Section 2
Continuous-Time Signals
EO 2402
Summer 2013

❖ Basic Signals

1. Complex Exponential Signal

$$x(t) = Ae^{at} \qquad a = \alpha + j\beta$$

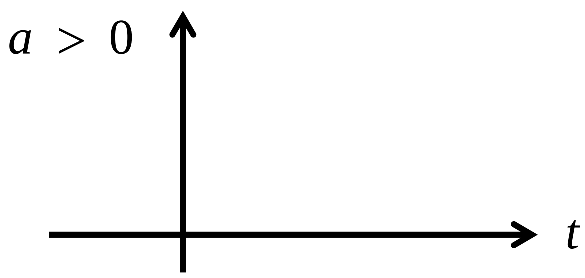
$$|A|e^{j\theta}$$

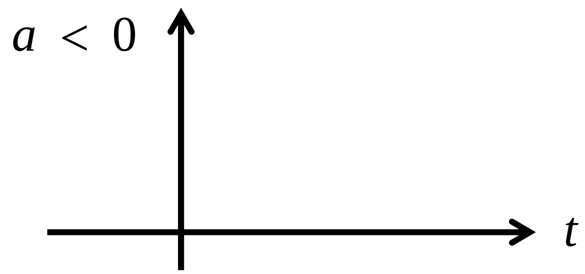
$$=$$

Complex Exponential Signals, cont'

$$x(t) = Ae^{at}$$

– A real, a real

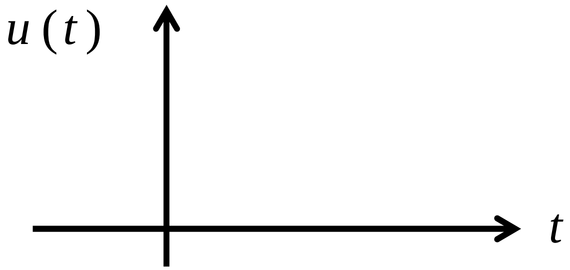




– A real, $a = \alpha + j\beta \rightarrow x(t) = Ae^{\alpha t}e^{j\beta t}$

2. Unit –Step Signal

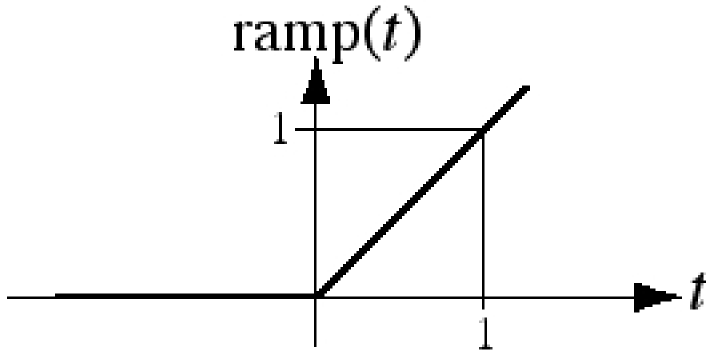
$u(t) =$



- Useful when defining input signals to systems that "turn on" at some time

3. Unit –ramp Signal

$$\text{ramp}(t) = \begin{cases} t, t > 0 \\ 0, t \leq 0 \end{cases}$$

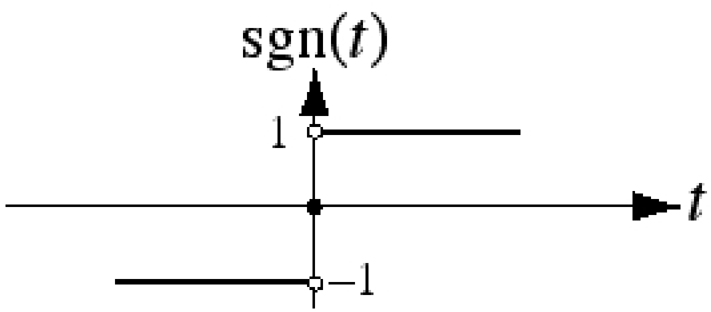


Note: The ramp signal is related to the unit step signal

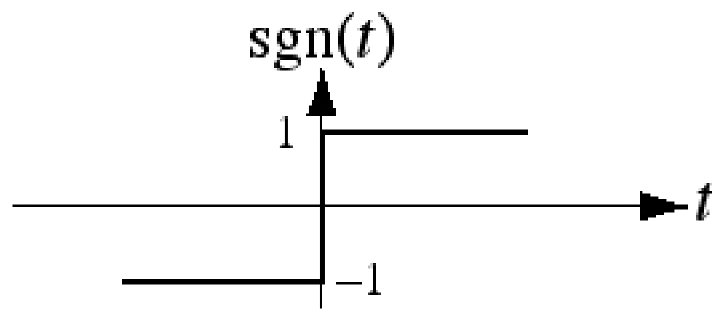
4. Signum function (Sign signal)

$$\operatorname{sgn}(t) = \begin{cases} 1 & , \ t > 0 \\ 0 & , \ t = 0 \\ -1 & , \ t < 0 \end{cases}$$

Precise Graph



Commonly-Used Graph

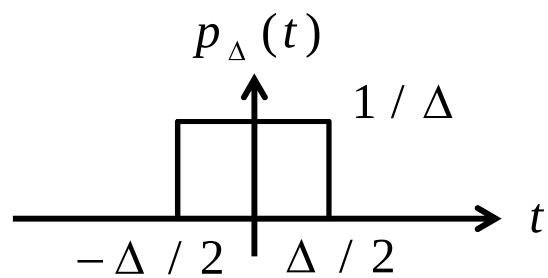


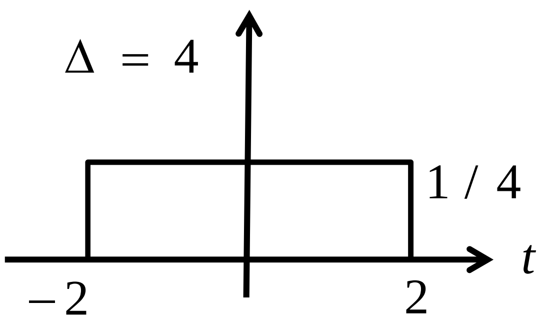
The signum function returns an indication of the sign of its argument.

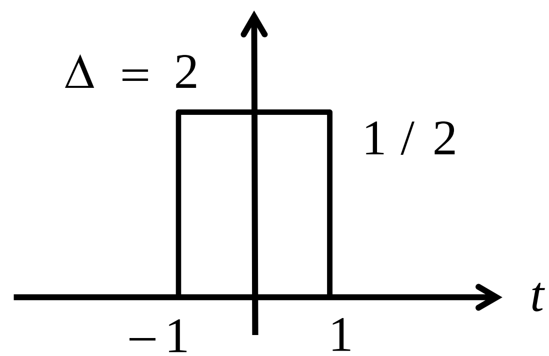
5. Impulse Signal $\delta(t)$

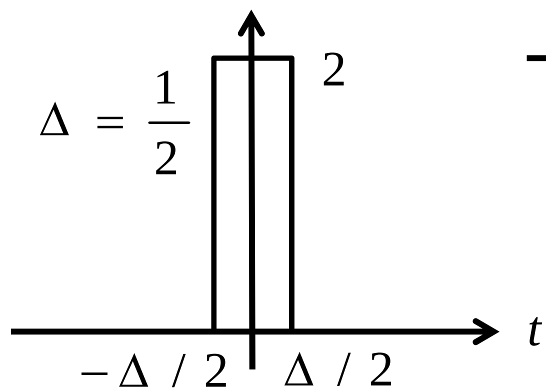
- Defined as a limit of $p_{\Delta}(t)$

$$p_{\Delta}(t) = \begin{cases} \frac{1}{\Delta}, & \frac{-\Delta}{2} \le t \le \frac{\Delta}{2} \\ 0 & 0\omega \end{cases}$$









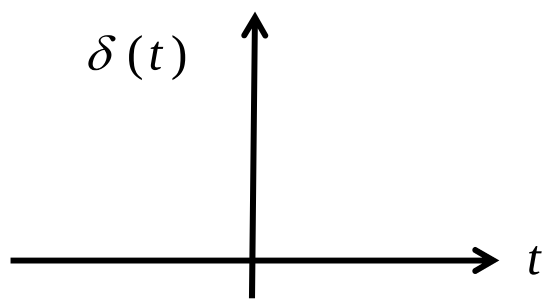
Impulse Signal, cont'

$\delta(t)$ is defined as:

$$\delta(t) = 0 \qquad t \neq 0$$

and $$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$

→ Intuitive definition: - concentrated at t=0 with
- unit area



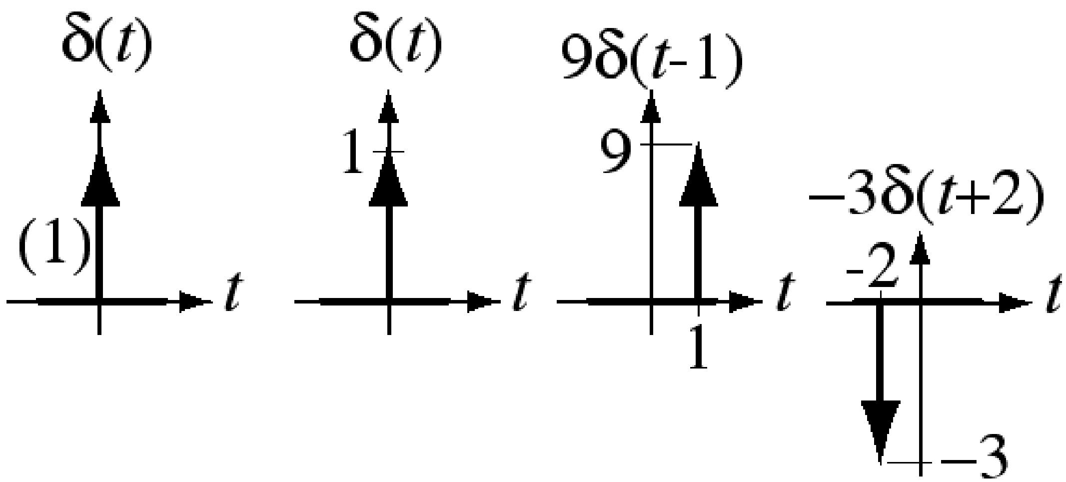
$$\Rightarrow \quad \delta(t - t_0) \neq 0 \qquad \text{for} \quad t - t_0 = 0$$

$$\rightarrow t = t_0$$

Note:

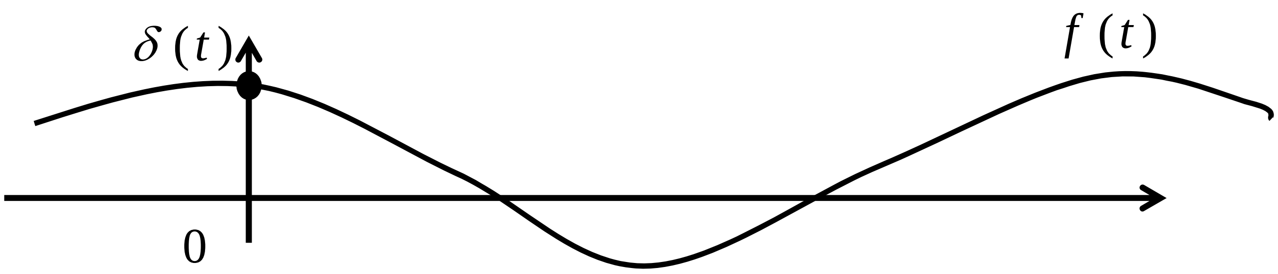
- The impulse is not a function in the ordinary sense because its value at the time of its occurrence is not defined.

- It is represented graphically by a vertical arrow. Its strength is either written beside it or is represented by its length.

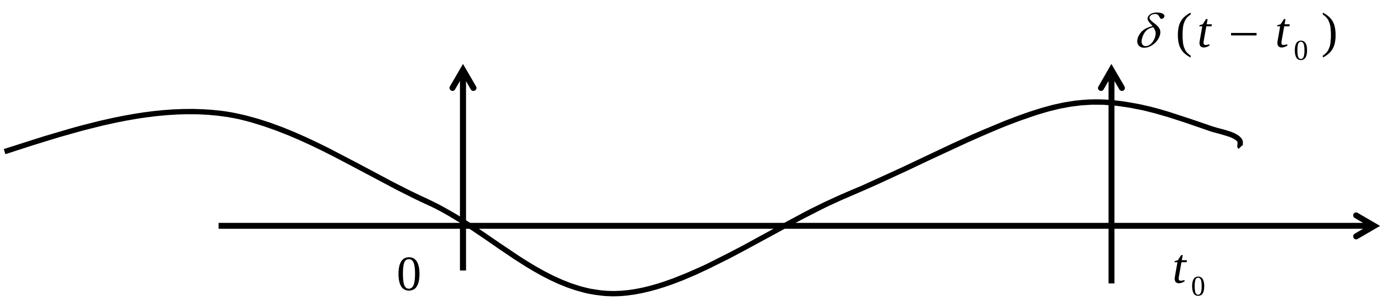


Impulse Signal, cont'

- Properties of $\delta(t)$



$$f(t)\,\delta(\mathrm{t}) = f(0)\,\delta(t)$$



$$f(t)\,\delta(t - t_0) = f(t_0)\,\delta(t - t_0)$$

Impulse Signal, cont'

$$\delta(t - t_0) = 0, \quad t \neq t_0$$ Non zero at one point only

$$\int_{-\infty}^{+\infty} \delta(t - t_0)dt = 1$$ Unit area

$$\int_{-\infty}^{+\infty} f(t)\delta(t - t_0)dt = f(t_0)$$ Sifting property

$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$$ Sampling property

$$\frac{du(t)}{dt} = \delta(t)$$ Derivative of unit step

$$\delta(at) = \frac{1}{|a|}\delta(t), \quad a \neq 0$$ Scaling property

- **Impulse Signal - Example**

$x(t) = e^{-2(t-1)}u(t-1)$, Compute and plot $\frac{d}{dt}x(t)$

❖ Examples:

$$A = \int_{-\infty}^{+\infty} \delta(\tau + 3)\, d\tau$$

$$B = \int_{-\infty}^{1} \delta(\tau + 3) e^{\tau}\, d\tau$$

$$C = \int_{-\infty}^{t-1} \delta(t + 3) e^{t}\, dt$$

$$D = \frac{d}{dt}\left\{ e^{-3t} u(t-1) \right\}$$

$$E = \int_{0}^{1} \sin(\tau) e^{-j\tau} \delta(\tau + 3)\, d\tau$$

$$F = \int_{2}^{+\infty} \cos(3\tau) \delta(\tau - 1)\, d\tau$$

Examples:

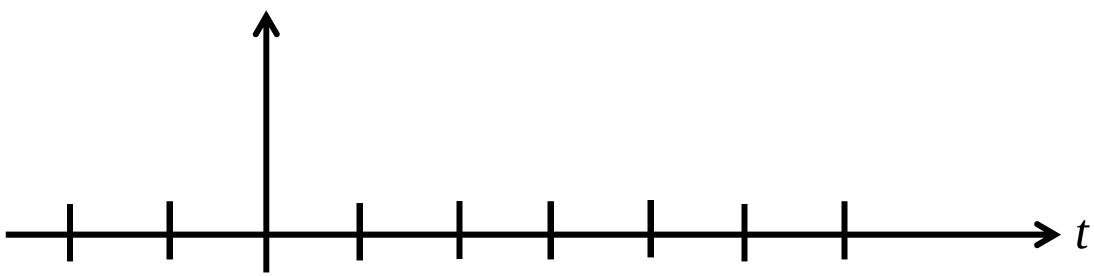
Assume $x(t) = u(t) - u(t-5)$

Plot $x(t), \quad \frac{dx(t)}{dt}, \quad x(2-t)$

5. Sampling Signal $\delta_{T_S}(t)$ (Unit Periodic Impulse)

- Definition: The sampling signal (unit periodic impulse) is defined as

$$\delta_{T_S}(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_S)$$



- Can be used to generate a sampled signal $x_s(t)$ from the analog version *x(t)*

$$x_s(t) = x(t).\delta_{T_S}(t)$$

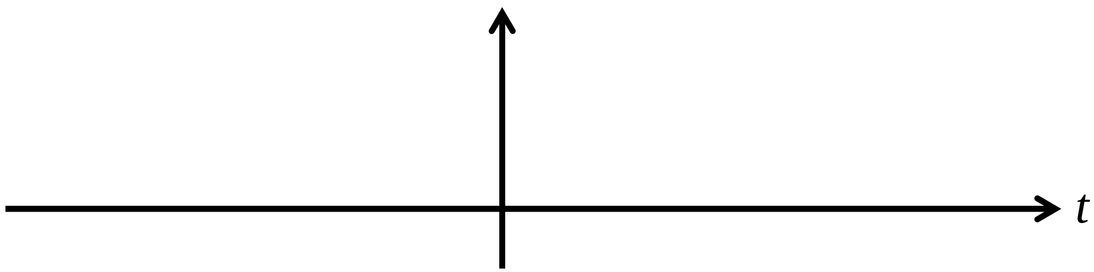
$$= x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_S)$$

$$=$$

6. Sinc Function

- Definition: The *sinc* function is defined as:

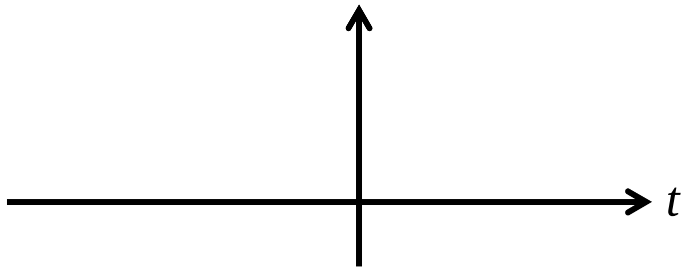
$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



- Behavior at $\pm\infty$, 0
- Zero crossing locations

7. Rectangle Function

$$\text{rect}(t) = \begin{cases} 1 & |t| \le 1/2 \\ 0 & ow \end{cases}$$



$$\text{rect}(t/T) = \begin{cases} 1 & |t| \le 1/2T \\ 0 & ow \end{cases}$$

NOTE: - area under $\text{rect}(t) =$

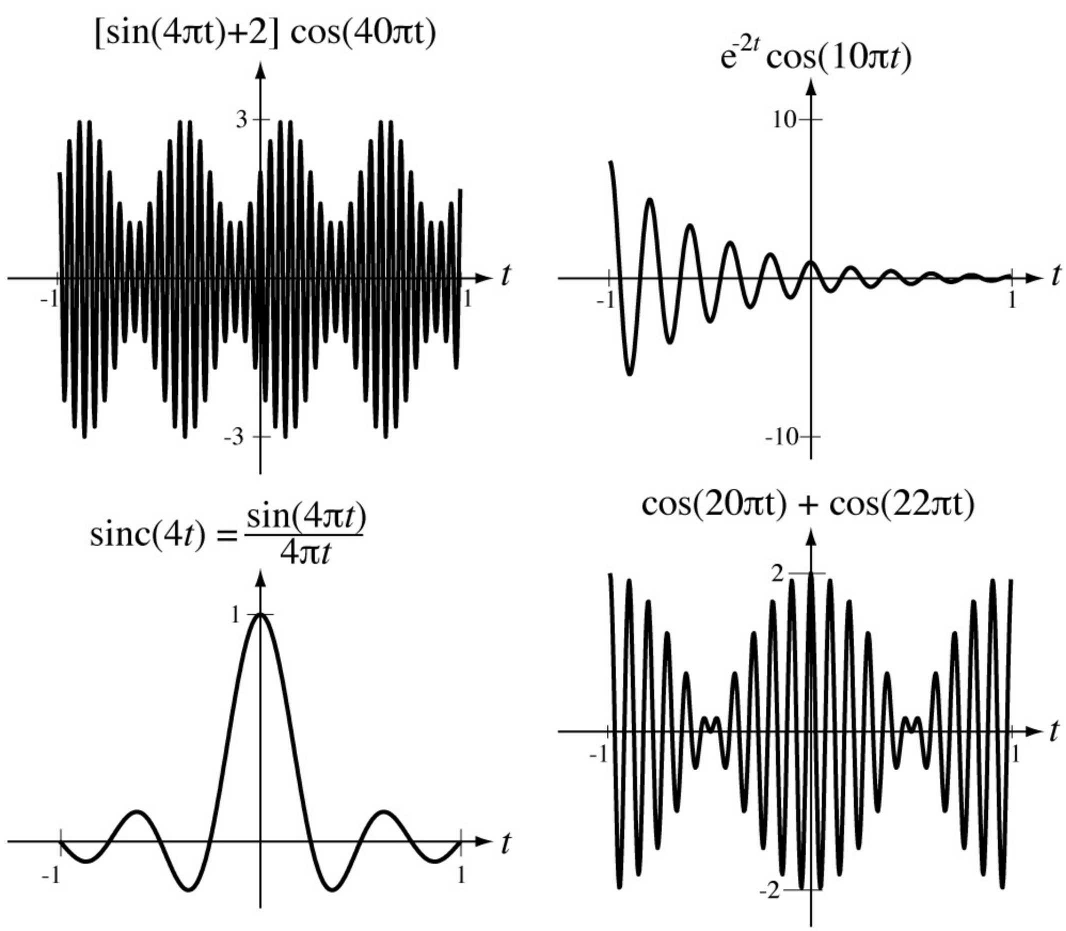
- $\text{rect}(t) =$

(Using unit steps)

❖ Examples:

Plot: $3rect\left(\frac{t+1}{4}\right), 4u(3-t), -5ramp(0.1t), -3\operatorname{sgn}(2t)$

❖ Combinations of Functions



❖ Continuous-Time (Analog) Signals Properties

- **Real/Complex Signal**

$$x(t) = \cos(2t + \pi / 3)$$

$$x(t) = (1 + j)\cos(2t + \pi / 3)$$

$$\text{Real}(x(t)) =$$

$$\text{Im}(x(t)) =$$

- **Periodic / Aperiodic Signals**

- Definition: An Analog Signal $x(t)$ is periodic with period T if

$$x(t) = x(t+T), \quad \forall t$$

Consequence: $x(t) = x(t+kT), \quad k \text{ integer}$

Example: $x(t) = A\cos(w_0 t + \theta)$

What is the Period T?

- **Do Periodic Signals Exist in Practical Applications?**

- **Periodic Signal Properties**

  - Assume $x(t)$ is periodic with period T

    Are signals below periodic?

$y(t) = A + x(t)$

$z(t) = x(t) + v(t)$, $v(t)$ periodic with period $T_2 = NT$

$w(t) = x(t) + b(t)$, $b(t)$ periodic with period T_2 which is not a multiple of T

- **Examples:** Are signals periodic?

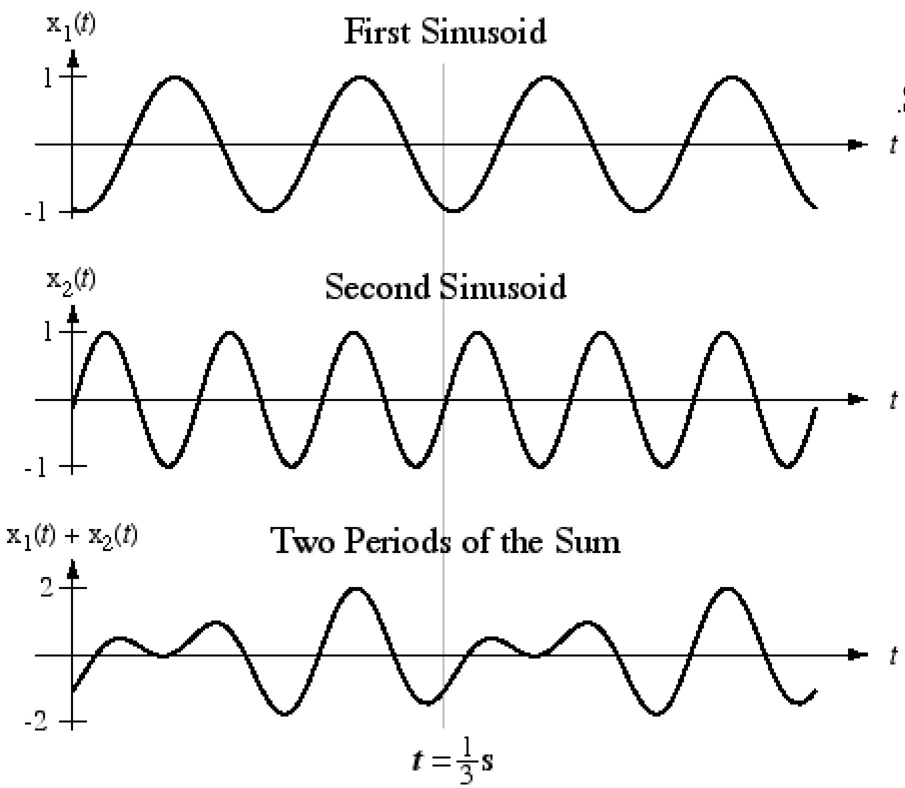
$$x(t) = \cos(4t) + \sin(8t + \pi/3)$$

$$y(t) = \cos(4t) + \sin(9t)$$

$$z(t) = e^{j2t} + e^{j\pi t}$$

- The period of the sum of periodic functions is the **least common multiple** of the periods of the individual functions summed.

- If the least common multiple is infinite, the sum function is aperiodic.



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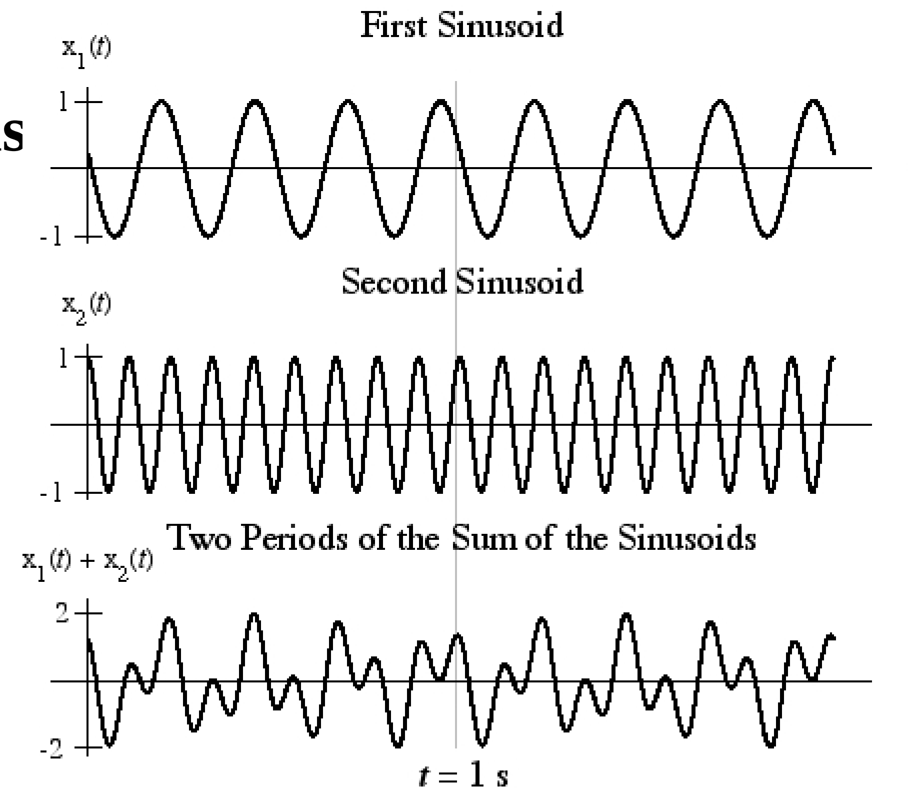
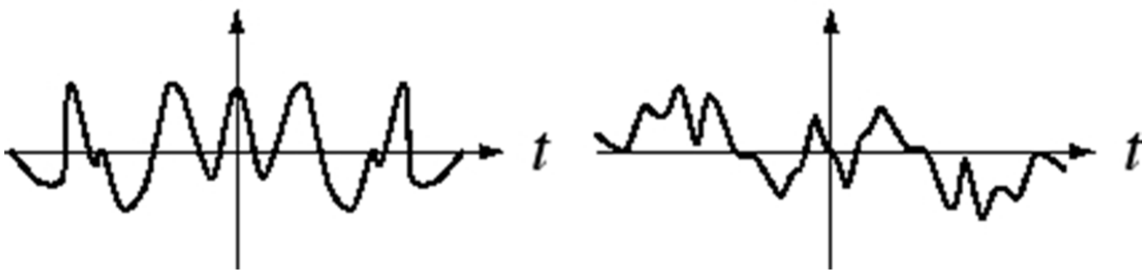


- **Even / Odd Signals**

- Definition: A Signal $x(t)$ is odd if $x(t) = -x(-t)$
A Signal $x(t)$ is even if $x(t) = x(-t)$



Property: Any signal $x(t)$ can be decomposed into a sum of even and odd components

$$x(t) = x_e(t) + x_o(t)$$

$x_e(t) = 0.5[x(t) + x(-t)]$

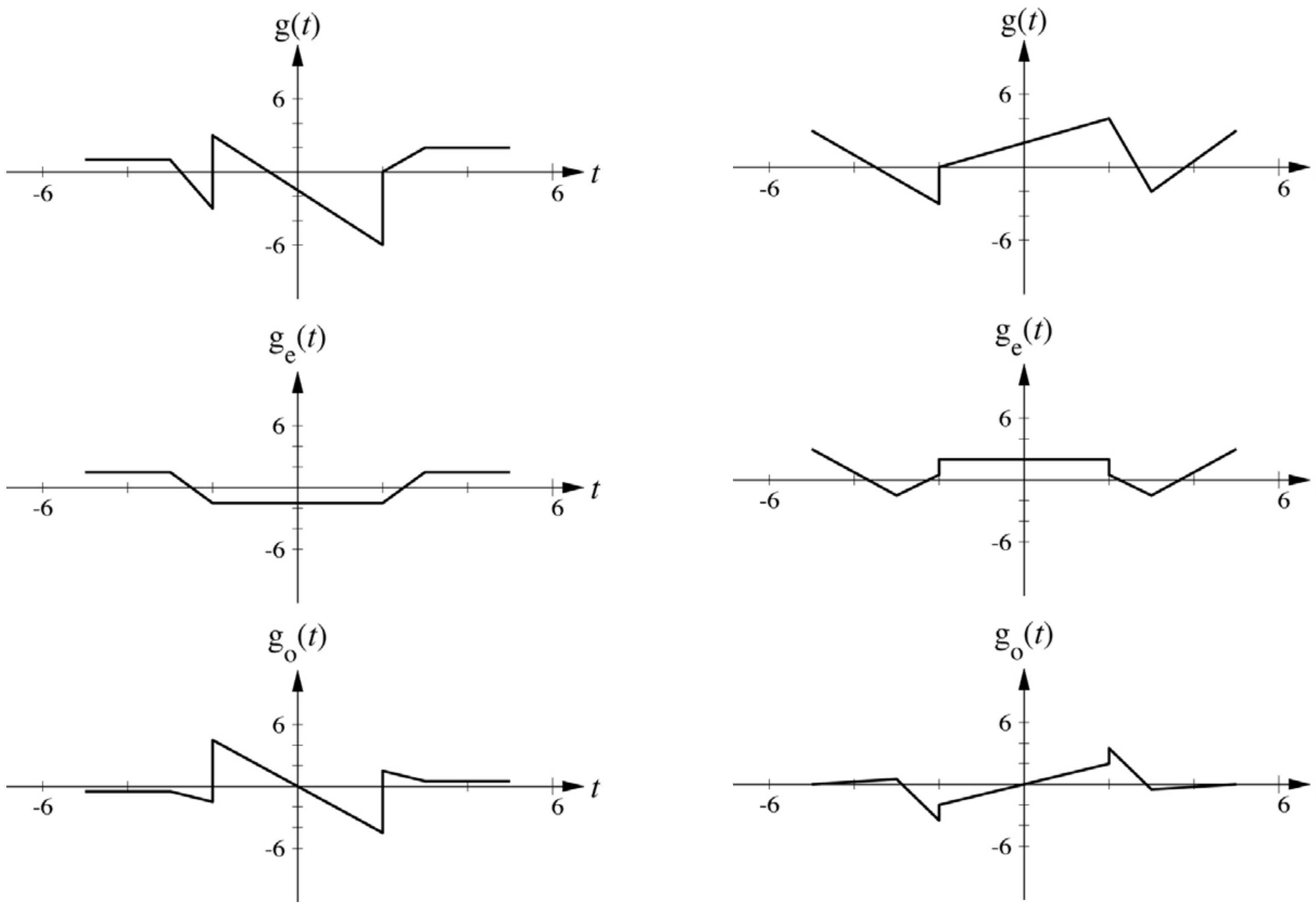
$x_o(t) = 0.5[x(t) - x(-t)]$

- **How to check $x_e(t)$ is even and $x_o(t)$ is odd**

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

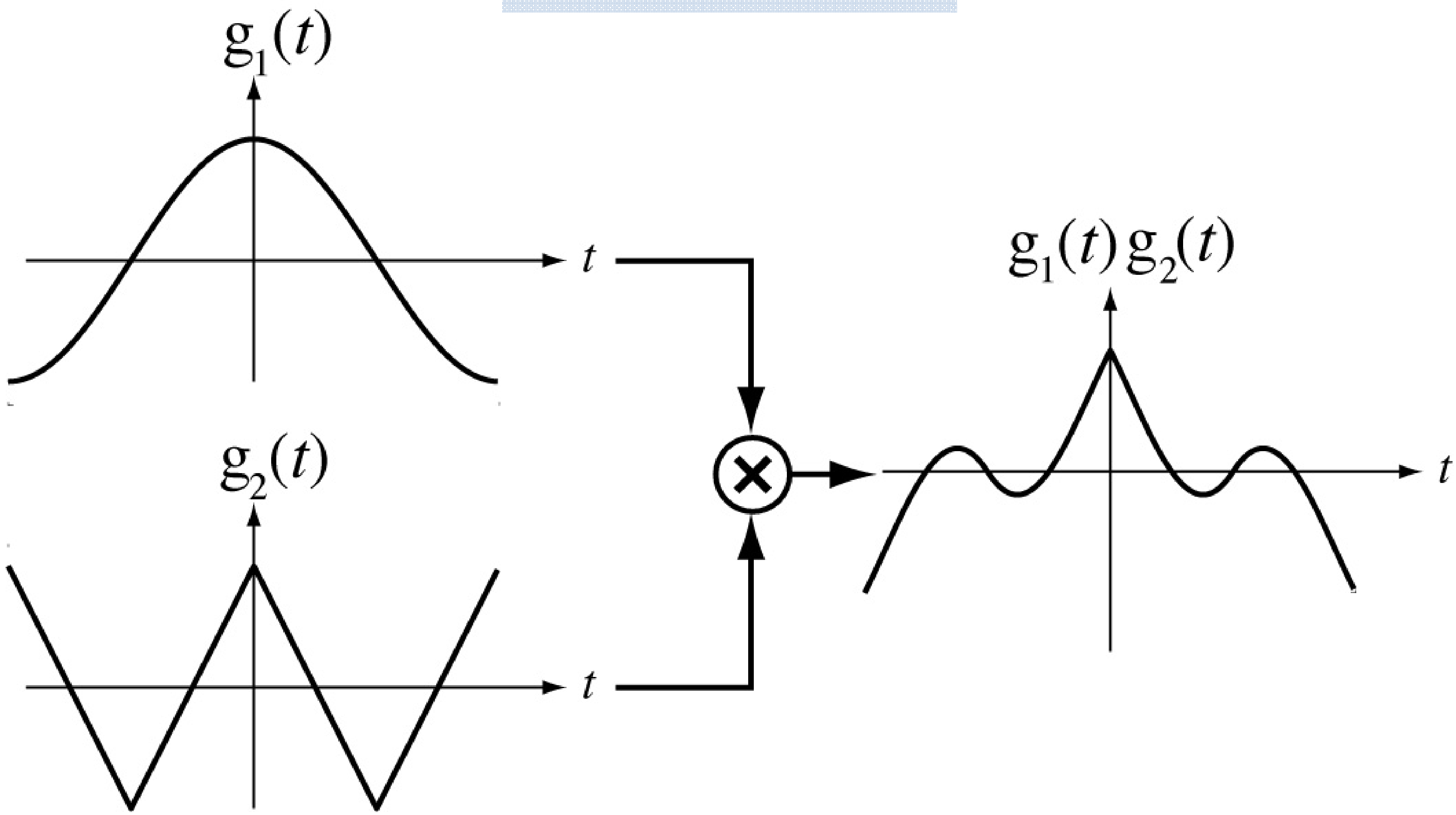
$$x_0(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

- **Even and Odd Parts of Functions**

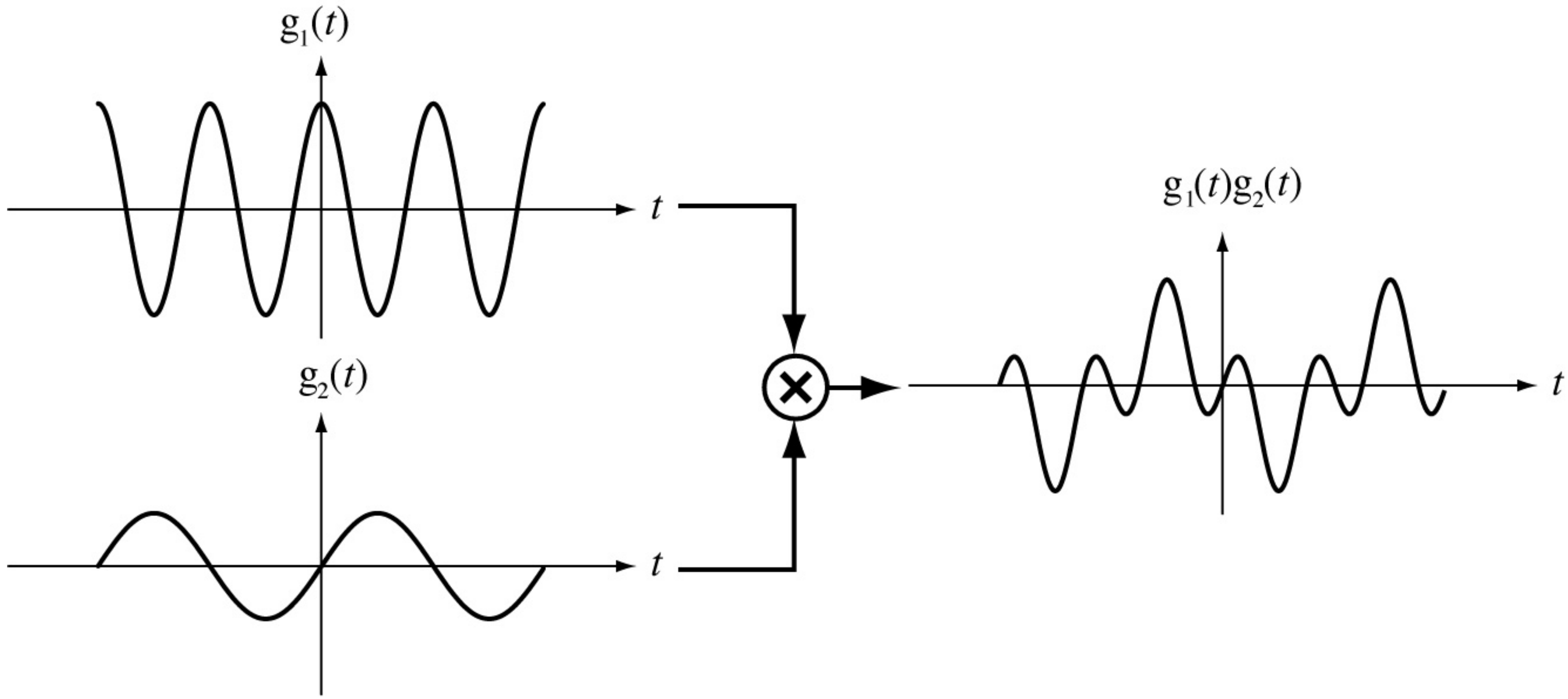


❖ Combinations of Odd and Even Functions

Two Even Functions

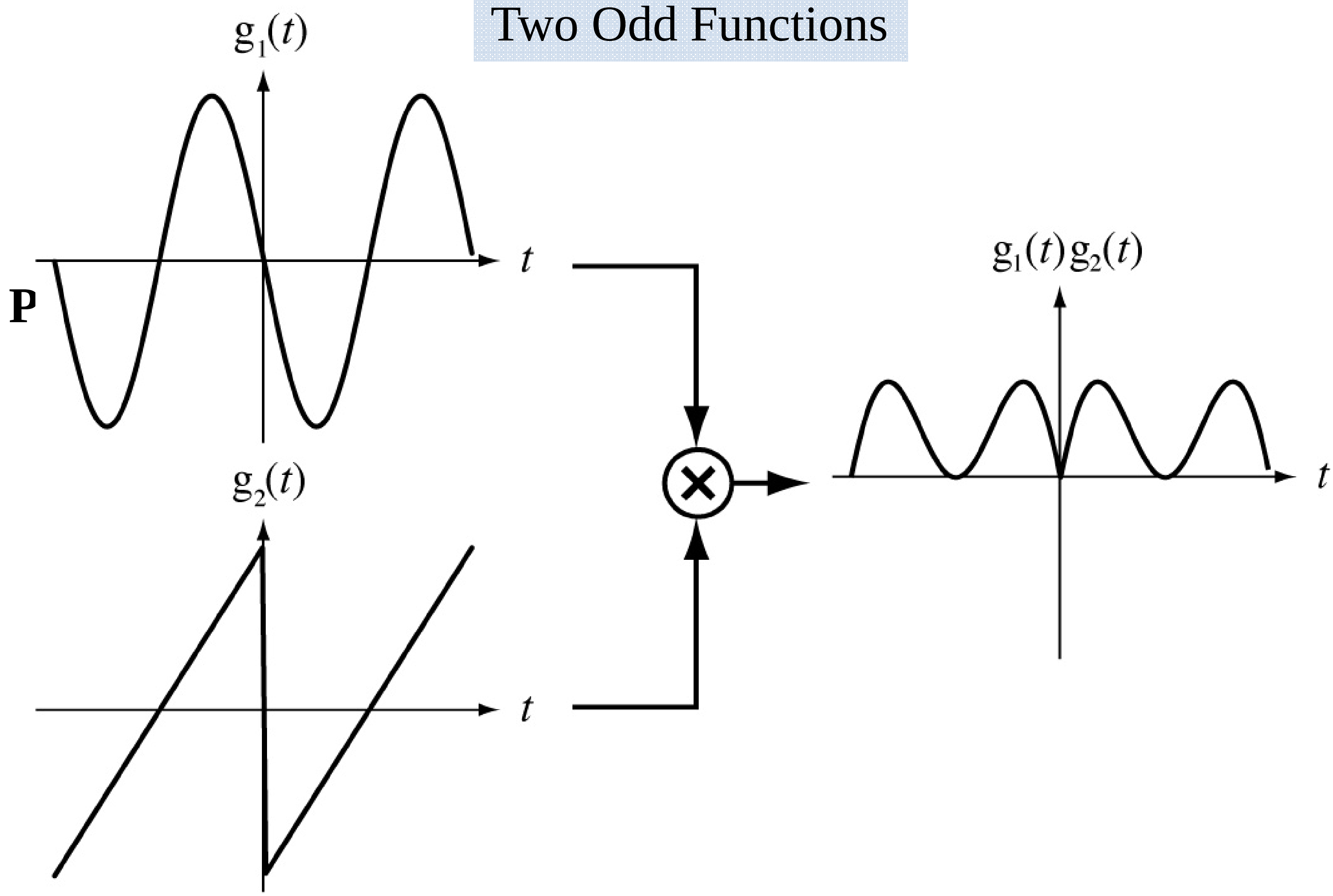


An Even Function and an Odd Function

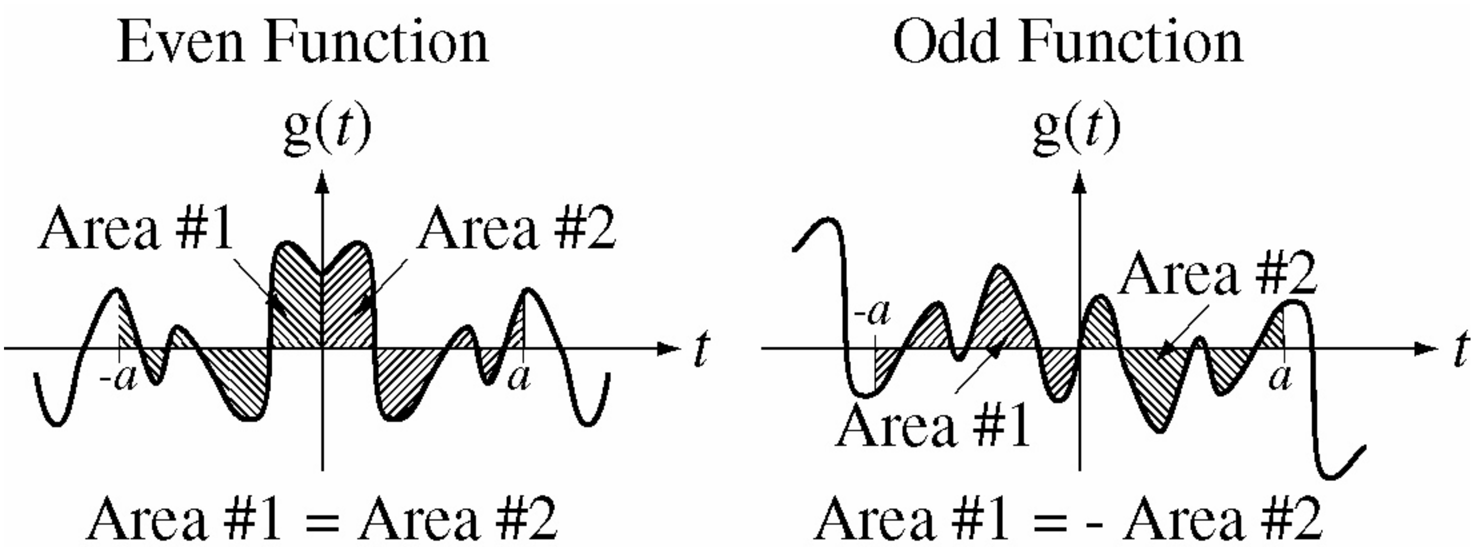


Products of Even and Odd Functions, cont'

Two Odd Functions



❖ Integrals of Even and Odd Functions (over symmetrical limits)



$$\int_{-a}^{a} g(t)dt = 2\int_{0}^{a} g(t)dt$$

$$\int_{-a}^{a} g(t)dt = 0$$

Integrals of Even and Odd Functions, cont'

Evaluate the integral

$$I = \int_{-10}^{10} 4\,\mathrm{rect}(t/8)\, e^{j2\pi t/16}\, dt$$

$$I = 4\int_{-4}^{4}\left[\underbrace{\cos(\pi t/8)}_{\text{even}} + j\underbrace{\sin(\pi t/8)}_{\text{odd}}\right]dt = 8\int_{0}^{4}\cos(\pi t/8)\,dt + \underbrace{j8\int_{-4}^{4}\sin(\pi t/8)\,dt}_{=0}$$

$$I = 8\left[\frac{\sin(\pi t/8)}{\pi/8}\right]_0^4 = \frac{64}{\pi}[1-0] = \frac{64}{\pi} \cong 20.372$$

❖ **Examples**:

For signals shown below: 1)find odd and even parts, 2) find integrals

$$x(t) = \sin(2\pi t), \qquad y(t) = \sin(2\pi t + \frac{\pi}{3})$$

$$z(t) = \begin{cases} 2\sin(2\pi t) & \text{t}>0 \\ 0 & \text{o}w \end{cases}$$

❖ Derivatives of Even and Odd Functions

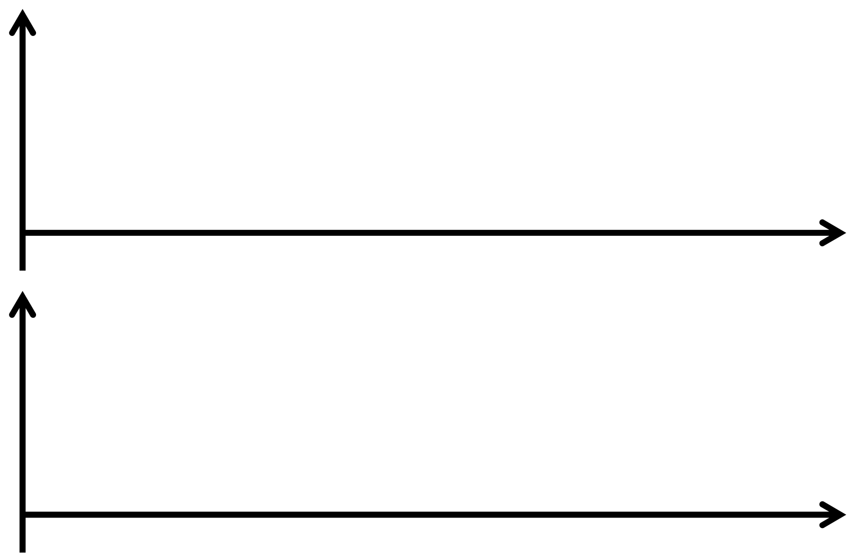
Property: Derivative of an even function is odd and vice versa

Proof:

❖ Signal Operations: Time Shift

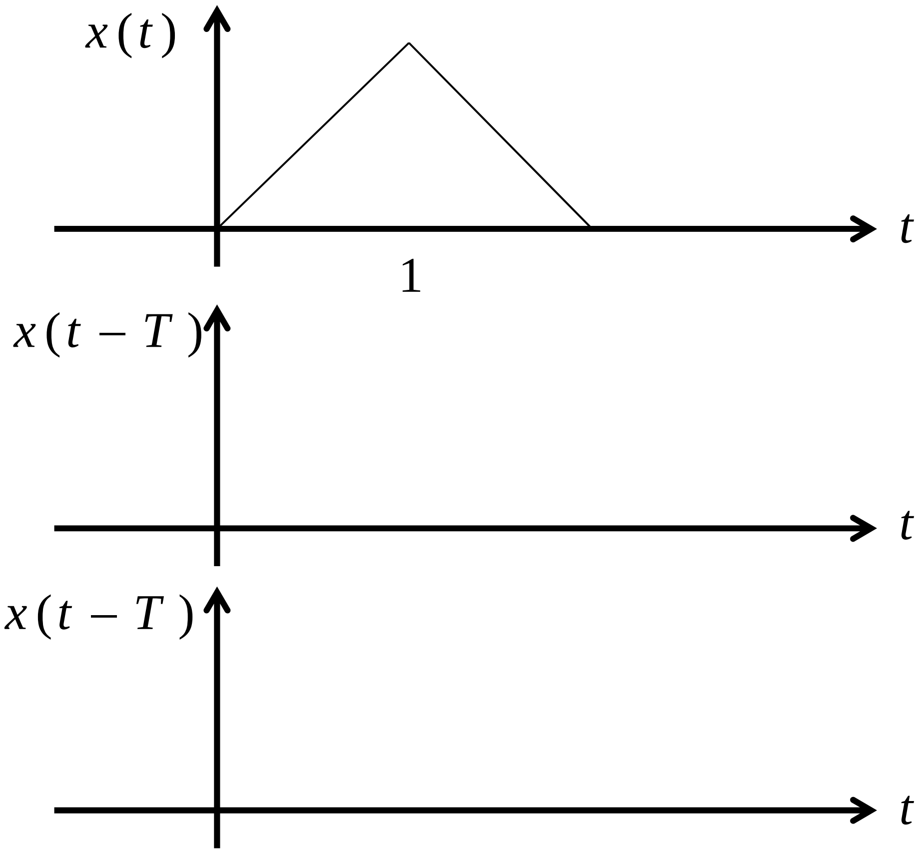
$$x(t) = \begin{cases} 2t & 0 \leq t \leq \dfrac{1}{2} \\ \dfrac{(4-2t)}{3} & \dfrac{1}{2} \leq t \leq 2 \\ 0 & \text{ow} \end{cases}$$

$$x_1(t) = x(t-2)$$



Signal Operations: Time Shift, cont'

- **Generic Time Shift** $x_1(t) = x(t - T)$

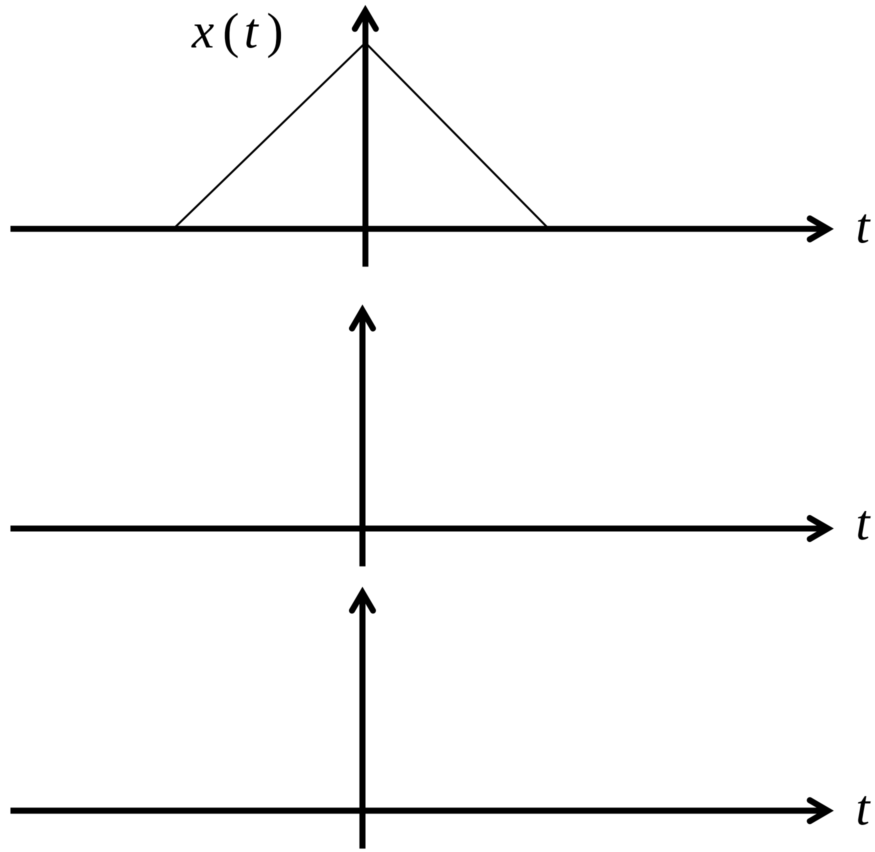


$T > 0$

$T < 0$

❖ Signal Operations: Time Scaling

$$x_1(t) = x(at)$$

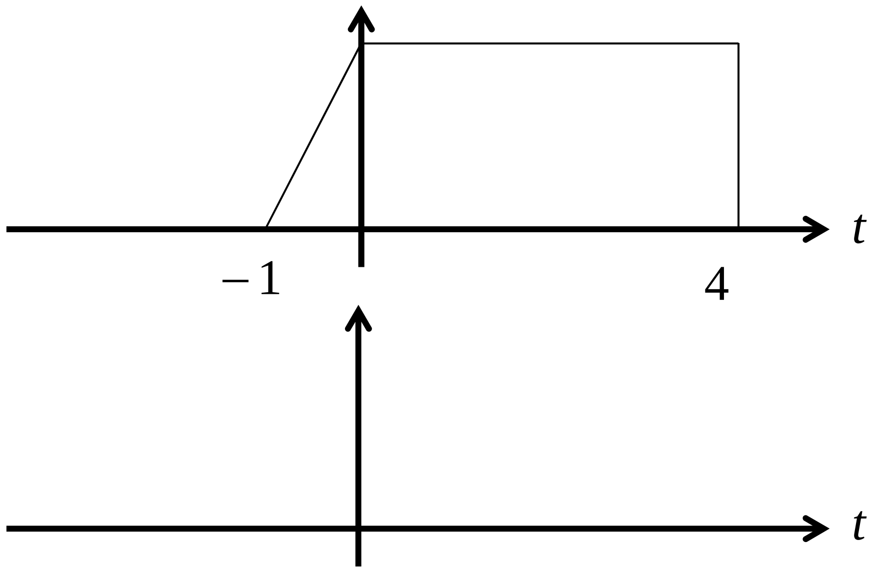


$$a = \frac{1}{2}$$

$$a = 2$$

❖ Signal Operation: Time Reversal

$$x_1(t) = x(-t)$$

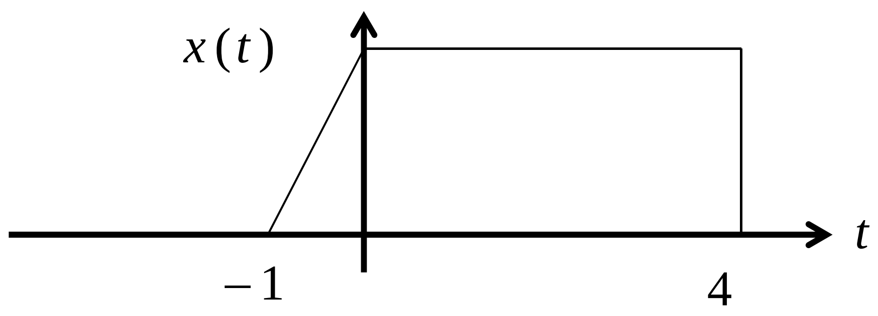


❖ Signal Operations: Combining Time Shifting & Scaling

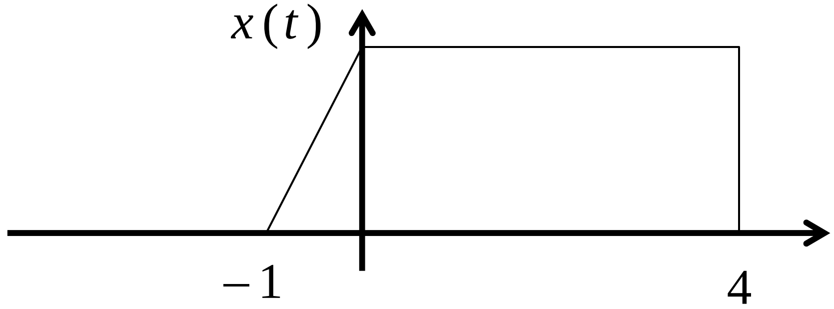
$$x_1(t) = x(at - b) \qquad a,\ b > 0$$
$$= x(a(t - b/a))$$

1. Scale $x(t)$ by a
2. Shift result by $\frac{b}{a}$

$$x_1(t) = x(2t - 4)$$



Combining Time Shifting & Scaling, cont'



$$x_1(t) = x(2t - 4)$$

❖ Signal Operation: Frequency Shifting

$$x(t) = e^{jw_oT}$$

$$y(t) = \underbrace{e^{jw_oT}.e^{j\phi t}} = e^{j\underbrace{(w_o+\phi)}t}$$

Called Modulation

New Frequency $w_0 + \phi$

❖ Energy / Power Signals

- Definition: The energy in a signal $x(t)$ over the interval $T = [t_1, t_2]$ is defined as

$$E_T = \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

Real Signal:
Complex Signal:

- Definition: The energy in a signal $x(t)$ is defined as:

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt$$

- Definition: A signal is called an Energy signal if it has finite energy.

Energy / Power Signals, cont'

- Definition: The power of a signal $x(t)$ over the interval $T = [t_1, t_2]$ is defined as

$$P_T = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

- Definition: The power of a signal $x(t)$ is defined as:

$$P_x = \lim_{T \to +\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

- Definition: A signal is called a power signal if it has finite power.

Energy / Power Signals, cont'

- Question: Can a signal have both finite energy and non zero power?

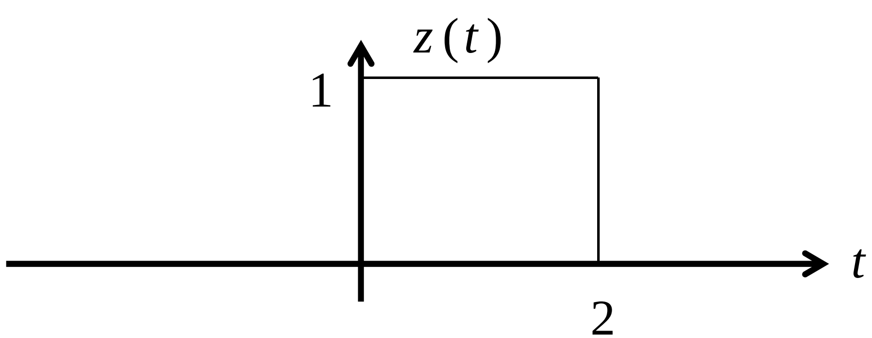
❖ Examples: Energy / Power Signals

- Find the energy and power of

$$- x(t) = \cos(\pi t + \frac{\pi}{3})$$

$$- y(t) = \begin{cases} 3e^{j\pi t/2}, 0 \le t \le 10 \\ 0 \quad\quad ow \end{cases}$$

$- z(t)$ defined as:



Energy / Power Signals, cont'

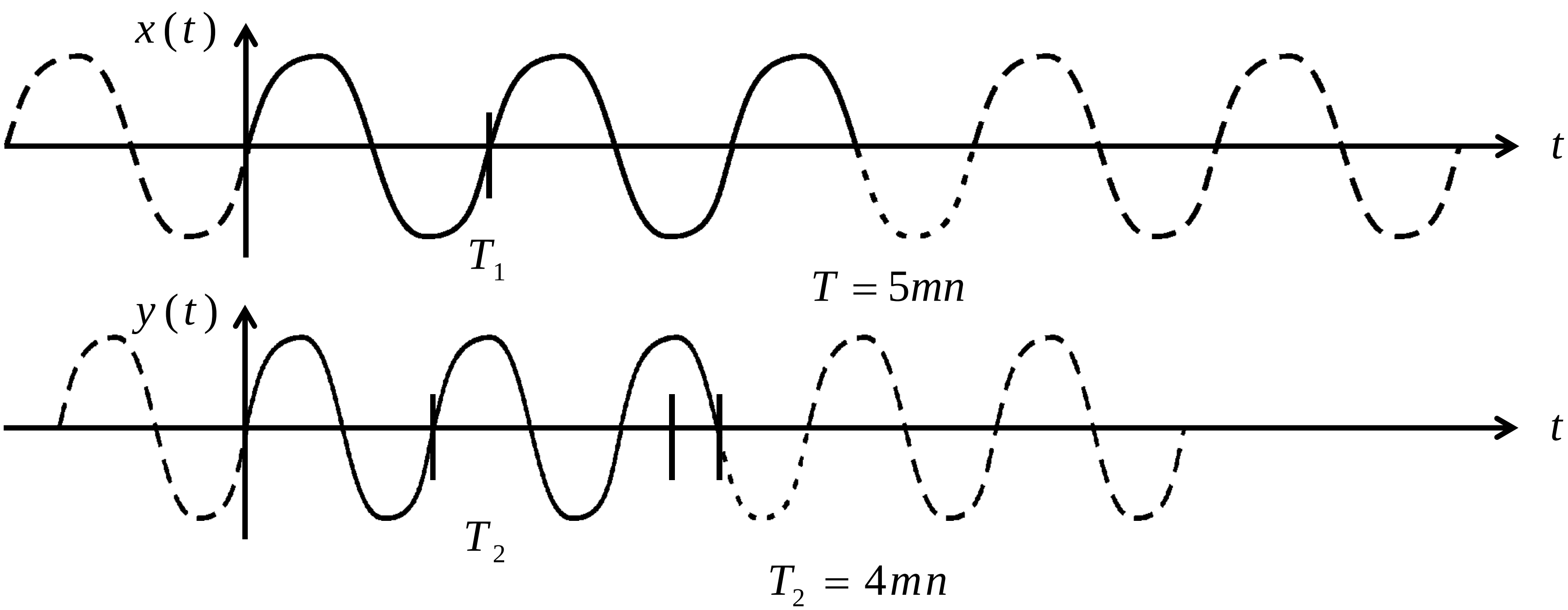
- Property If $x(t) = \sum_k A_k \cos(\omega_k t)$

$$P_x = \sum_k P_{x_k}$$

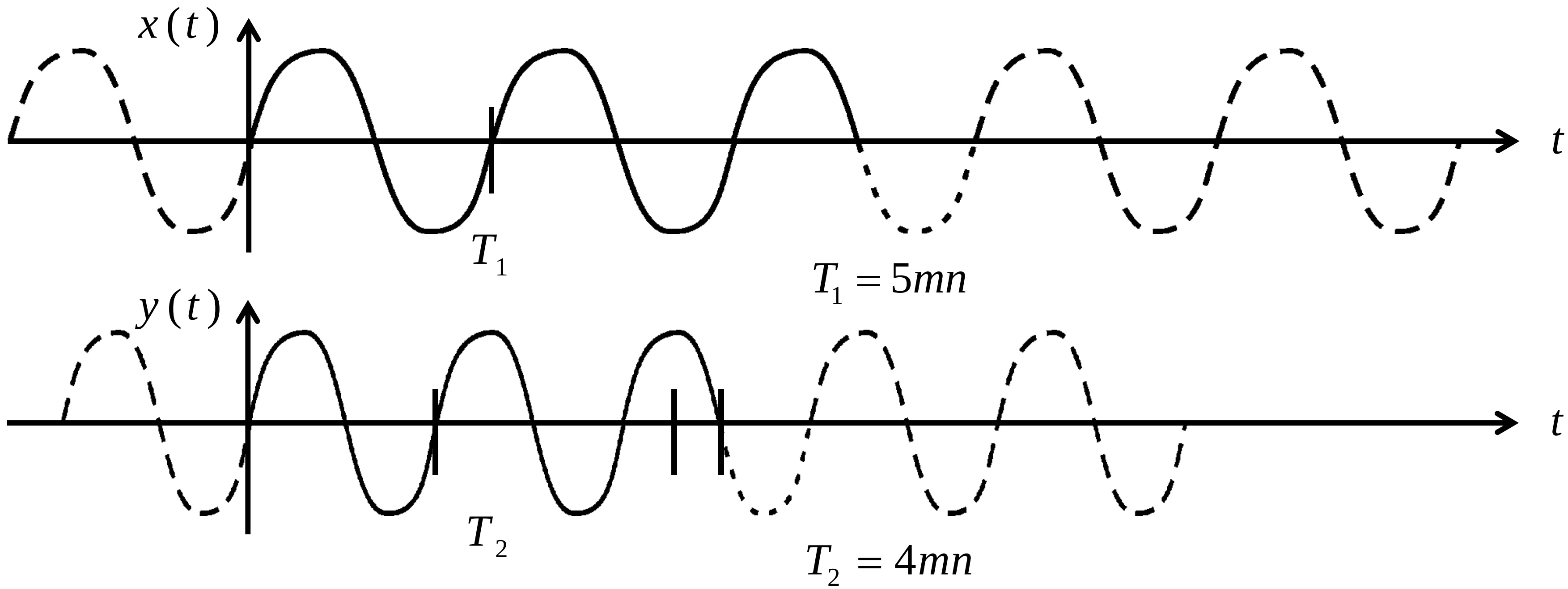
$$P_{x_k} = \text{Power}\left\{ A_k \cos(\omega_k t) \right\} =$$

❖ Example: Tone Crunch

Assume you are given a tone frequency f_0 *Hz* of duration to 5*mn*. Assume a full number of periods within 5mn. Assume you want to crunch the signal into a 4*mn* window. How does that affect the signal frequency?



Example cont', Tone Crunch



Conclusion: If $x(t)$ is periodic with period T_1, the time scaled signal $x(at)$ is also periodic with period T_2=

- **Example: Amplitude Modulation**