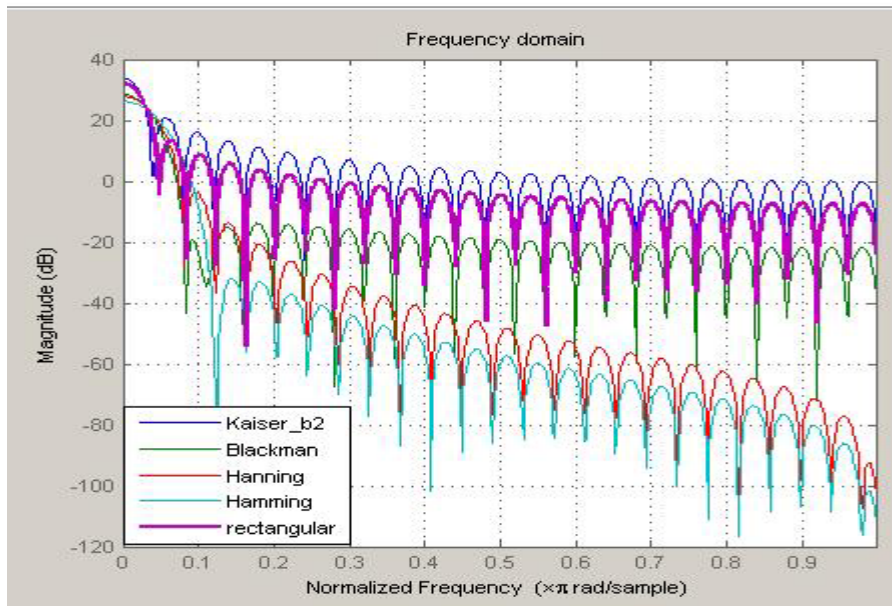
## ❖ Windows/Frames Effects



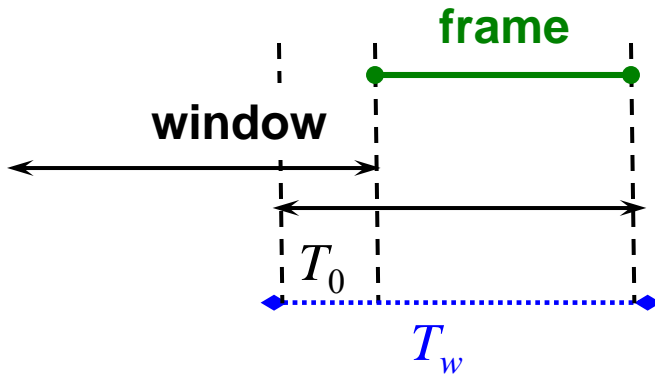
- Purpose of windows

- Stationarity issues

Time-frequency resolution trade-off



## Definition of window/frame

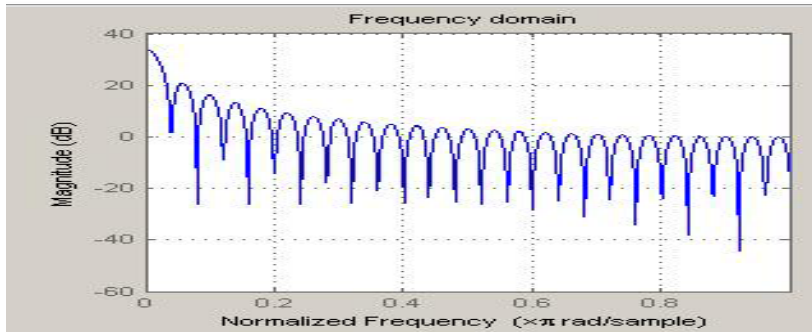


❖ Example: for speech applications, frame duration usually between 10-20 ms, and frame duration less than 8 ms usually not used.

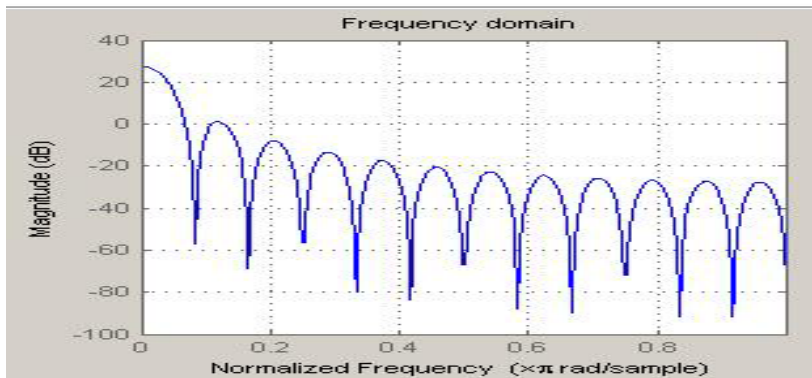
❖ Percentage overlap defined by

$$\% \text{ overlap} = \frac{T_0}{T_w} \times 100\%$$

## ❖ Windows / Properties

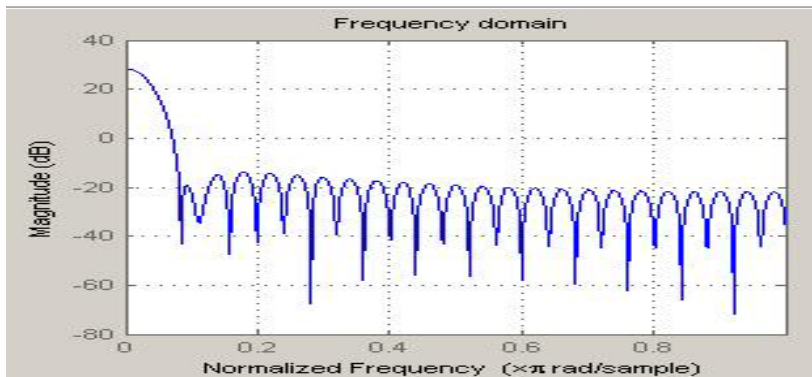


**Rectangular window**



**Triangular window (Bartlett)**

$$w(n) = \begin{cases} 2n/M & 0 \leq n \leq M/2 \\ 2 - 2n/M & M/2 \leq n \leq M \end{cases}$$

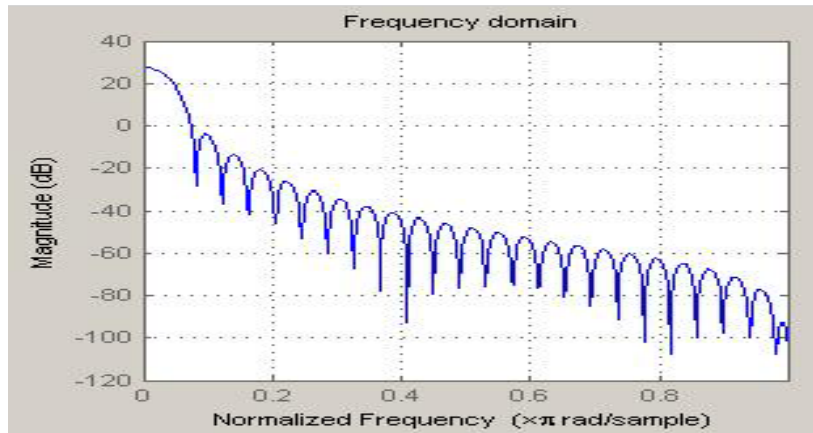


**Hamming window**

$$w(n) = 0.54 - 0.46 \cos(2\pi n/M); 0 \leq n \leq M$$

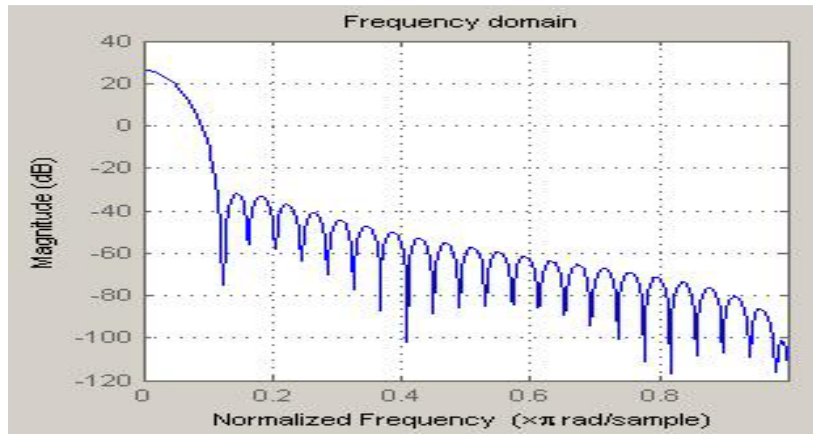
Window length  $M=50$

## ❖ Windows / Properties, cont'



### Hanning window

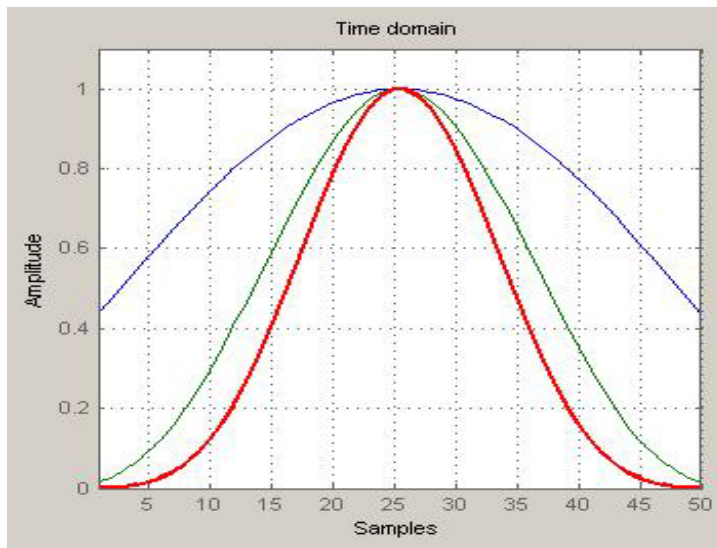
$$w(n) = \begin{cases} 0.5 - 0.5 \cos(2\pi n / M) & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$



### Blackman window

$$w(n) = 0.42 - 0.5 \cos\left(\frac{2\pi n}{M}\right) + 0.08 \cos\left(\frac{4\pi n}{M}\right), 0 \leq n \leq M$$

Window length  $M=50$



**Kaiser Window** (actually, a family of windows)

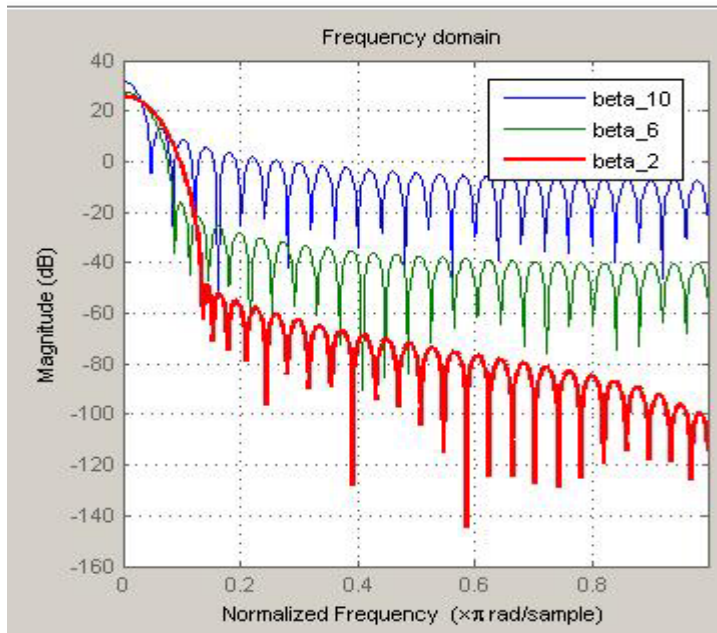
Designed to be maximally concentrated around  $\omega = 0$

$$w(n) = \begin{cases} \frac{I_0 \left[ \beta \left( 1 - \left[ (n - \alpha) / \alpha \right]^2 \right)^{1/2} \right]}{I_0(\beta)}, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

with  $\alpha = \frac{M}{2}$

Kaiser window:

- Has 2 adjustable parameters:  
→  $M$  and  $\beta$
- Used to trade sidelobes for mainlobe width.

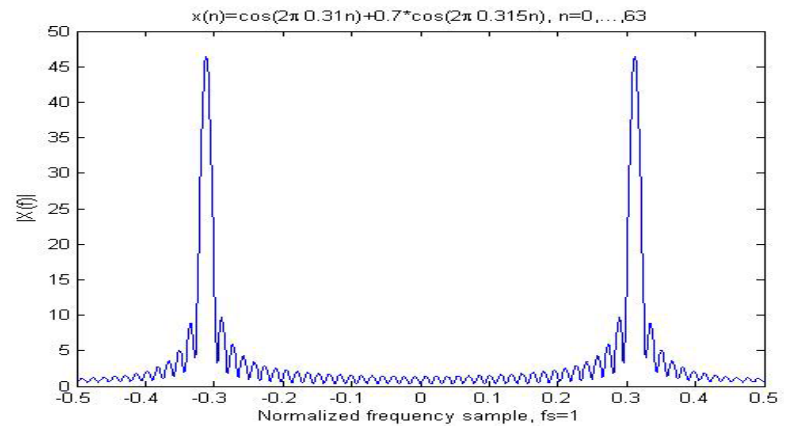
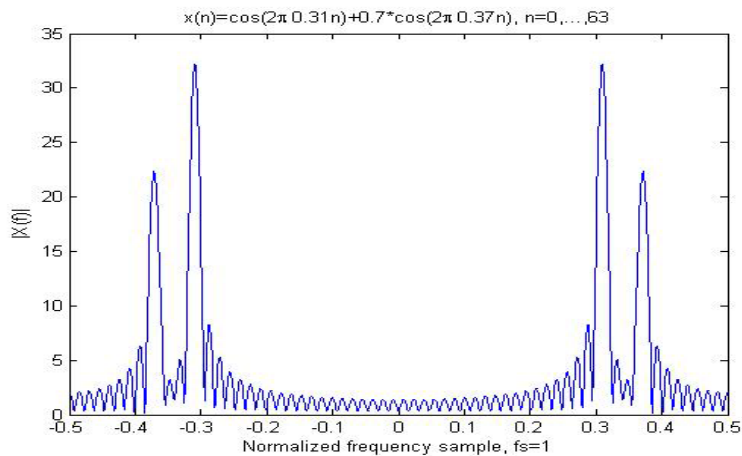
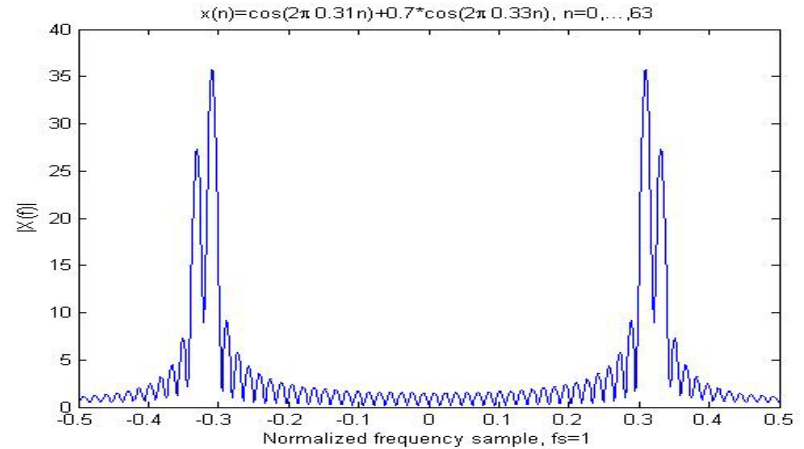
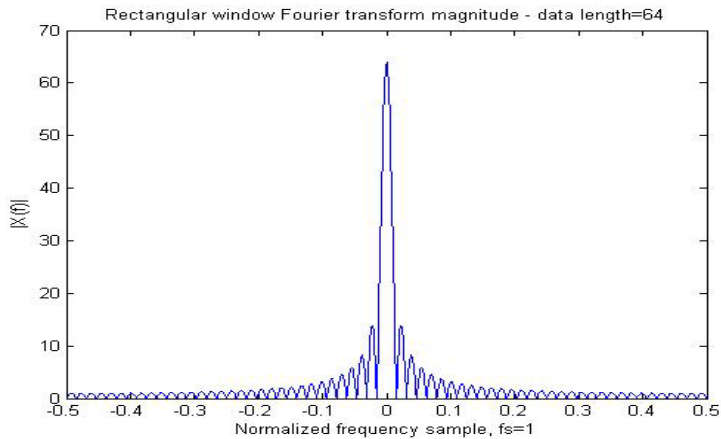


❖ **Comparison of commonly used windows, from [1]**

<b>Window Type</b>	<b>Relative Peak Sidelobe Amplitude</b>	<b>Approximate Mainlobe Width</b>	<b>Equivalent Kaiser Window</b>
<b>Rectangular</b>	<b>-13</b>	<b><math>4\pi/(M+1)</math></b>	<b>0</b>
<b>Bartlett</b>	<b>-25</b>	<b><math>8\pi/M</math></b>	<b>1.33</b>
<b>Hamming</b>	<b>-41</b>	<b><math>8\pi/M</math></b>	<b>4.86</b>
<b>Hanning</b>	<b>-31</b>	<b><math>8\pi/M</math></b>	<b>3.86</b>
<b>Blackman</b>	<b>-57</b>	<b><math>12\pi/M</math></b>	<b>7.04</b>

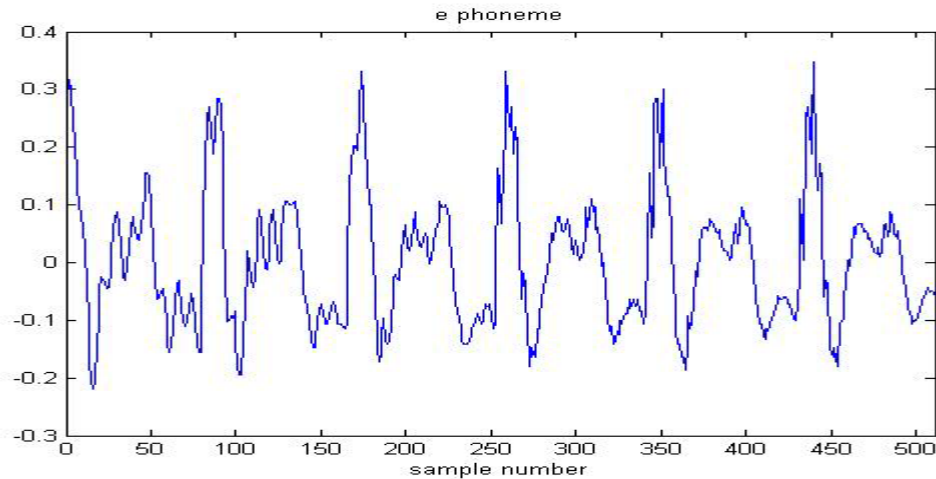
# ❖ Resolution issues

Controlled by length and type of window NOT by the FFT length.

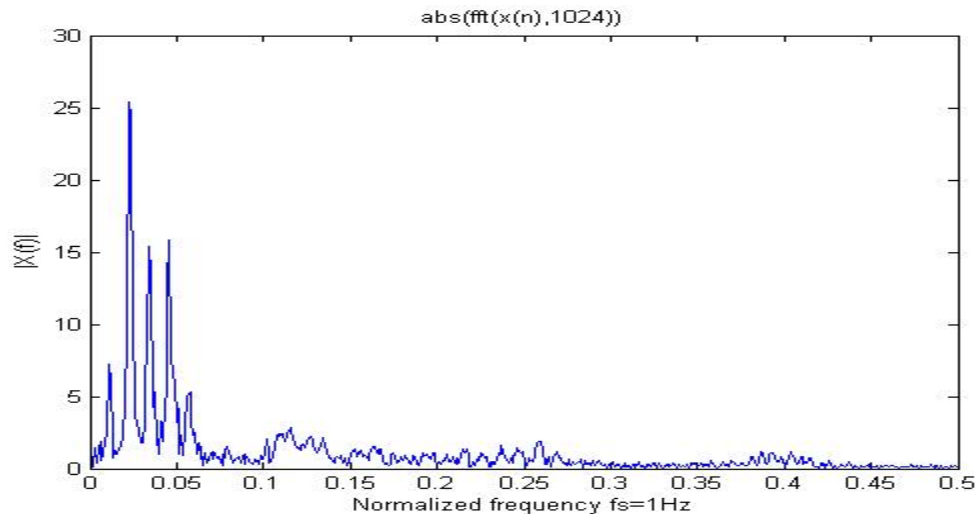




## ❖ Practical Applications of the Fourier Transform (length effects)

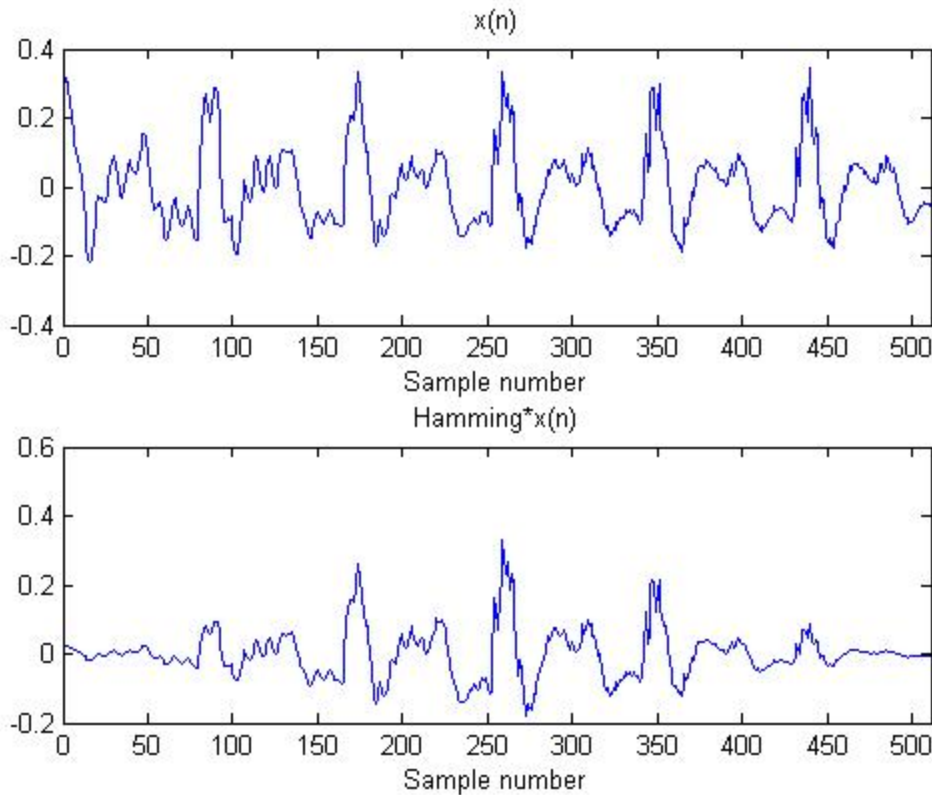


A segment of a vowel extracted with a rectangular window.



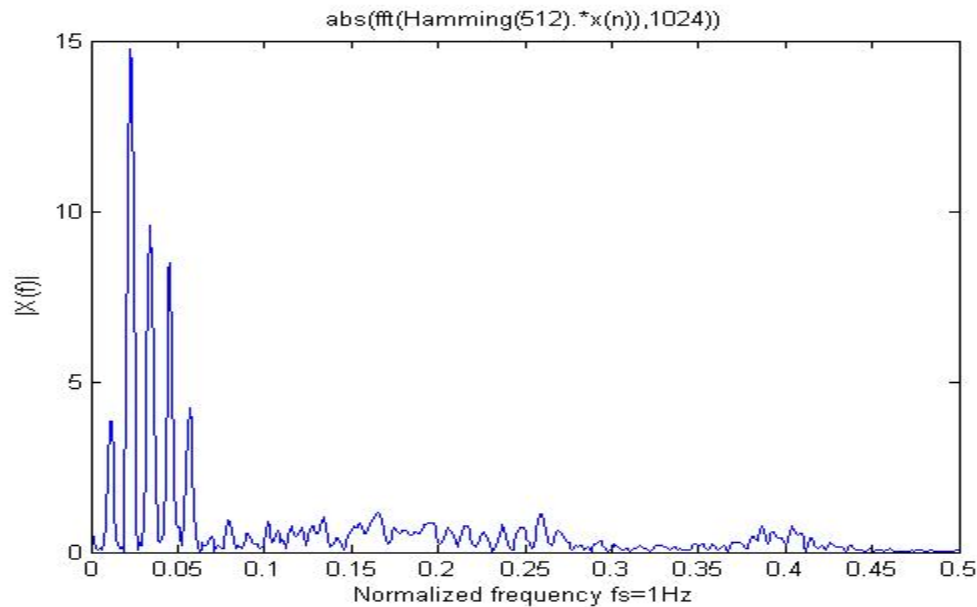
The amplitude spectrum using a rectangular window. Calculated using MATLAB: `abs(fft(sig))`.

- **Practical Applications of the Fourier Transform (length effects), cont'**

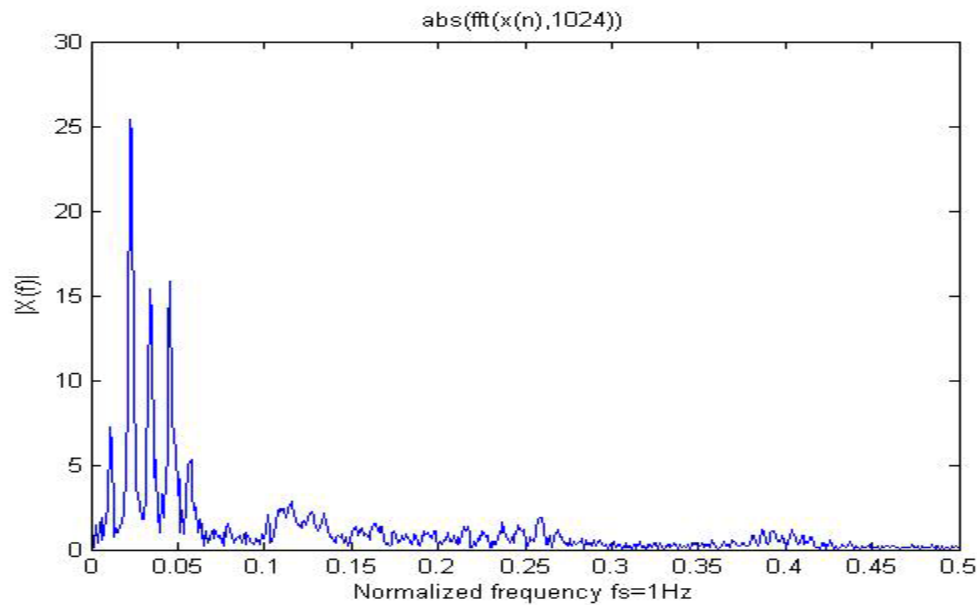


A segment of a vowel  
extracted with a  
Hamming window.

Calculated using  
MATLAB:  
`hamming(512) .* sig;`

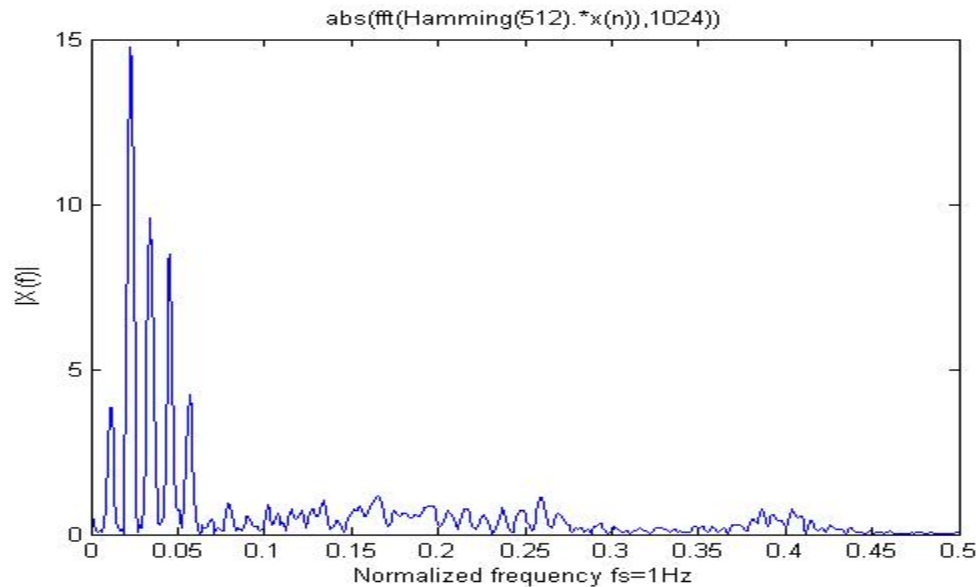


The amplitude spectrum  
using a Hamming window.  
Calculated using MATLAB



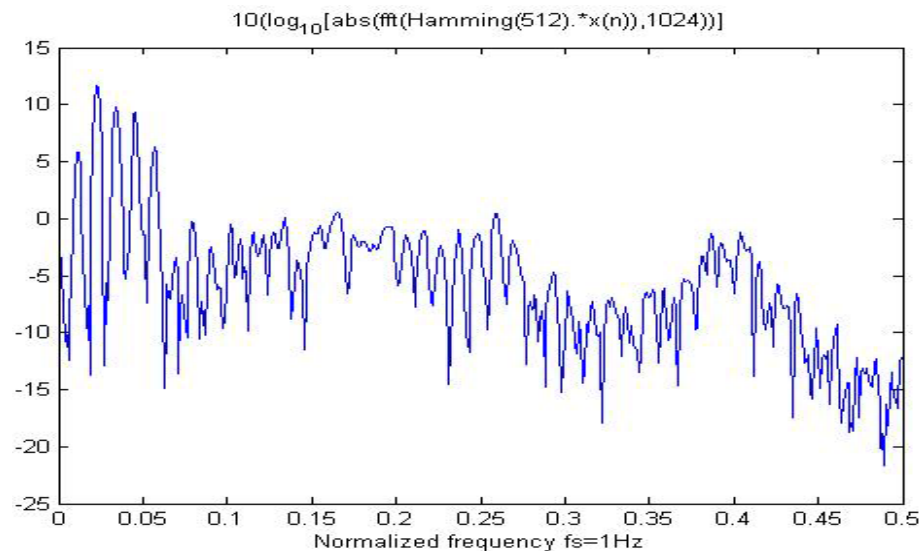
The amplitude spectrum  
using a rectangular window.  
Calculated using MATLAB

**Impact of the DB scale:** The power spectrum displayed on a dB scale. Calculated using MATLAB:  $10 \log_{10}(\text{abs}(\text{fft}(\text{hamming}(512) .* \text{sig}, 1024)))$

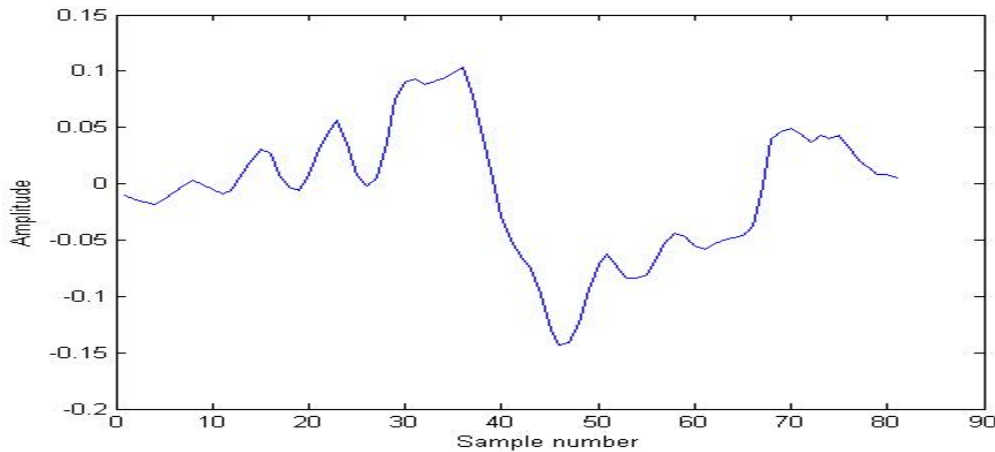


The amplitude spectrum using a Hamming window.

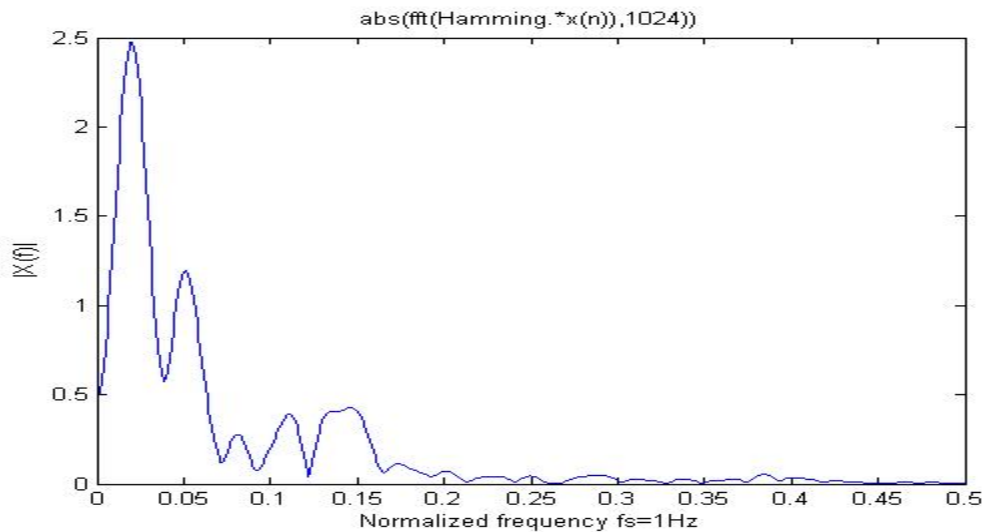
Calculated using MATLAB:  
`abs(fft(hamming(512) .*  
sig,1024)).`



Power spectrum displayed on a dB scale. Calculated using MATLAB:  
`10log10(abs(fft(hamming(512) .*  
sig)),1024)`

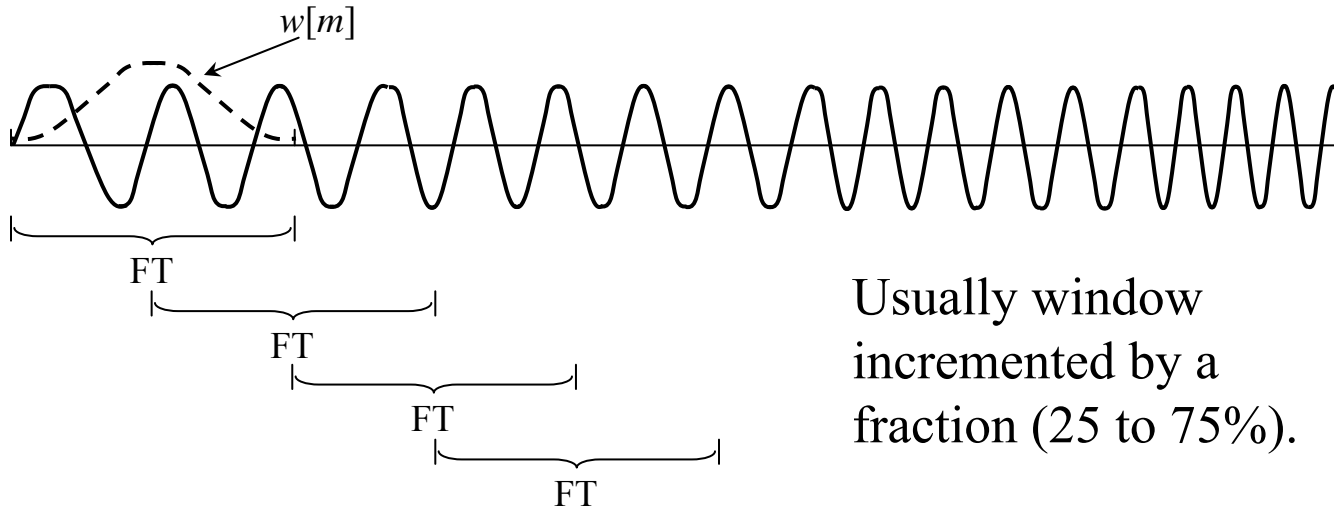


A segment of a vowel extracted with a Hamming window of length 81.

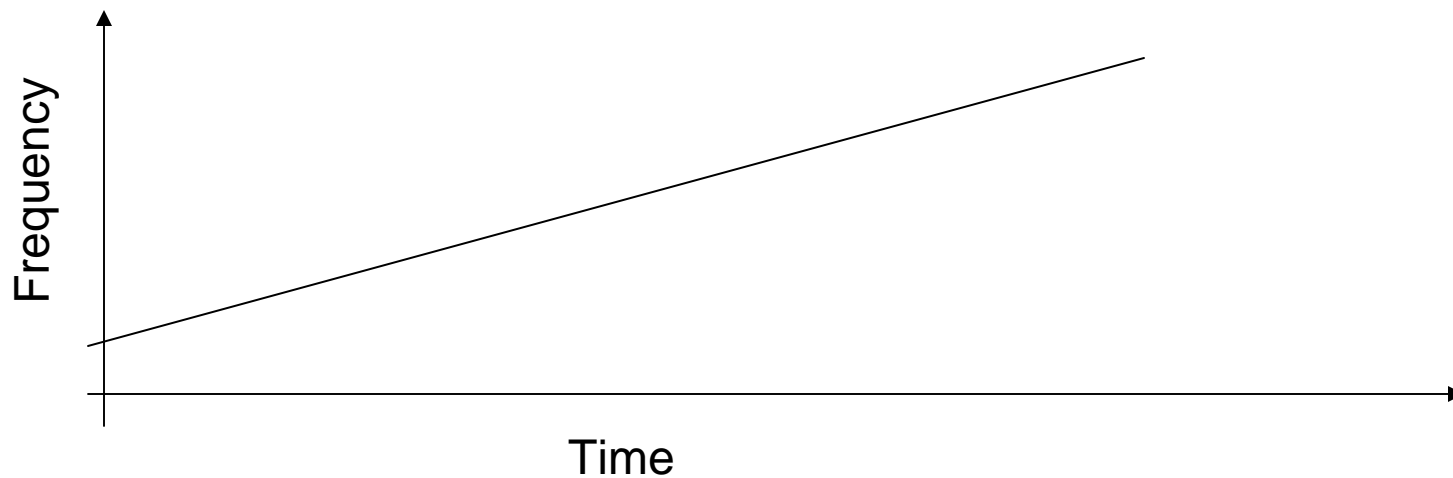


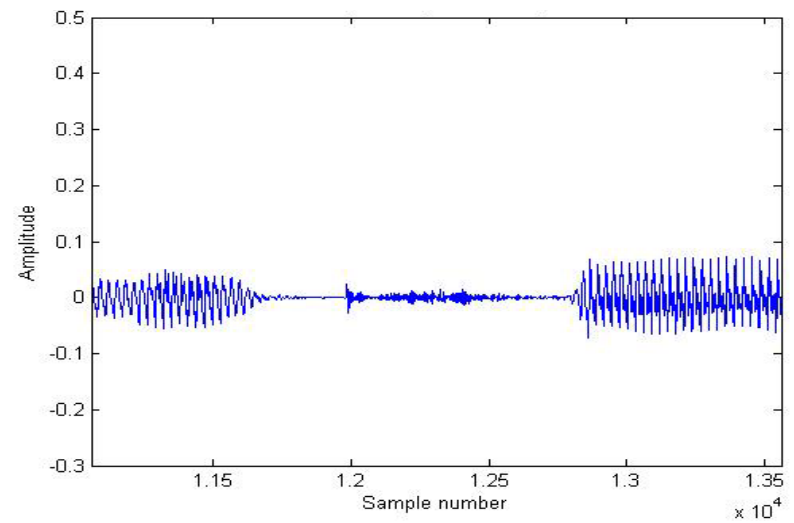
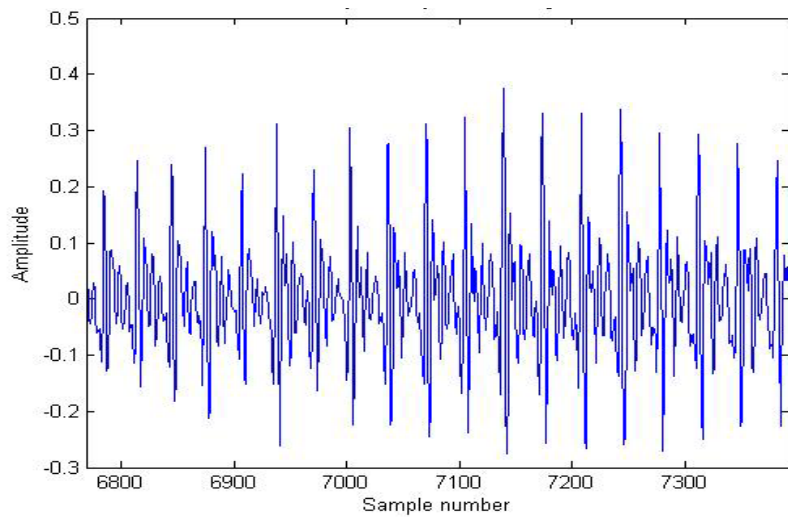
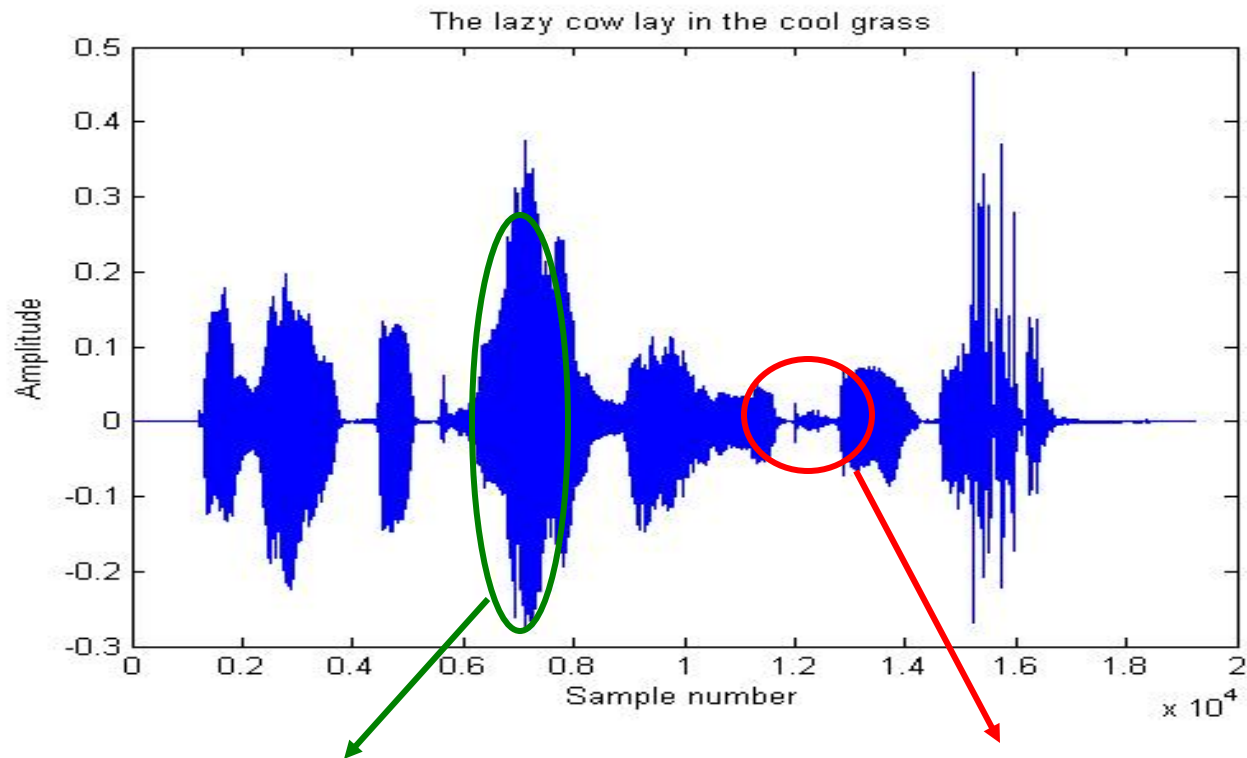
The amplitude spectrum using a Hamming window of length 81, fft zero-padded to 1024.

## ❖ Sliding Window FT (spectrogram)



Usually window incremented by a fraction (25 to 75%).





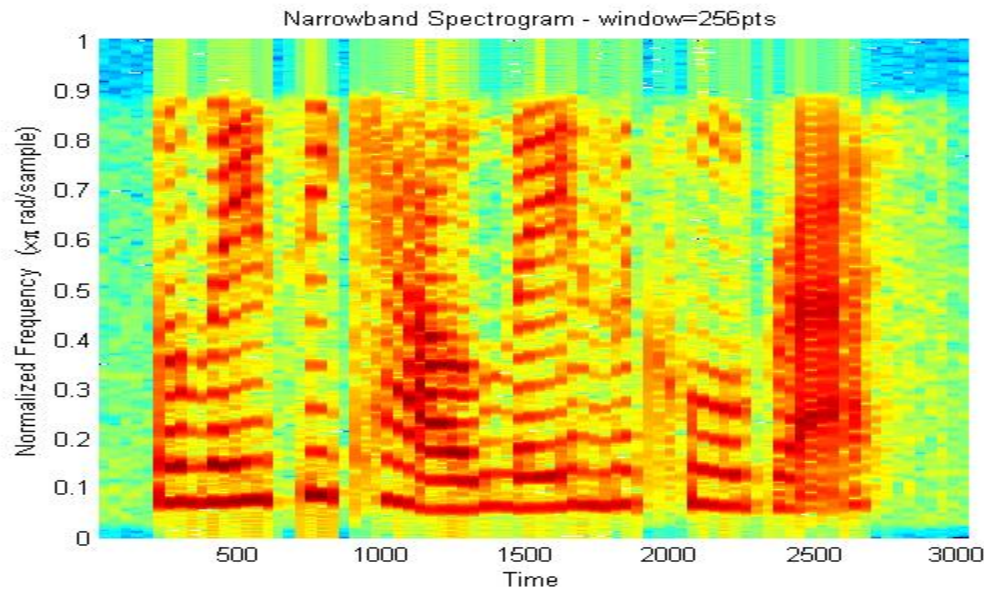
## ❖ Spectrogram – Windowing Effects

- Window type affects the spectrum shape on each window.
- Window size affects the spectrum smoothness.  
→ see next pages
- $x(t) \rightarrow X(t, f)$

In speech processing applications:

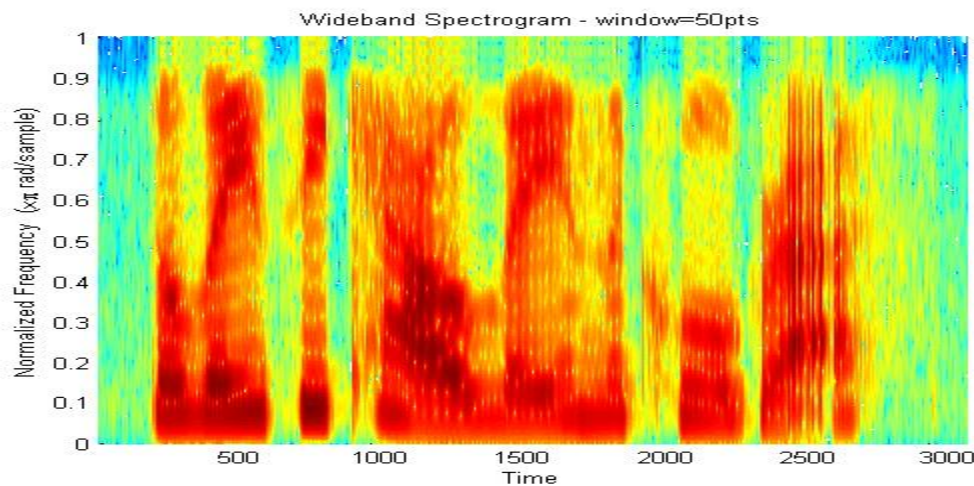
- Short window length ( $\sim 15$  ms) → wideband spectrogram
- Long window length ( $\sim 50$  ms) → narrowband spectrogram





Narrowband spectrogram

**The lazy cow lay in the cool grass**



Wideband spectrogram

# References

[1] *Discrete-time Signal Processing*, A. Oppenheim & R. Schaffer, Prentice Hall, 1989.

[2] *Discrete Time Processing of Speech Signals*, J. Deller et al, Macmillan, 1993.