

The Military Bulk Fuel Distribution Problem

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ABSTRACT

Maintaining U.S. military operations worldwide requires vast quantities of fuel to support aircraft, ships, and ground equipment. Military planners routinely evaluate the requirements for moving and storing fuel and the feasibility of satisfying these demands, particularly for illustrative scenarios or to prepare for potential conflicts. This article integrates several decades of applied research into a sequence of models representing best practices for assessing capability and planning investment in bulk fuel operations. Our case study shows how such modeling can aid an analyst in understanding the capabilities of a supply chain and/or advise investment in relevant infrastructure to support such operations.

INTRODUCTION

The United States military operates around the globe with (as of February 2024) up to 170,000 of its active duty personnel stationed outside of the United States ([USA Facts, 2024](#)). Maintaining U.S. military operations worldwide is an essential part of the U.S. goal to promote and strengthen democracy. Overseas military installations and equipment they support (e.g., diesel-powered ships, fighter jets) require substantial fuel. In fiscal year (FY) 2023, the U.S. Department of Defense (DOD) used 70.8 million barrels of fuel across the agency, which equates to approximately 194,000 barrels, or 194 Mbbl, per day ([U.S. Department of Defense, 2024](#)). (Each Mbbl, the basic unit for measuring crude oil, is one thousand standard barrels of 42 U.S. gallons.)

The U.S. military routinely evaluates plans for potential conflict, including requirements to move and store fuel. DOD Instruction 4140.25 “establishes policy, assigns responsibilities, and provides procedures for sizing, acquiring, and managing petroleum operating stocks and prepositioned war reserve stocks (PWRS)” ([U.S. Department of Defense, 2015](#)). For any given planning scenario, there are requirements that specify the fuel type, amount, location, and date (typically at daily resolution). Logistics planners take these requirements and seek a feasible delivery plan with available assets. The modeling presented in this article has been used to assist planners with this task.

Specifically, our modeling assists in answering several strategic questions: Are the available transportation and

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OR METHODS:

Inventory Optimization, Linear/Integer Programming, Data Science

storage assets sufficient to satisfy a schedule of fuel requirements? If not feasible, what changes to initial inventory, storage capacity, demand, and/or assets produce a feasible plan? Once feasible, how robust (or vulnerable) is this capability to the loss of one or more assets in the fuel system? Knowing existing vulnerabilities is crucial in justifying the need for improvements.

Our modeling has typically focused on three fuel types (F76, JP5, and JP8), with the greatest military requirements being delivered via several different transportation modes. F76 (or NATO F-76, also called Diesel Fuel Marine [DFM]) is a naval distillate typically used in shipboard diesel and gas turbines. JP8 (Jet Propellant-8, also called NATO F-34) is a kerosene-based jet fuel used by the U.S. Air Force and U.S. Army, similar to conventional petroleum-derived middle distillate aviation fuel (i.e., Jet A, Jet A-1) but with additives for de-icing and corrosion inhibitors. JP5 (Jet Propellant-5, also called NATO F-44) is a kerosene-based jet fuel with a higher flash point for carrier-based aircraft. JP5 and JP8 are sometimes substitutable but must be stored and transported separately (see [U.S. Department of Defense, 2013](#), for details on the specified use of each fuel type).

Historically, the DOD has operated tankers of different sizes. T5 tankers (decommissioned in 2019) carried multiple commodities with a total capacity of up to 235 Mbbls, whereas smaller T1 tankers and barges have a capacity of approximately 30 Mbbls of fuel each. Single commodity railcars and tank trucks have carrying capacity of 276 bbls and 119 bbls, respectively. Pipelines provide an additional delivery mode.

Military operations conducted over vast areas of air and sea require large amounts of fuel. This is particularly true in the area around the Indian and Pacific Oceans (known as the Indo-Pacific). The quantities of fuel are so large that simply distributing sufficient quantities to military locations is nontrivial. Bulk fuel planning is further complicated by competition and conflict among the many different nations that host installations and service ports, as well as uncertainty in the availability of commercial fuels ([McLemore and Alderson, 2018](#)).

Broadly speaking, there are two extreme strategies for fuel storage and delivery. One strategy involves prepositioning all the necessary fuel at forward operating locations. This strategy suffers for several reasons. First, large amounts of fuel translate to extremely high inventory costs that can be financially prohibitive. Second, fixed fuel locations are attractive targets for adversaries who seek to interfere with U.S. military operations, or they could become assets to be seized during a conflict. Finally, there are environmental risks of storing large amounts of fuel, as evidenced by the November 2021 leak from the Red Hill Bulk Fuel Storage Facility in Hawaii, which affected approximately 93,000 U.S. Navy water system users ([U.S. Environmental Protection Agency, 2024](#)).

At the other extreme, one can imagine a zero inventory strategy involving just-in-time delivery. This strategy suffers because it is susceptible to uncertainties in demands and transportation times. Somewhere in the middle is a strategy that moves and stores the right amounts of fuel so that they are available at the right places at the right times. What constitutes the “right” amount is a strategic question, but one that can be addressed using operational models.

This article integrates several decades of applied research into a sequence of models representing best practices for assessing capability and planning investment in bulk fuel operations. After briefly reviewing the relevant historical literature, we provide a baseline model that captures what we believe to be essential features of the problem. We then describe a series of model extensions that can be instrumented to address specific features of interest. We illustrate the setup and execution of our model with a notional case study, and we use this case study to demonstrate how such modeling can aid an analyst in understanding the capabilities of a supply chain and/or advise investment in relevant infrastructure to support such operations.

BACKGROUND

Some of the earliest articles in the newly established operations research (OR) journals considered military applications with fuel delivery being among these applications. Specifically, [Dantzig and Fulkerson \(1954\)](#) show how to formulate a linear program to minimize the number of tankers required to deliver Navy fuel subject to a required pickup and delivery schedule. Despite the early

introduction of optimization-based modeling to military bulk fuel distribution problems, there is little in the openly published OR literature that reports on real-world application. In this section, we review the few that do report real-world application and provide a summary of some seminal ship routing and scheduling survey articles.

The vast majority of the OR literature on ship routing and scheduling consider commercial applications. [Christiansen et al. \(2004, 2013\)](#) and [Ronen \(1983, 1993\)](#) provide surveys of this literature that were published roughly every decade and collectively cover almost 300 articles. In the most recent of these surveys, [Christiansen et al. \(2013, p. 468\)](#) group this vast literature into liner, industrial, and tramp operational modes: “Liner vessels follow a fixed route according to a published schedule trying to maximize profit, similar to a public bus service. An industrial operator owns the cargo and controls the ships, trying to minimize the cost of delivering the cargoes, similar to a private fleet. In a tramp operation the vessels follow the available cargoes (some of which may be optional), trying to maximize profit, similar to a taxi cab.” Within these modes, industrial and more specifically the maritime inventory routing problem most closely resembles models covered in this article. [Christiansen et al. \(2013\)](#) define the maritime inventory routing problem as “a planning problem where an actor has the responsibility for both the inventory management at one or both ends of the maritime transportation legs, and for the ships’ routing and scheduling.” See [Li et al. \(2022\)](#) for a recent maritime inventory routing problem application for liquefied natural gas. In contrast to the typical commercial maritime inventory routing problem, our modeling must consider multiple modes of delivery, different commodities, and a primary objective that seeks to maximize demand satisfaction with minimal delays.

[Brown and Carlyle \(2008\)](#) and [Brown et al. \(2017\)](#) report on real-world application of integer linear programming to plan use of the Navy Combat Logistics Force (CLF). The CLF is a heterogeneous fleet of about 30 ships that provide delivery at sea of multiple commodities (e.g., aircraft fuel, ordnance, food). [Brown and Carlyle \(2008\)](#) report on the application of their modeling to plan strategic fleet decisions and investment in relevant infrastructure to support operations. [Brown et al. \(2017\)](#) report on the real-world daily operation of their models for peacetime operations and convey some distinguishing differences between routing and scheduling of the CLF and civilian ships. Our effort shares some of these distinguishing differences such as “our ship routes are not pre-cataloged in standard shipping guides as are those for commercial vessels” ([Brown et al., 2017, p. 680](#)). In contrast to their CLF modeling, we consider fuel delivered by multiple modes as required for land bases operating overseas during peacetime and in preparation for potential conflict scenarios. Other real-world scheduling of Navy ships include [Salmerón et al. \(2011\)](#), who report on the use of an integer linear program to plan humanitarian missions. Unlike our focus on fuel, they schedule the best use of engagement teams to multiple locations subject to limited resources (including time).

[Salmerón et al. \(2009\)](#) formulate and implement a stochastic integer linear program to plan shipment of military cargo packages for conflict scenarios. Their modeling assumes a fixed delivery schedule is required for cargo packages that include pickup and delivery pairings. They also include probabilistic information on the time and location of potential enemy attacks on delivery locations.

Since the late 1990s, the authors have advised more than 20 Naval Postgraduate School theses that have modeled worldwide fuel distribution, many of which are focused on the Indo-Pacific region. See [Alderson et al. \(2017\)](#) for more details on this collection. Additional theses at the Army War College (e.g., [Thoreson, 2013](#); [Woodard, 2013](#)) focus on the Army force structure needed to support inland bulk fuel distribution.

FORMULATION: BASELINE MODEL

Following the guidelines described in [Brown and Dell \(2007\)](#) and [Teter et al. \(2016\)](#), we present a baseline operator model with indexed sets, parameters, variables, objective, and constraints. Despite its relative simplicity, this baseline linear programming model has been very effective at helping planners answer strategic questions.

Indices and Sets

$t \in \mathcal{T}$	Ordinal time period (alias t')
$n \in \mathcal{N}$	Node (aliases i, j)
$(i, j) \in \mathcal{A}$	Directed arcs representing physical product flow from node i to node j
$(i, t, j, t') \in \mathcal{F}$	Allowable flows on arcs (i, j) , departing at node i at time period t and arriving at the node j at some later time period, $t' \geq t$
$p \in \mathcal{P}$	Product (e.g., F76, JP5, JP8)

Parameters [units]

dem_{nt}^p	Demand for product p at node n at start of time t (nonnegative)	[bbbls]
sup_{nt}^p	Supply of product p at node n at start of time t (nonnegative)	[bbbls]
$init_n^p$	Initial inventory of product p in node n at $t = 0$ (nonnegative)	[bbbls]
$ncap_n^p$	Maximum storage capacity of product p at node n	[bbbls]
$acap_{ijt'}^p$	Maximum flow capacity of product p on (i, t, j, t')	[bbbls]
$cost_{ijt'}^p$	Cost per unit to move product p on (i, t, j, t')	[cost/bbl]
$nper_n^p$	Maximum number of days allowed to hold a backlog of product p at node n	[integer]
$dpen_{nt}^p$	Penalty for not satisfying demand of product p at node n at time t	[cost/bbl]
$bpen_{nt}^p$	Penalty for accumulating backlog demands for product p at node n at time t	[cost/bbl]
$epen_{nt}^p$	Penalty for excessive (lost) product p at node n at time t	[cost/bbl]

Variables [units]

$F_{ijt'}^p$	Flow of product p from node i at time t to node j at time t'	[bbbls]
C_{nt}^p	Carryover inventory of product p at node n held over from time period $t-1$ into time t	[bbbls]
B_{nt}^p	Backlog of cumulative unmet demands of product p at node n at end of time period t	[bbbls]
U_{nt}^p	Unmet demand of product p at node n at end of time period t	[bbbls]
E_{nt}^p	Excess product p at node n at end of time period t	[bbbls]

Formulation of Operator's Model

For a fixed product p , we have the following:

$$\min_{F, C, B, U, E} \sum_{(i, t, j, t') \in \mathcal{F}} cost_{ijt'}^p F_{ijt'}^p \quad (\text{A01})$$

$$+ \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} (bpen_{nt}^p B_{nt}^p + upen_{nt}^p U_{nt}^p + epan_{nt}^p E_{nt}^p), \quad (\text{A02})$$

subject to:

$$\begin{aligned} & \sum_{\substack{(j, t'): \\ (n, t, j, t') \in \mathcal{F}}} F_{ntjt'}^p - \sum_{\substack{(i, t'): \\ (i, t', n, t) \in \mathcal{F}}} F_{it'nt}^p + C_{nt}^p - C_{n(t-1)}^p \\ & + B_{n(t-1)}^p - B_{nt}^p - U_{nt}^p + E_{nt}^p = sup_{nt}^p - dem_{nt}^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, \end{aligned} \quad (\text{A1})$$

$$\sum_{\substack{(j,t'): \\ (n,t,j,t') \in \mathcal{F}}} F_{ntjt'}^p + C_{nt}^p \leq \sum_{\substack{(i,t'): \\ (i,t',n,t) \in \mathcal{F}}} F_{it'nt}^p + C_{n(t-1)}^p + sup_{nt}^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, \quad (A2)$$

$$F_{ijjt'}^p \leq acap_{ijjt'}^p \quad \forall (i,t,j,t') \in \mathcal{F}, \quad (A3)$$

$$B_{nt}^p \leq \sum_{t-npn^p+1}^t dem_{nt}^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, \quad (A4)$$

$$U_{nt}^p \leq dem_{nt}^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, \quad (A5)$$

$$C_{nt}^p \leq ncap_n^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, \quad (A6)$$

$$F_{ijjt'}^p \geq 0 \quad \forall (i,t,j,t') \in \mathcal{F}, \quad (A7)$$

$$U_{nt}^p \geq 0, E_{nt}^p \geq 0, B_{nt}^p \geq 0 \quad \forall n \in \mathcal{N}, t \in \mathcal{T}. \quad (A8)$$

Discussion

The objective function includes (A01) the cost of moving product through the network and (A02) penalties for backlogged demand, unmet demand, and excess fuel that cannot be used or stored, respectively. Following that, we require that $upen_n > bpen_n$ such that having unmet demand is always more costly than backlogging it to be satisfied later. Given that this system “pulls” product to satisfy demand, we do not expect there to be excess fuel that would need to be dumped without use.

Constraints (A1) ensure balance of flow at each node at each time period, as Figure 1 illustrates. Constraints (A2) further restrict outbound physical flows and carryover inventory to be less than the inbound physical flows, carryover, and supply (i.e., variables for backlog or unmet demand cannot be used to generate artificial flows elsewhere). Constraints (A3) enforce capacity limits for each product on each arc. Constraints (A4) limit the amount of backlogged demand for product p at node n to at most npn^p periods. Constraints (A5) bound the amount of unmet demand at each node to the actual demand. Constraints (A6) limit the amount of unmet demand at each node to the actual demand. Constraints (A7) limit the amount of product that can be carried forward in time to the holding capacity at that node. Stipulations (A7) and (A8) enforce nonnegativity of all variables.

In the parlance of recent inventory modeling (e.g., Adak and Mahapatra, 2022), we are implementing a model with partial backlogs and lost sales and have been using such modeling for

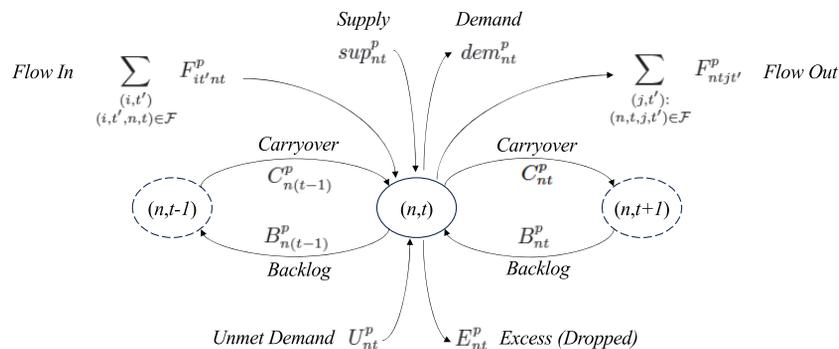


Figure 1. A graphical representation of constraint (A1), balance of flow. For a given product p , flow moves in space and time between nodes indexed by (n,t) .

decades. In a military context, fuel can be needed for specific time-sensitive missions. This mission could potentially be delayed for a short period of time but not indefinitely. Keeping the length of the delay as a user input has proven to support planners tasked with real-world satisfaction of fuel.

There are several cost parameters ($cost_{ijt}^p$, $bpen_{nt}^p$, $upen_{nt}^p$, $epen_{nt}^p$) required to support our modeling. Typically, $cost_{ijt}^p$ is just the cost of transit between a pair of nodes. The index t allows for the inclusion of coefficients that discount costs in time and can model the “fog-of-far-future planning” (e.g., Brown et al., 2004), where more importance is given to early decisions. This “cost” could have an associated risk element that represents possible loss of an asset for a given route. In general, $bpen_{nt}^p < upen_{nt}^p$. We would backlog demand that could be satisfied by future delivery, so $nper_n^p \cdot bpen_{nt}^p < upen_{nt}^p$. Satisfying demand is paramount, so even a backlog should be avoided if possible ($cost_{ijt}^p < bpen_{nt}^p$). Lastly, $epen_{nt}^p$ is a minor consideration that should rarely happen because other costs discourage distribution of excess product.

This baseline model provides a relatively complete representation for studying many operational issues, particularly the basic movement and storage of fuels to satisfy a particular schedule of demands over a fixed time horizon. However, the real world often poses additional constraints or questions to be addressed. Fortunately, this baseline model can be extended to address these issues. Each of the following subsections provides model extensions that can be added individually or together. The resulting formulation gets increasingly detailed, largely due to the overhead associated with managing the supporting input data, but not fundamentally more complicated.

EXTENSION: SAFETY STOCK LEVELS

It is common to have requirements for minimum fuel quantities, often called “safety stocks.” These can be implemented by creating penalties in the objective for any shortfalls below a specified inventory level. We augment our baseline model with the following additional parameters, variables, objective terms, and constraints.

Additional Parameters [units]

$safe_n^p$	Minimum fractional amount of storage capacity for product p to be maintained as safety stock at node n	[fraction]
$spen_n^p$	Per-unit penalty for falling below safety stock level of product p at node n	[cost/bbl]

Additional Variables [units]

S_{nt}^p	Amount below safety stock level of product p at node n at end of time t	[bbls]
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Additional Term in Objective Function

$$+ \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} spen_n^p S_{nt}^p \quad (\text{A02.1})$$

Additional Constraints

subject to:

$$S_{nt}^p + C_{nt}^p \geq safe_n^p ncap_n^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, p \in \mathcal{P}, \tag{A5.1}$$

$$S_{nt}^p \geq 0 \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, p \in \mathcal{P}. \tag{A8.1}$$

Discussion

The term (A02.1) is an additive penalty at each node for falling below the specified safety stock level. The decimal added in the equation level (e.g., “.1” added to A02.1) suggests the location where the equation would occur in the original baseline model (after A02). In general, we require $spen < bpen < upen$, such that the first priority is minimizing unmet demand, then satisfying backlogged demand, and then maintaining safety stock levels. That is, we would always dip into safety stock before not satisfying a requirement, even if only temporarily. Constraints (A5.1) calculate safety stock shortfalls as a fraction of the storage capacity at each node. Stipulation (A8.1) maintains the nonnegativity of these shortfall variables. The addition of safety stock penalties also helps to avoid “end effects” in fixed time horizon models. That is, instead of a fuel schedule that ends with zero inventory at the end of the given time horizon, the model is incentivized to end with the specified safety stock level.

EXTENSION: INTERDICTION

When considering the strategic risk to the movement and storage of fuels, it is often important to consider the implications of potential *interdiction* to the supply network. The following extension implements *cost-based interdiction* that makes using an interdicted arc or node prohibitively expensive.

Additional Parameters [units]

$apen_{ij}$	(Artificial) Per-unit penalty for flow on an interdicted arc (i, j)	[cost/ bbl]
$npen_n$	(Artificial) Per-unit penalty for using interdicted node n	[cost/ bbl]
$\hat{X}_{ijt'}$	Indicates movement of flow from node i at time t to node j at time t' has been interdicted (=1), 0 otherwise	[binary]
\hat{Y}_n	Indicates whether a node has been interdicted (=1), 0 otherwise, preventing use of node n	[binary]

Additional Term in Objective Function

$$\sum_{(i,t,j,t') \in \mathcal{F}} \left(cost_{ij}^p + apen_{ij} \hat{X}_{ijt'} + npen_i \hat{Y}_i^p + npen_j \hat{Y}_j^p \right) F_{ijt'}^p. \tag{A01r1}$$

The equation label suffix “r1” signifies replacing (A01) with (A01r1) in the original baseline model.

Discussion

The updated objective (A01r1) adds the additional cost of moving flow along interdicted arcs or those incident to an interdicted node. The intent is that these incremental costs are sufficiently high to prevent the optimal solution from including such movement.

There is a long history of interdiction models for assessing the impact of losing one or more arcs in a network flow problem; for example, see Harris and Ross (1955) as documented by Schrijver (2002), as well as the treatment in Alderson et al. (2013). In practice, the interdiction of an arc could

result from a deliberate event (e.g., adversarial attack) or a nondeliberate one (e.g., accident, failure, or extreme weather event), and the model here is instrumented to be agnostic to the cause. In practice, interdiction could mean a complete interruption of flow or a partial blockage, resulting in higher costs. Interdiction can be applied to nodes or arcs without loss of generality.

There is often a desire to discover the combination of one or more components that results in a worst-case degradation in capability. The study of worst-case interdiction began with seminal work by Wood (1993) and its connection to game theory (Washburn and Wood, 1995). Over the last 30 years, this work on so-called “attacker-defender (AD) models” has since expanded in application to include critical infrastructure systems (e.g., Brown et al., 2006; Alderson et al., 2014). The use of indicator variables here, $\hat{X}_{ijt'}$ and \hat{Y}_m , makes the formulation suitable for inclusion in AD models documented in the literature (e.g., Alderson et al., 2015).

While it is possible to restrict flow by setting arc capacities to zero (i.e., modifying constraint (A3) so that the interdiction indicator variable sets arc capacity to zero, $F_{ijt'}^p \leq (1 - \hat{X}_{ijt'}) \text{acap}_{ijt'}^p$ for all $(i, t, j, t') \in \mathcal{F}$), this can cause problems when solving bi-level AD formulations because intermediate solutions can become infeasible. Instead, solutions to cost-based interdiction formulations are always feasible, although perhaps expensive (see also the discussion in Alderson et al., 2014). Note that indexing for $\hat{X}_{ijt'}$ allows for interdiction at the level of individual flows in space and time (i.e., for specific shipments). If interdiction affects all flows along arc (i, j) , then a simplified indicator \hat{X}_{ij} suffices.

EXTENSION: MULTIPLE MODES

Delivering fuel to installations requires multiple modes, and we have typically modeled five different ones: tanker, barge, rail, truck, and pipeline. The modeling is made easier when we have only one mode of delivery possible between a pair of locations. In such cases, the delivery capacity is simply what is provided by the mode, and there is no need to have the mode explicitly represented in the model. Explicit mode representation is needed when mode options between node pairs exist, and the modeling helps answer how best to employ these modes. For example, are rail cars needed, or is an existing pipeline sufficient to deliver between two locations?

Additional Index

$m \in \mathcal{M}$ Mode of transport (e.g., tanker, barge, rail, truck, pipeline)

Revised Indices and Sets

$(i, j, m) \in \mathcal{A}$	Directed arcs representing physical product flow from node i to node j using model m
$(i, t, j, t', m) \in \mathcal{F}$	Allowable flows on arcs (i, j) , departing at node i at time period t and arriving at the node j at some later time period, $t' \geq t$ using model m

Revised Parameters [units]

$\text{acap}_{ijt'm}^p$	Maximum flow capacity of product p on (i, t, j, t', m)	[bbbls]
$\text{cost}_{ijt'm}^p$	Cost per unit to move product p on (i, t, j, t', m)	[cost/bbl]

Revised Variables [units]

$F_{ijt'm}^p$	Flow of product p from node i at time t to node j at time t' using mode m	[bbbls]
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Revised Formulation

$$\min_{F,C,B,U,E} \sum_{(i,t,j,t',m) \in \mathcal{F}} \text{cost}_{ijt'm}^p F_{ijt'm}^p \tag{A01r2}$$

$$+ \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} (bpen_{nt}^p B_{nt}^p + upen_{nt}^p U_{nt}^p + epen_{nt}^p E_{nt}^p). \tag{A02}$$

subject to:

$$\sum_{(j,t'): (n,t,j,t',m) \in \mathcal{F}} F_{ntjt'm}^p - \sum_{(i,t'): (i,t',n,t,m) \in \mathcal{F}} F_{it'ntm}^p + C_{nt}^p - C_{n(t-1)}^p + B_{n(t-1)}^p - B_{nt}^p - U_{nt}^p + E_{nt}^p = \text{sup}_{nt}^p - \text{dem}_{nt}^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, \tag{A1r1}$$

$$\sum_{(j,t'): (n,t,j,t',m) \in \mathcal{F}} F_{ntjt'm}^p + C_{nt}^p \leq \sum_{(i,t'): (i,t',n,t,m) \in \mathcal{F}} F_{it'ntm}^p + C_{n(t-1)}^p + \text{sup}_{nt}^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, \tag{A2r1}$$

$$F_{ijt'm}^p \leq \text{acap}_{ijt'm}^p \quad \forall (i, t, j, t', m) \in \mathcal{F}. \tag{A3r1}$$

Discussion

Very little has changed from the baseline model. The expression in (A01r2) represents the cost to flow commodities over multiple transport modes, with penalties in (A02) unchanged. Balance of flow in (A1r1) and outbound flow restrictions (A2r1) now reflect individual modes. There are now upper bounds on flows per mode in (A3r1).

EXTENSION: FUEL SUBSTITUTIONS AND FUEL FLOW CAPACITY

Up to now, it has been assumed that fuel products do not interact, and therefore, each fuel product can be solved independently. This assumes fixed transport capacity for each fuel type and allows for no fuel substitutions. In reality, JP5 and JP8 are substitutable, and more broadly each fuel type can compete for transport or storage capacity. It is possible to adjust the capacity for each fuel type subject to some overall limit. To do so, we augment our baseline model with the following additional parameters, variables, objective terms, and constraints.

Additional Parameters [units]

$tcap_{ijt'}$	Total capacity to be allocated for all products on (i, t, j, t')	[bbbls]
$rpen_{ijt'}^p$	Penalty for substituting product p' for product p at time t	[cost/bbbl]

Additional Variables [units]

$R_{it'jt}^{p'p}$	Amount of product p' from node i at time t' sent to node j at time t as product p	[bbbls]
$A_{ijt'}^p$	Allocated capacity for product p on (i, t, j, t')	[bbbls]

Revised Formulation

$$\min_{F,C,B,U,E,R} \sum_{p \in \mathcal{P}} \sum_{(i,t,j,t') \in \mathcal{F}} \left(\text{cost}_{ijt'}^p F_{ijt'}^p + \sum_{p' \neq p} \text{rpen}_{ijt'}^p R_{ijt'}^{p'p} \right) \quad (\text{A01r3})$$

$$+ \sum_{p \in \mathcal{P}} \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} (b\text{pen}_{nt}^p B_{nt}^p + u\text{pen}_{nt}^p U_{nt}^p + e\text{pen}_{nt}^p E_{nt}^p), \quad (\text{A02r1})$$

subject to

$$\begin{aligned} & \sum_{\substack{(j,t'): \\ (n,t,j,t') \in \mathcal{F}}} \left(F_{ntjt'}^p + \sum_{p' \neq p} R_{ntjt'}^{pp'} \right) - \sum_{\substack{(i,t'): \\ (i,t',n,t) \in \mathcal{F}}} \left(F_{it'nt}^p + \sum_{p' \neq p} R_{it'nt}^{p'p} \right) \\ & + C_{nt}^p - C_{n(t-1)}^p + B_{n(t-1)}^p - B_{nt}^p - U_{nt}^p + E_{nt}^p \\ & = \text{sup}_{nt}^p - \text{dem}_{nt}^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, p \in \mathcal{P}, \end{aligned} \quad (\text{A1r2})$$

$$\sum_{\substack{(j,t'): \\ (n,t,j,t') \in \mathcal{F}}} F_{ntjt'}^p + C_{nt}^p \leq \sum_{\substack{(i,t'): \\ (i,t',n,t) \in \mathcal{F}}} F_{it'nt}^p + C_{n(t-1)}^p + \text{sup}_{nt}^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, p \in \mathcal{P}, \quad (\text{A2r2})$$

$$F_{ijt'}^p \leq A_{ijt'}^p \quad \forall (i,t,j,t') \in \mathcal{F}, p \in \mathcal{P}, \quad (\text{A3r2})$$

$$\sum_{p \in \mathcal{P}} A_{ijt'}^p \leq \text{tcap}_{ijt'} \quad \forall (i,t,j,t') \in \mathcal{F}, \quad (\text{A3.1})$$

$$B_{nt}^p \leq \sum_{t-n\text{per}_n^p+1}^t \text{dem}_{nt}^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, p \in \mathcal{P}, \quad (\text{A4r1})$$

$$U_{nt}^p \leq \text{dem}_{n,t-n\text{per}_n^p+1}^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, p \in \mathcal{P}, \quad (\text{A5r1})$$

$$C_{nt}^p \leq \text{ncap}_n^p \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, p \in \mathcal{P}, \quad (\text{A6r1})$$

$$F_{ijt'}^p \geq 0 \quad \forall (i,t,j,t') \in \mathcal{F}, p \in \mathcal{P}, \quad (\text{A7r1})$$

$$U_{nt}^p \geq 0, E_{nt}^p \geq 0, B_{nt}^p \geq 0 \quad \forall n \in \mathcal{N}, t \in \mathcal{T}, p \in \mathcal{P}. \quad (\text{A8r1})$$

Discussion

There is a small cost associated with substituting fuel products. Given the construct we have here, we should ensure $\text{rpen}_{ijt'}^p > \text{cost}_{ijt'}^p$, where the difference is the small substitution cost. Constraints (A3r2) and (A3.1) replace the previous constraint with fixed capacity to allow the model to decide the best way to allocate capacity subject to a total amount available. The inclusion of safety stock or interdiction variables adds further details but would not fundamentally complicate the formulation.

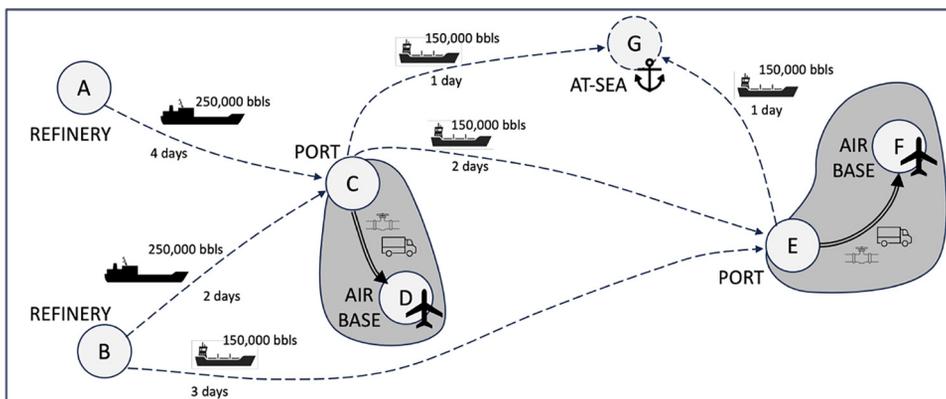


Figure 2. Notional military fuel supply chain network.

CASE STUDY

The baseline model and its extensions are designed to assess the capability of a bulk fuel system to meet a schedule of demands over space and time. In this section, we present a notional case study of a military system over a 30-day time horizon and exercise our model to analyze it. We begin with an overall system description and then develop corresponding data for our model.

Supply Chain Network

Figure 2 depicts a notional system in which we consider the movement and storage of two types of fuel—aviation fuel (denoted JET) and diesel fuel (denoted DSL)—to two remote island locations. Throughout this case study, we measure capacity in barrels (bbls). In this example, these two fuel types originate at both Refinery A and Refinery B. Commercial Port C is located on an island and can receive large (250,000 bbl, similar to commercial or T5 class) tanker deliveries. On that island also is Air Base D, which is supplied by truck (for DSL) and pipe (for JET) from Port C. On a separate island is commercial Port E, which can receive only smaller (150,000 bbl, similar to T1 class) tanker deliveries. Also on this island is Air Base F, which is fed by trucks (for DSL) and pipes (for JET). In this example, we also consider At-Sea Location G, a replenishment site in open water visited by military oiler ships originating from commercial ports C or E.

Storage capacity is a key element in terms of the ability to preposition fuel in anticipation of future demands. Table 1 displays the assumed storage capacity for each key transshipment and demand location. We assume that storage capacity at the refineries is sufficiently large not to affect the ability to deliver fuel. Commercial ports C and E report only total overall storage capacities. However, the storage capacities at air bases D and F are known based on individual storage tanks. At-Sea Location G has zero inventory capacity (although ships obviously carry their own). We

Table 1. Storage capacity, baseline demand, target safety stock level, and initial inventory levels by location.

Location	DSL				JET			
	Storage capacity	Daily demand	Safety level	Init. inv.	Storage capacity	Daily demand	Safety level	Init. inv.
C	500,000 bbls total	None	25%	125,000 bbls	100,000 bbls total	None	25%	50,000 bbls
D	6 tanks @ 5,000 bbls = 30,000 bbls total	1,000 bbls	75%	22,500 bbls	6 tanks @ 15,000 bbls = 90,000 bbls total	10,000 bbls	75%	67,500 bbls
E	375,000 bbls total	None	25%	93,750 bbls	75,000 bbls total	None	25%	37,500 bbls
F	5 tanks @ 5,000 bbls = 25,000 bbls total	1,000 bbls	75%	18,750 bbls	6 tanks @ 15,000 bbls = 90,000 bbls total	7,500 bbls	75%	67,500 bbls
G	No storage	5,000 bbls	n/a	0	No storage	2,500 bbls	n/a	0

Table 2. Transport capacity and delivery schedule.

From	To	Days travel	Depart day	Arrive day	DSL capacity	JET capacity
A	C	4	1, 9, 17, 25	5, 13, 21, 29	125,000 bbls	125,000 bbls
B	C	2	1, 5, 9, 13, 17, 21, 25	3, 7, 11, 15, 19, 23, 27	125,000 bbls	125,000 bbls
B	E	3	2, 8, 14, 20, 26	5, 11, 17, 23, 29	75,000 bbls	75,000 bbls
C	D	1	Daily	Daily	1,200 bbls	50,000 bbls
C	E	2	1, 5, 9, 13, 17, 21, 25	3, 7, 11, 15, 19, 23, 27	75,000 bbls	75,000 bbls
C	G	1	Even periods 2–28	Odd periods 3–29	75,000 bbls	75,000 bbls
E	F	1	Daily	Daily	1,200 bbls	50,000 bbls
E	G	1	Odd periods 1–29	Even periods 2–30	75,000 bbls	75,000 bbls

assume that refineries A and B each have a daily production capacity of 250,000 bbls each for DSL and JET (so production capacity is not a limitation here).

The different transportation modalities determine delivery capacity. We assume large (250,000 bbl) ships are dedicated to round-trip deliveries between A-C and B-C. We assume smaller (150,000 bbl) ships are dedicated to round-trip deliveries between B-E, C-E, C-G, and E-G. [Table 2](#) displays these assumed one-way travel times, capacities, and schedules of arrivals and departures. Importantly, we assume that each ship is partitioned 50-50 between DSL and JET fuel capacity. For example, each shipment from A to C has a fixed capacity of 125,000 bbls for DSL and JET fuel. We assume that the terrestrial transport of JET fuel (from C to D and from E to F) is via pipeline with a daily capacity of 50,000 bbls. In contrast, diesel delivery is via truck with a daily capacity of 1,200 bbls (10 round trips at 120 bbls per truck).

Baseline System Demands

[Table 1](#) also shows the baseline daily fuel demand per location, which we review and discuss in more detail. For each air base, we assume that the rate of sorties flown by aircraft dictates the baseline fuel demand for JET. Smaller aircraft (fixed-wing fighters and rotary-wing) can use 50–100 bbls per sortie, whereas larger bombers can use more than 500 bbls per sortie. For simplicity of demonstration, we assume a single fuel type JET is sufficient for land-based and ship-based aircraft. Moreover, we assume that initial inventories are equal to safety stock levels. We assume that system demands at the air bases can tolerate a single one-day backlog, while the at-sea location can tolerate a backlog of two days.

At Air Base D, we assume a baseline daily demand of 1,000 bbls of DSL, dictated by heavy equipment needs (e.g., trucks and generators). Moreover, we assume a daily demand of 10,000 bbls of JET, sufficient to support approximately one hundred sorties. This air base has a maximum storage capacity for DSL equivalent to 30 days of supply at baseline demand (with a 50% safety stock target equivalent to 15 days of supply). Current operations support the daily delivery (from Port C) of 1,200 bbls of DSL by truck. This air base has a maximum storage capacity for JET equivalent to 9 days of supply at baseline demand (with a 75% safety stock target equivalent of 6.75 days of supply). It can receive a maximum of 50,000 bbls of JET daily via pipeline delivery.

At Air Base F, we assume the same baseline daily demand of 1,000 bbls of DSL but a demand of only 7,500 bbls of JET daily. This air base has a maximum storage capacity for DSL equivalent to 25 days of supply at baseline daily demand (and a safety stock target equivalent to 12.5 days of supply). It can receive a maximum of 1,200 bbls of JET via truck delivery. This air base has a maximum storage capacity for JET equivalent to 12 days of supply at baseline daily demand (and a safety stock target equivalent to nine days of supply). It can receive a daily maximum of 50,000 bbls of JET via pipeline delivery.

At At-Sea Location G, demand for fuel comes primarily from the use of DSL as a ship propellant (e.g., for destroyers and cruisers) and the use of JET by the aircraft carried by ships (e.g., aboard aircraft carriers). We assume a baseline daily demand of 5,000 bbls of DSL and 2,500 bbls of JET.

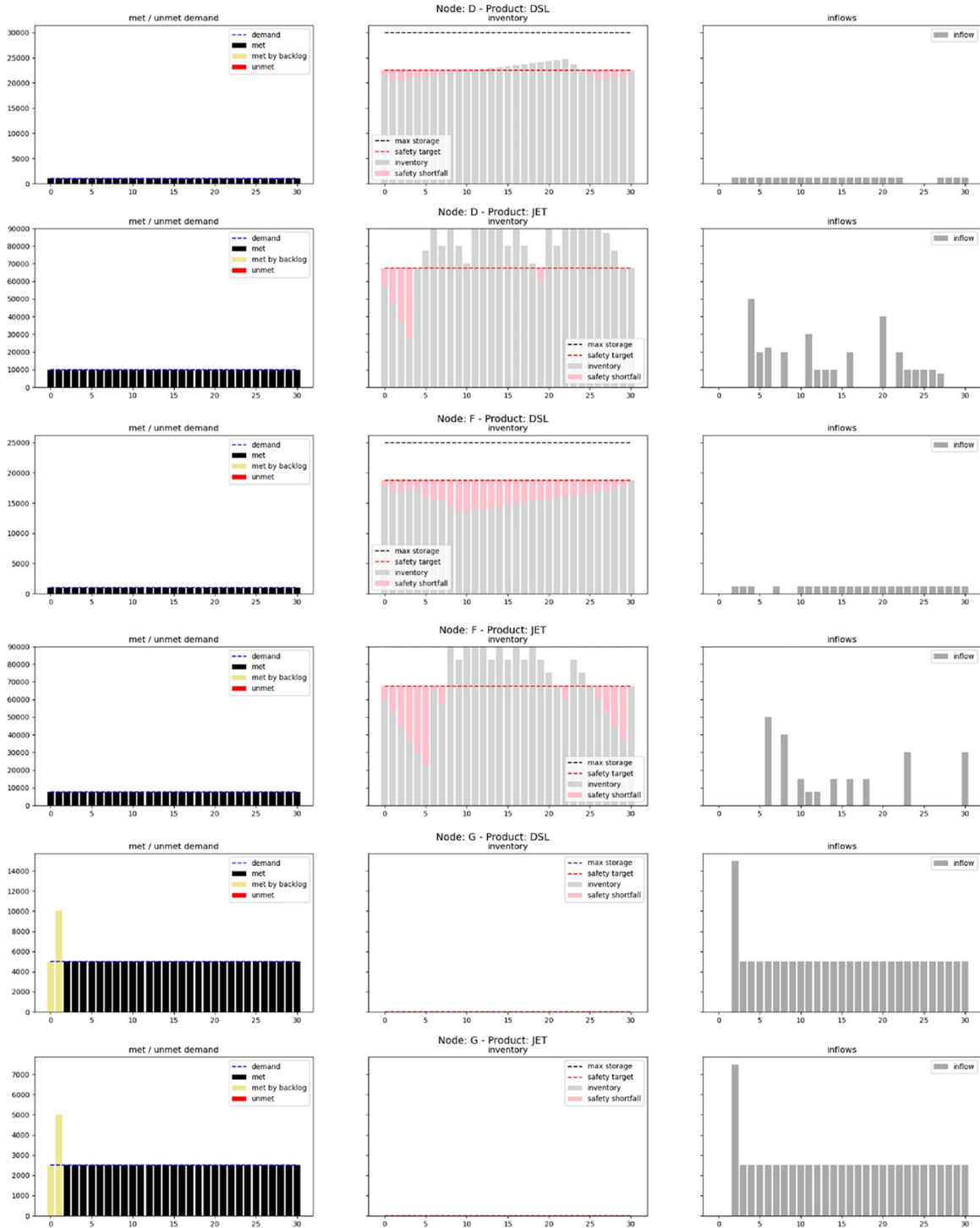


Figure 3. Results for normal operations. For locations D, F, G, and both DSL and JET, we illustrate the daily demand (left column), inventory (center column), and fuel inflows (right column).

MODEL IMPLEMENTATION AND RESULTS

We implement the model in the Python programming language (Python Software Foundation, 2024) using the Pyomo optimization package (Hart et al., 2017). Input data is managed via comma-separated value (CSV) files (see the appendix). Model execution is orchestrated using a Jupyter notebook (Project Jupyter, 2020) with numerical results and graphics charts embedded

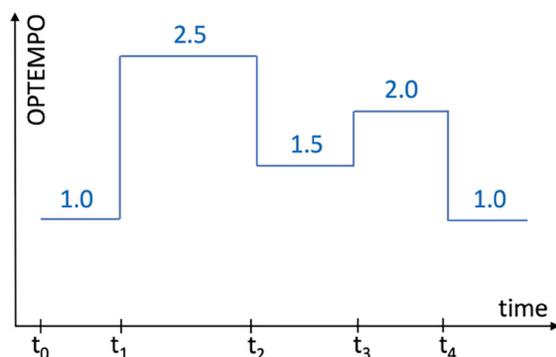


Figure 4. Notional change in demand during the scenario as a function of OPTEMPO.

into a single end-to-end workflow (Perkel, 2018). Each run of the model on a MacBook Pro laptop (with 2.7 GHz CPU and 16 GB of memory) completes within a few seconds.

Figure 3 illustrates model output for demand locations D, F, and G. There is one row for each combination of location and fuel type. The left column illustrates demand by time period. We observe demands are constant for the entire 30-day horizon at their nominal levels, and these are all satisfied. An exception is the first two periods for At-Sea Location G; this is a model artifact— fuel delivery can't get there fast enough, but demand is satisfied as backlog. More specifically, we observe that the demand in period 0 is satisfied as a backlog, and the spike in backlog in period 1 indicates that backlog is carried backward for an additional period. The middle column illustrates inventory by time period. Inventory levels fluctuate around the target safety stock level (except for At-Sea Location G, which has zero inventory capacity; again, ships carry their own inventory). The right column illustrates product inflow (receipts from other nodes) by time period. Delivery to some locations is relatively constant, while delivery to others can fluctuate.

SCENARIO ANALYSIS

In this section, we consider several model excursions that change the input data to study specific scenarios of interest.

Increased Demand: High OPTEMPO Surge

We consider a notional surge of activity to support a military exercise. We construct the overall demand according to an operational tempo (OPTEMPO) that serves as a multiplier for a baseline level of activity. Figure 4 displays the notional surge in activity for the exercise. During an initial period, $[t_0, t_1]$ overall activity is at a baseline level, then an initial surge $[t_1, t_2]$ increases the demand for fuel to 2.5 times the baseline. An intermediate phase $[t_2, t_3]$ consists of a relative lull down to 1.5 times the baseline and is then followed by a second surge $[t_3, t_4]$, which increases to 2.0 times the baseline. Finally, at t_4 , the demand returns to its baseline rate. For both air bases and at-sea locations, we assume that increases in OPTEMPO affect DSL and JET demand the same.

The primary question of interest for this scenario is: Can the supply chain support the notional surge in fuel demand? Figure 5 illustrates the results. As before, each row represents a combination of location and fuel type. Columns are associated with demand (left column), product inventory (middle column), and product inflows (right column). In particular, we observe the increased demand profile from Figure 4 in all combinations of location and fuel.

Overall, we observe no unmet demands over this 30-day scenario. However, both air bases have their inventories fall far below target safety stock levels, with the inventory of DSL nearing zero toward the end of the scenario. Nonetheless, most planners would conclude that the system can satisfy the planned surge in demands.

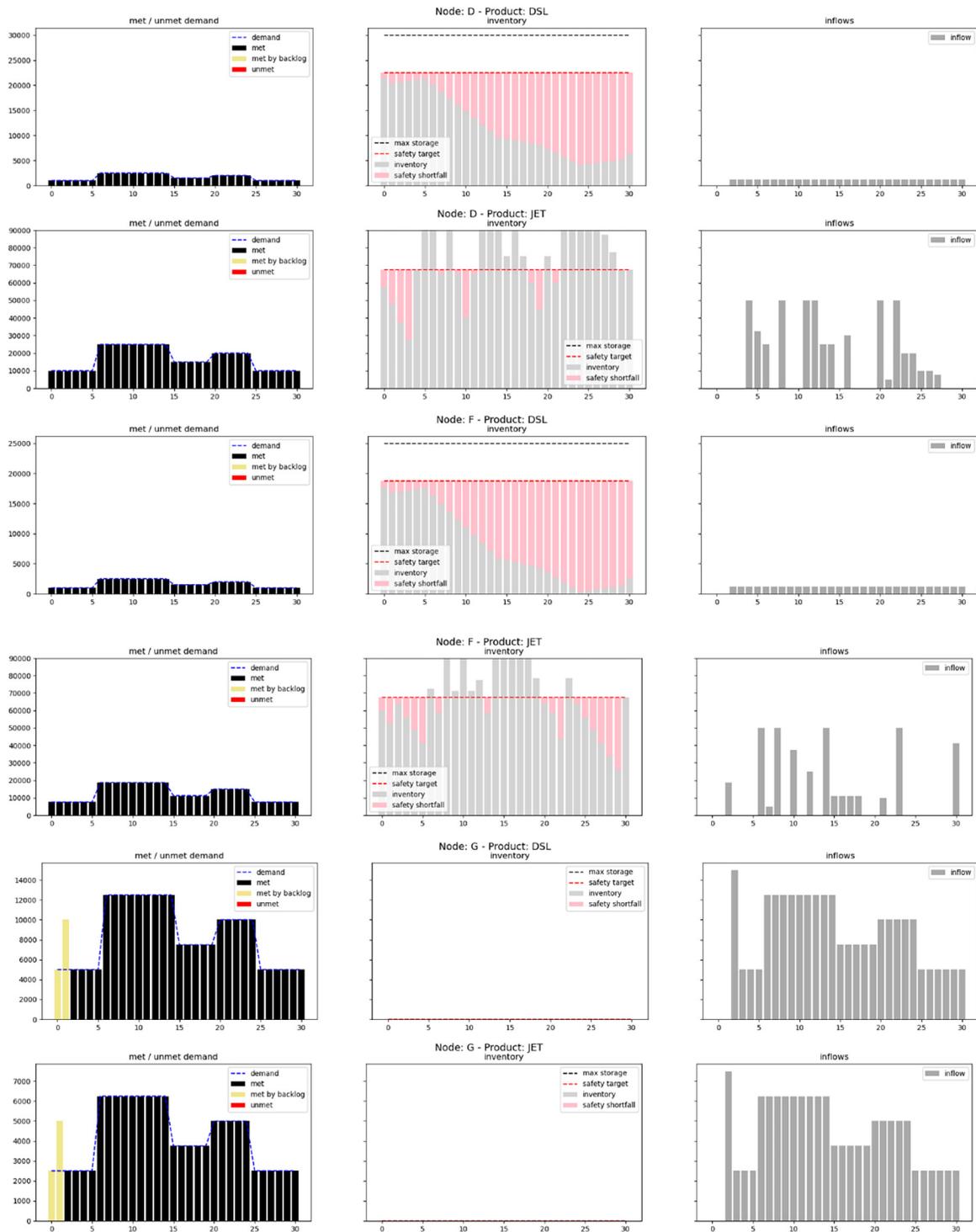


Figure 5. Results for scenario involving a surge in demands. Increased demands (left column) follow the OPTEMPO surge from Figure 4. Inventory levels (center column) fall far below target safety stock levels but do not result in unmet demand.

Loss of Pipeline

Next, we consider the potential loss in the pipeline from Port C supplying JET to Air Base D. We imagine a scenario where the pipeline is down for repair and makeshift delivery has

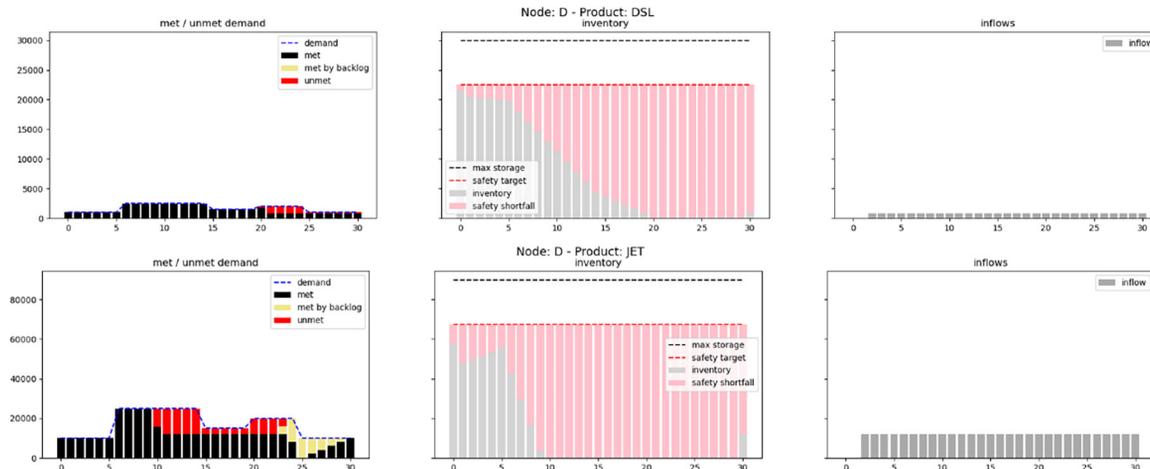


Figure 6. Results for a scenario involving loss of pipeline to D. Reduced delivery capacity for both DSL and JET results in unmet demands during the latter half of the notional exercise.

been implemented using a fleet of trucks. The result is a decrease in daily delivery capacity for JET from 50,000 bbls to only 12,000 bbls. We assume that the makeshift capacity to deliver JET came at the expense of reduced transport capacity for DSL, which has now dropped to 800 bbls/day.

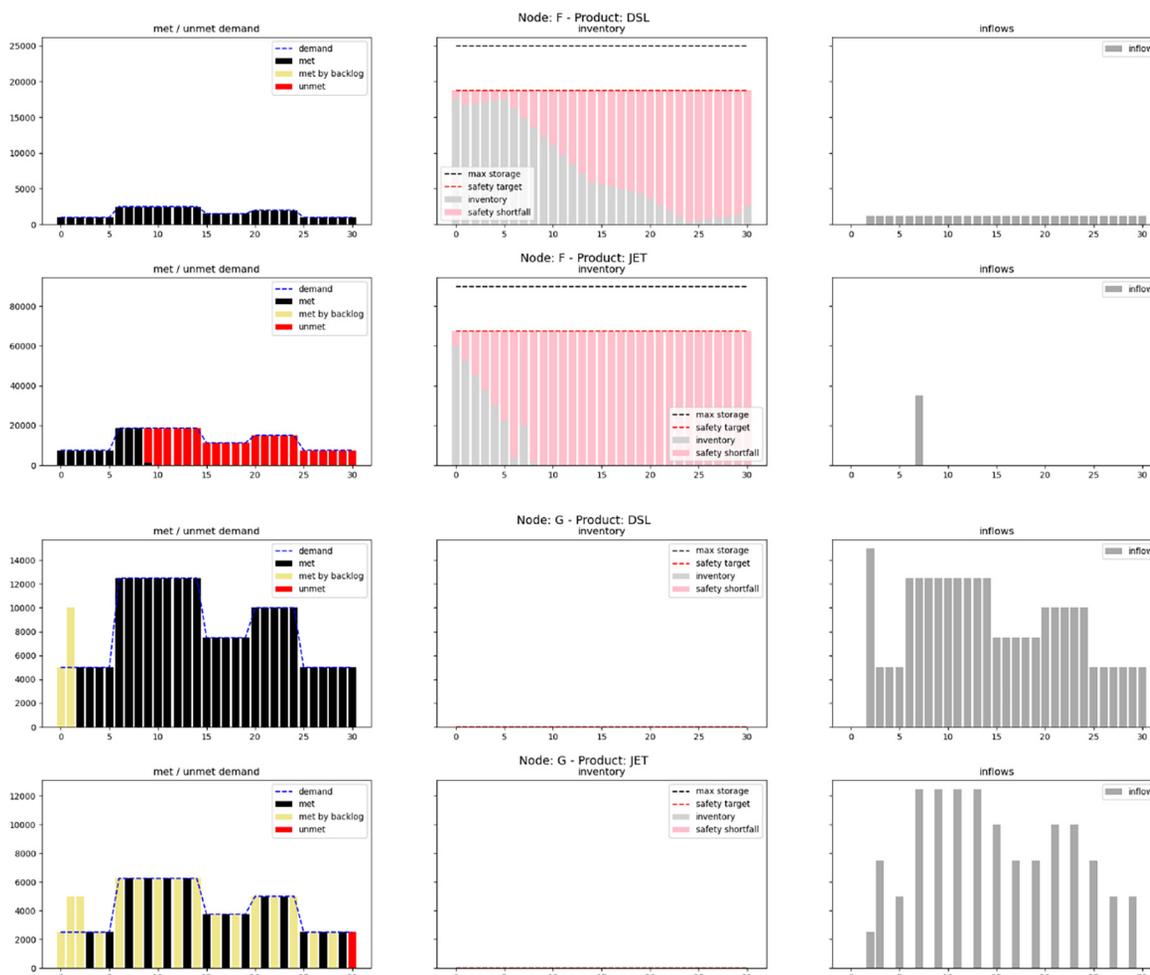


Figure 7. Results for a scenario involving loss of shipments from Refinery B to Port E. Air Base F runs out of JET after Day 8, while At-Sea Location G avoids unmet demands only via backlogged demands and shipments from Port C.

Figure 6 shows the impact for the loss of this pipeline. We observe that we are no longer able to satisfy the surge in demand for JET. Fuel inflows are at their maximum for each period, yet there are unmet demands between periods 9–23, with additional demands backlogged between periods 22–29. The decrease in DSL delivery capacity has also come at a penalty; we also observe unmet demand in periods 21–24.

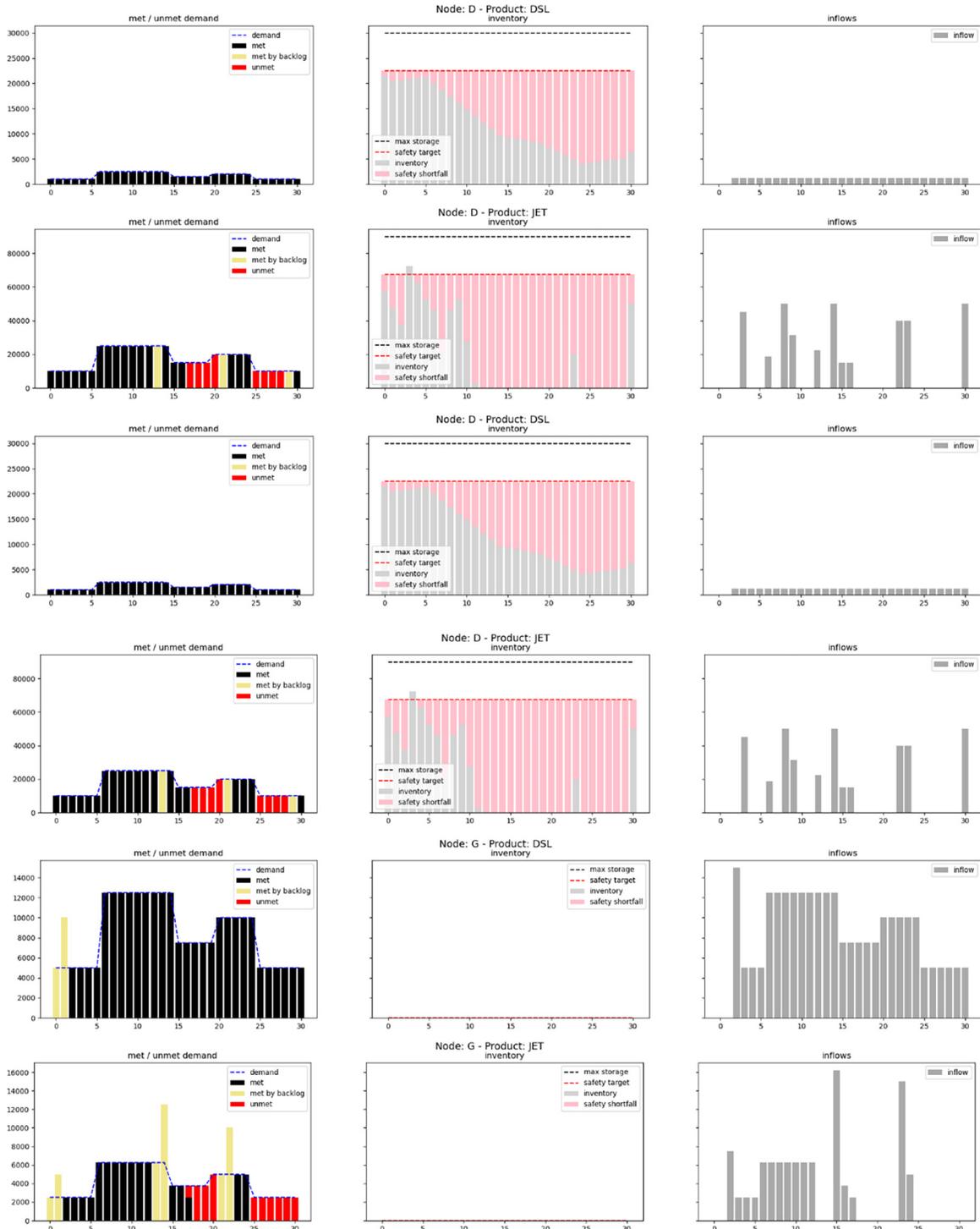


Figure 8. Results for a scenario involving loss of access to Refinery B. Transport of fuel from Refinery A is insufficient to support the demand for JET at all locations.

Loss of Deliveries B-to-E

Here, we consider another question: How would the loss of shipments from B-E affect the ability of the system to support the surge? This might occur if the dedicated ship used for transport became unavailable or if the contract with Refinery B had simply expired. [Figure 7](#) illustrates the capability of the system in the absence of this scheduled shipment.

This scenario has no appreciable impact on the ability to satisfy demands at Air Base D (therefore, not shown); however, there is a significant disruption to demand at Air Base F and At-Sea Location G. We observe that Air Base F is effectively starved of resupply for JET and can sustain itself only while the initial inventory lasts. In contrast, At-Sea Location G can satisfy JET demand with deliveries originating from Port C, but with backlogged demand every other day, following an every-other-day delivery schedule.

Loss of Refinery B

Here, we consider a more extreme scenario: How would the loss of access to Refinery B altogether affect the ability of the system to support the surge? [Figure 8](#) illustrates.

We observe that the loss of Refinery B affects the system's ability to satisfy the needs for JET at all demand locations. The profile of unmet and backlogged demand at nodes D, F, and G share a similar profile that coincides with the delivery schedule from Refinery A, which now is the sole source of supply and has insufficient capacity to support all operations.

CONCLUSIONS AND MORE EXTENSIONS

The baseline linear program and its extensions, as presented in this paper, are what we have found to be the most relevant and useful for a variety of practical planning problems. Our goal has been to provide a complete package of mathematical models, scenario data, and codes for solving (available online; contact the author). As presented here, we typically start with a feasibility assessment of a starting scenario and then proceed to multiple scenario excursions.

The models can be used to support what-if analysis for any scenario affecting supplies, demands, storage capacity, inventories, or transport capacities. Planners need to understand the impact of losing key assets to potential attrition in combat, along with possible mitigations to assure mission success. For example, the models can be used to complete an energy supportability analysis, part of the logistics supportability analysis for a given operational plan. Moreover, public works officers face similar questions: for example, what are the operational implications of taking an infrastructure asset out of service for maintenance or replacement?

We acknowledge there are many additional extensions that we have sometimes found useful. We conclude with a list of some of these additional extensions together with some relevant references.

Delivery Schedules. The models here produce network flows, assumed to be continuous and separable, that follow a predefined schedule of available (tanker) shipments. In practice, shipments tend to be discrete and nonseparable (i.e., sending a fractional tanker is impossible, and shipments must be fully loaded before they make economic sense). A model for generating optimal delivery schedules (schedules of discrete shipments) requires integer variables that typically result in issues of tractability. There is a large literature describing such models (e.g., [Christiansen et al. 2013](#)) with reported military fuels implementations (e.g., [Noble, 2000](#); [Brown et al., 2017](#)).

Stochastic Component Failures. In practice, the availability of individual components in the fuel storage and delivery system is uncertain. Components fail at random, accidents happen, and weather can delay or cancel individual shipments. It becomes possible to characterize the reliability of individual components and characterize the aggregate ability to deliver fuel in support of a given schedule of demands. Moreover, it becomes important to consider online readjustment of flows in response to disruption, perhaps using a cascade optimization approach (e.g., [Cabana, 2018](#)).

Stochastic Scenario Planning. We frequently face large-scale structural uncertainty in the future. Which operational plans will need to be executed? Where will the demands for operations be located? Will commercial supplies be available in sufficient quantities to support on-demand purchases? For these types of concerns, it is typically insufficient to add “Gaussian noise” to parameters or demands. Instead, we must consider highly structured future alternatives. We can use stochastic optimization techniques to consider tradeoffs between radically different future scenarios and have done so for military applications (e.g., Devlin, 2001; Carline, 2013).

Tradespace Analysis. Beyond these specific model features or techniques, there is an ongoing need to explore the trade space of solutions for managing bulk fuels in the presence of uncertainty. We emphasize optimization here as a tool, not simply to find the best operating solution for any particular situation but also to evaluate this evolving trade space. The design of the bulk fuel resupply system involves tradeoffs between optimizing for efficiency and positioning for flexibility in responding to changes that arise. Therefore, it becomes important to exercise these models in a manner that supports timely exploration of tradeoffs between, for example, the size, number, and capacity of storage tanks versus the number of tankers for frequent delivery. Accordingly, there is an opportunity to employ a design-of-experiments approach to explore the relationships in this trade space (e.g., Beaumont, 2017).

Bulk Fuel Analysis at Game Speed. The ability to exercise these models and explore their results quickly creates additional opportunities for the integration of logistics models in planning and wargaming exercises. Supporting live tabletop exercises “at game speed” – this requires not only that we can solve the model in a relatively short amount of time, but that we have the databases and interfaces to change the data and manage the outputs in a timely manner – changes the nature of planning. Adoption and integration of these modeling tools into wargames and other planning exercises remain an opportunity for additional exploration.

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APPENDIX: CASE STUDY INPUT DATA

Input data is tabular, organized according to index sets, and stored as separate CSV files. Node data is given in [Table A.1](#) and arc data is given in [Table A.2](#).

Table A.1. Input data for nodes, indexed by (n, p) .

Node	Product	ncap	init	dpen	bpen	epen	safe	spen	nper	yhat
A	DSL	250000	250000	100000	1000	10	0	100	0	0
B	DSL	250000	250000	100000	1000	10	0	100	0	0
C	DSL	500000	125000	100000	1000	10	0	100	0	0
D	DSL	30000	22500	100000	1000	10	0.75	100	1	0
E	DSL	375000	187500	100000	1000	10	0.5	100	0	0
F	DSL	25000	18750	100000	1000	10	0.75	100	1	0
G	DSL	0	0	100000	1000	10	0	100	0	0
A	JET	250000	125000	100000	1000	10	0	100	0	0
B	JET	250000	125000	100000	1000	10	0	100	0	0
C	JET	100000	50000	100000	1000	10	0	100	0	0
D	JET	90000	45000	100000	1000	10	0.5	100	1	0
E	JET	75000	37500	100000	1000	10	0.5	100	0	0
F	JET	90000	45000	100000	1000	10	0.5	100	1	0
G	JET	0	0	100000	1000	10	0	100	0	0

Table A.2. Space-time arc data indexed by (i, t, j, t', p) .

From	To	t	tp	Product	acap	cost	apen	xhat
A	C	1	5	DSL	125000	1	10000	0
A	C	9	13	DSL	125000	1	10000	0
A	C	17	21	DSL	125000	1	10000	0
A	C	25	29	DSL	125000	1	10000	0
B	C	1	3	DSL	125000	1	10000	0
B	C	5	7	DSL	125000	1	10000	0
B	C	9	11	DSL	125000	1	10000	0
B	C	13	15	DSL	125000	1	10000	0
B	C	17	19	DSL	125000	1	10000	0
B	C	21	23	DSL	125000	1	10000	0
B	C	25	27	DSL	125000	1	10000	0
B	E	2	5	DSL	75000	2	10000	0
B	E	8	11	DSL	75000	2	10000	0
B	E	14	17	DSL	75000	2	10000	0
B	E	20	23	DSL	75000	2	10000	0
B	E	26	29	DSL	75000	2	10000	0
C	D	1	2	DSL	1200	5	10000	0
C	D	2	3	DSL	1200	5	10000	0
C	D	3	4	DSL	1200	5	10000	0
C	D	4	5	DSL	1200	5	10000	0
C	D	5	6	DSL	1200	5	10000	0
C	D	6	7	DSL	1200	5	10000	0
C	D	7	8	DSL	1200	5	10000	0
C	D	8	9	DSL	1200	5	10000	0
C	D	9	10	DSL	1200	5	10000	0
C	D	10	11	DSL	1200	5	10000	0
C	D	11	12	DSL	1200	5	10000	0
C	D	12	13	DSL	1200	5	10000	0
C	D	13	14	DSL	1200	5	10000	0
C	D	14	15	DSL	1200	5	10000	0
C	D	15	16	DSL	1200	5	10000	0
C	D	16	17	DSL	1200	5	10000	0

Table A.2. *Continued*

From	To	t	tp	Product	acap	cost	apen	xhat
C	D	17	18	DSL	1200	5	10000	0
C	D	18	19	DSL	1200	5	10000	0
C	D	19	20	DSL	1200	5	10000	0
C	D	20	21	DSL	1200	5	10000	0
C	D	21	22	DSL	1200	5	10000	0
C	D	22	23	DSL	1200	5	10000	0
C	D	23	24	DSL	1200	5	10000	0
C	D	24	25	DSL	1200	5	10000	0
C	D	25	26	DSL	1200	5	10000	0
C	D	26	27	DSL	1200	5	10000	0
C	D	27	28	DSL	1200	5	10000	0
C	D	28	29	DSL	1200	5	10000	0
C	D	29	30	DSL	1200	5	10000	0
C	E	1	3	DSL	75000	2	10000	0
C	E	5	7	DSL	75000	2	10000	0
C	E	9	11	DSL	75000	2	10000	0
C	E	13	15	DSL	75000	2	10000	0
C	E	17	19	DSL	75000	2	10000	0
C	E	21	23	DSL	75000	2	10000	0
C	E	25	27	DSL	75000	2	10000	0
C	G	2	3	DSL	75000	2	10000	0
C	G	4	5	DSL	75000	2	10000	0
C	G	6	7	DSL	75000	2	10000	0
C	G	8	9	DSL	75000	2	10000	0
C	G	10	11	DSL	75000	2	10000	0
C	G	12	13	DSL	75000	2	10000	0
C	G	14	15	DSL	75000	2	10000	0
C	G	16	17	DSL	75000	2	10000	0
C	G	18	19	DSL	75000	2	10000	0
C	G	20	21	DSL	75000	2	10000	0
C	G	22	23	DSL	75000	2	10000	0
C	G	24	25	DSL	75000	2	10000	0
C	G	26	27	DSL	75000	2	10000	0
C	G	28	29	DSL	75000	2	10000	0
E	F	1	2	DSL	1200	5	10000	0
E	F	2	3	DSL	1200	5	10000	0
E	F	3	4	DSL	1200	5	10000	0
E	F	4	5	DSL	1200	5	10000	0
E	F	5	6	DSL	1200	5	10000	0
E	F	6	7	DSL	1200	5	10000	0
E	F	7	8	DSL	1200	5	10000	0
E	F	8	9	DSL	1200	5	10000	0
E	F	9	10	DSL	1200	5	10000	0
E	F	10	11	DSL	1200	5	10000	0
E	F	11	12	DSL	1200	5	10000	0
E	F	12	13	DSL	1200	5	10000	0
E	F	13	14	DSL	1200	5	10000	0
E	F	14	15	DSL	1200	5	10000	0
E	F	15	16	DSL	1200	5	10000	0
E	F	16	17	DSL	1200	5	10000	0
E	F	17	18	DSL	1200	5	10000	0
E	F	18	19	DSL	1200	5	10000	0
E	F	19	20	DSL	1200	5	10000	0
E	F	20	21	DSL	1200	5	10000	0
E	F	21	22	DSL	1200	5	10000	0
E	F	22	23	DSL	1200	5	10000	0
E	F	23	24	DSL	1200	5	10000	0

Table A.2. *Continued*

From	To	t	tp	Product	acap	cost	apen	xhat
E	F	24	25	DSL	1200	5	10000	0
E	F	25	26	DSL	1200	5	10000	0
E	F	26	27	DSL	1200	5	10000	0
E	F	27	28	DSL	1200	5	10000	0
E	F	28	29	DSL	1200	5	10000	0
E	F	29	30	DSL	1200	5	10000	0
E	G	1	2	DSL	75000	2	10000	0
E	G	3	4	DSL	75000	2	10000	0
E	G	5	6	DSL	75000	2	10000	0
E	G	7	8	DSL	75000	2	10000	0
E	G	9	10	DSL	75000	2	10000	0
E	G	11	12	DSL	75000	2	10000	0
E	G	13	14	DSL	75000	2	10000	0
E	G	15	16	DSL	75000	2	10000	0
E	G	17	18	DSL	75000	2	10000	0
E	G	19	20	DSL	75000	2	10000	0
E	G	21	22	DSL	75000	2	10000	0
E	G	23	24	DSL	75000	2	10000	0
E	G	25	26	DSL	75000	2	10000	0
E	G	27	28	DSL	75000	2	10000	0
E	G	29	30	DSL	75000	2	10000	0
A	C	1	5	JET	125000	1	10000	0
A	C	9	13	JET	125000	1	10000	0
A	C	17	21	JET	125000	1	10000	0
A	C	25	29	JET	125000	1	10000	0
B	C	1	3	JET	125000	1	10000	0
B	C	5	7	JET	125000	1	10000	0
B	C	9	11	JET	125000	1	10000	0
B	C	13	15	JET	125000	1	10000	0
B	C	17	19	JET	125000	1	10000	0
B	C	21	23	JET	125000	1	10000	0
B	C	25	27	JET	125000	1	10000	0
B	E	2	5	JET	75000	2	10000	0
B	E	8	11	JET	75000	2	10000	0
B	E	14	17	JET	75000	2	10000	0
B	E	20	23	JET	75000	2	10000	0
B	E	26	29	JET	75000	2	10000	0
C	D	1	2	JET	50000	5	10000	0
C	D	2	3	JET	50000	5	10000	0
C	D	3	4	JET	50000	5	10000	0
C	D	4	5	JET	50000	5	10000	0
C	D	5	6	JET	50000	5	10000	0
C	D	6	7	JET	50000	5	10000	0
C	D	7	8	JET	50000	5	10000	0
C	D	8	9	JET	50000	5	10000	0
C	D	9	10	JET	50000	5	10000	0
C	D	10	11	JET	50000	5	10000	0
C	D	11	12	JET	50000	5	10000	0
C	D	12	13	JET	50000	5	10000	0
C	D	13	14	JET	50000	5	10000	0
C	D	14	15	JET	50000	5	10000	0
C	D	15	16	JET	50000	5	10000	0
C	D	16	17	JET	50000	5	10000	0
C	D	17	18	JET	50000	5	10000	0
C	D	18	19	JET	50000	5	10000	0
C	D	19	20	JET	50000	5	10000	0
C	D	20	21	JET	50000	5	10000	0

Table A.2. *Continued*

From	To	t	tp	Product	acap	cost	apen	xhat
C	D	21	22	JET	50000	5	10000	0
C	D	22	23	JET	50000	5	10000	0
C	D	23	24	JET	50000	5	10000	0
C	D	24	25	JET	50000	5	10000	0
C	D	25	26	JET	50000	5	10000	0
C	D	26	27	JET	50000	5	10000	0
C	D	27	28	JET	50000	5	10000	0
C	D	28	29	JET	50000	5	10000	0
C	D	29	30	JET	50000	5	10000	0
C	E	1	3	JET	75000	2	10000	0
C	E	5	7	JET	75000	2	10000	0
C	E	9	11	JET	75000	2	10000	0
C	E	13	15	JET	75000	2	10000	0
C	E	17	19	JET	75000	2	10000	0
C	E	21	23	JET	75000	2	10000	0
C	E	25	27	JET	75000	2	10000	0
C	G	2	3	JET	75000	2	10000	0
C	G	4	5	JET	75000	2	10000	0
C	G	6	7	JET	75000	2	10000	0
C	G	8	9	JET	75000	2	10000	0
C	G	10	11	JET	75000	2	10000	0
C	G	12	13	JET	75000	2	10000	0
C	G	14	15	JET	75000	2	10000	0
C	G	16	17	JET	75000	2	10000	0
C	G	18	19	JET	75000	2	10000	0
C	G	20	21	JET	75000	2	10000	0
C	G	22	23	JET	75000	2	10000	0
C	G	24	25	JET	75000	2	10000	0
C	G	26	27	JET	75000	2	10000	0
C	G	28	29	JET	75000	2	10000	0
E	F	1	2	JET	50000	5	10000	0
E	F	2	3	JET	50000	5	10000	0
E	F	3	4	JET	50000	5	10000	0
E	F	4	5	JET	50000	5	10000	0
E	F	5	6	JET	50000	5	10000	0
E	F	6	7	JET	50000	5	10000	0
E	F	7	8	JET	50000	5	10000	0
E	F	8	9	JET	50000	5	10000	0
E	F	9	10	JET	50000	5	10000	0
E	F	10	11	JET	50000	5	10000	0
E	F	11	12	JET	50000	5	10000	0
E	F	12	13	JET	50000	5	10000	0
E	F	13	14	JET	50000	5	10000	0
E	F	14	15	JET	50000	5	10000	0
E	F	15	16	JET	50000	5	10000	0
E	F	16	17	JET	50000	5	10000	0
E	F	17	18	JET	50000	5	10000	0
E	F	18	19	JET	50000	5	10000	0
E	F	19	20	JET	50000	5	10000	0
E	F	20	21	JET	50000	5	10000	0
E	F	21	22	JET	50000	5	10000	0
E	F	22	23	JET	50000	5	10000	0
E	F	23	24	JET	50000	5	10000	0
E	F	24	25	JET	50000	5	10000	0
E	F	25	26	JET	50000	5	10000	0
E	F	26	27	JET	50000	5	10000	0
E	F	27	28	JET	50000	5	10000	0

Table A.2. *Continued*

From	To	t	tp	Product	acap	cost	apen	xhat
E	F	28	29	JET	50000	5	10000	0
E	F	29	30	JET	50000	5	10000	0
E	G	1	2	JET	75000	2	10000	0
E	G	3	4	JET	75000	2	10000	0
E	G	5	6	JET	75000	2	10000	0
E	G	7	8	JET	75000	2	10000	0
E	G	9	10	JET	75000	2	10000	0
E	G	11	12	JET	75000	2	10000	0
E	G	13	14	JET	75000	2	10000	0
E	G	15	16	JET	75000	2	10000	0
E	G	17	18	JET	75000	2	10000	0
E	G	19	20	JET	75000	2	10000	0
E	G	21	22	JET	75000	2	10000	0
E	G	23	24	JET	75000	2	10000	0
E	G	25	26	JET	75000	2	10000	0
E	G	27	28	JET	75000	2	10000	0
E	G	29	30	JET	75000	2	10000	0