Optimal Resource Allocation for Recovery from Multimodal Transportation Disruptions

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Transportation disruptions
Previous studies

• Disrupt one or more transportation modes
• Determine alternate transportation routes for each firm
• Calculate additional transportation cost or economic impact


Research goals

• Develop optimal resource allocation model to repair disrupted transportation infrastructure
  – Given firms’ alternate transportation routes
  – Given additional costs or delays experienced by firms

• Calculate optimal allocation as function of parameters (e.g., initial inoperability, effectiveness of allocation, additional costs and delays)

• Explore whether decision changes over time
Static model

\[ \text{minimize} \sum_{i=1}^{n} f_i(q) \]

\[ \sum_{i=1}^{n} g_i(q) \]

subject to \( q_j = \hat{q}_j \exp(-k_j z_j - k_0 z_0) \)

\[ z_0 + \sum_{j=1}^{m} z_j \leq Z \]

\[ z_j \geq 0, \ z_0 \geq 0 \]
Illustrative example

<table>
<thead>
<tr>
<th>Mode</th>
<th>Inoperability ($q_j$)</th>
<th>$k_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.3</td>
<td>1</td>
</tr>
<tr>
<td>Railroad</td>
<td>0.3</td>
<td>1</td>
</tr>
<tr>
<td>Highway</td>
<td>0.3</td>
<td>1</td>
</tr>
</tbody>
</table>
### Illustrative example

#### Linear cost function for firm $i$

$$f_i(q) = c_{i,w} q_w + c_{i,r} q_r + c_{i,h} q_h$$

<table>
<thead>
<tr>
<th></th>
<th>$c_{i,w}$</th>
<th>$c_{i,r}$</th>
<th>$c_{i,h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Firm 2</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Firm 3</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

#### Linear delay function for firm $i$

$$g_i(q) = d_{i,w} q_w + d_{i,r} q_r + d_{i,h} q_h$$

<table>
<thead>
<tr>
<th></th>
<th>$d_{i,w}$</th>
<th>$d_{i,r}$</th>
<th>$d_{i,h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Firm 2</td>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Firm 3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Modeling philosophy

1. Pareto front

2. Solution if $z_0 = 0$

3. Conditions when $z_0 = 0$

4. Tradeoffs

5. Impact of allocation with respect to time
Modeling philosophy

1. Pareto front
2. Solution if $z_0 = 0$
3. Conditions when $z_0 = 0$
4. Tradeoffs
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Pareto front

Create Pareto front for cost and delay

\[
\text{minimize } \alpha \sum_{i=1}^{n} f_i(q) + (1 - \alpha) \sum_{i=1}^{n} g_i(q) \quad 0 \leq \alpha \leq 1
\]

- tradeoff parameter between cost and delay
- cost for firm \( i \)
- vector (length \( m \)) of inoperability for each transportation infrastructure, mode, route
- delay for firm \( i \)
Pareto front for different budgets

\[
\text{minimize } \alpha \sum_{i=1}^{n} f_i(q) + (1 - \alpha) \sum_{i=1}^{n} g_i(q)
\]
Modeling philosophy

1. Pareto front
2. Solution if $z_0 = 0$
3. Conditions when $z_0 = 0$
4. Tradeoffs
5. Impact of allocation with respect to time
If $z_0 = 0$

Change in objective function per change in inoperability

$$x_j = \alpha \sum_{i=1}^{n} \frac{\partial f_i}{\partial q_j} + (1 - \alpha) \sum_{i=1}^{n} \frac{\partial g_i}{\partial q_j}$$

Optimal allocation to transportation $j$

$$z_j^* = \frac{1}{k_j} \log \left( \frac{\hat{q}_j k_j x_j}{\lambda^*} \right)$$

Lagrange multiplier for budget constraint
Optimal allocation for railroad and highway

Comparing allocation to railroad and highway

Allocate more to highway

Equal allocation

Allocate more to railroad

MacKenzie and Barker, Optimal Resource Allocation for Multimodal Transportation Disruptions
Modeling philosophy

1. Pareto front

2. Solution if $z_0 = 0$

3. Conditions when $z_0 = 0$

4. Tradeoffs

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When is $z_0 > 0$?

$z_0 > 0$ if and only if $k_0 \geq k_0^*$

$$k_0^* = \frac{\lambda^* \sum_{j=1}^{m} \hat{q}_j \exp(-k_j z_j^*) x_j}{\sum_{j=1}^{m} \hat{q}_j \exp(-k_j z_j^*) x_j}$$

Lagrange multiplier for budget constraint

Optimal allocation to transportation $j$

Initial inoperability for transportation $j$

Change in objective function per change in inoperability

Effectiveness of general allocation
Modeling philosophy

1. Pareto front

2. Solution if \( z_0 = 0 \)

3. Conditions when \( z_0 = 0 \)

4. Tradeoffs

5. Impact of allocation with respect to time
Impact of $k_0$ on optimal allocation

For larger budgets, allocate higher proportion of budget to $z_0$
If $z_0 > 0$

The inoperability for transportation $j$ as a function of $z_0$ is given by:

$$\hat{q}_j(z_0) = \hat{q}_j \exp(-k_0 z_0)$$

Calculate $z_j^*$ as a function of $z_0$.

Optimal allocation when $k_0 = 0.4$
Alternative interpretation for $Z_0$

- $Z_0$ could represent preparedness activities in advance of the disruption
- Initial inoperability decreases as $Z_0$ increases
- But model assumes that planners can prepare for transportation disruption with certainty
Tradeoff between $k_0$ and budget

Contour plot of objective function

Lower cost and less delay
Modeling philosophy

1. Pareto front
2. Solution if \( z_0 = 0 \)
3. Conditions when \( z_0 = 0 \)
4. Tradeoffs with budget
5. Impact of allocation with respect to time
Discrete time dynamic model

minimize \( J = \sum_{t=0}^{t_f} \left[ \alpha \sum_{i=1}^{n} f_i(q(t)) + (1 - \alpha) \sum_{i=1}^{n} g_i(q(t)) \right] \)

subject to \( q_j(t + 1) = q_j(t) \exp \left( -k_j(t)z_j(t) - k_0(t)z_0(t) \right) \)

\[ \sum_{t=0}^{t_f} \left[ z_0(t) + \sum_{j=1}^{m} z_j(t) \right] \leq Z \]

\( z_j(t) \geq 0, \quad z_0(t) \geq 0 \)

\( q(0) = \hat{q} \)
Dynamic models and effect of time

![Graph showing the effectiveness of resource allocation over time (in months). The graph indicates a flat line, suggesting no change in effectiveness over time.]
Dynamic models and effect of time
Dynamic models and effect of time

Effectiveness of resource allocation

Time (months)
Discrete time dynamic model

\[
\begin{align*}
\text{minimize } J &= \sum_{t=0}^{t_f} \left[ \alpha \sum_{i=1}^{n} f_i(q(t)) + (1 - \alpha) \sum_{i=1}^{n} g_i(q(t)) \right] \\
\text{subject to } q_j(t + 1) &= q_j(t) \exp \left( -tk_jz_j(t) - tk_0z_0(t) \right) \\
\sum_{t=0}^{t_f} \left[ z_0(t) + \sum_{j=1}^{m} z_j(t) \right] &\leq Z \\
z_j(t) &\geq 0, \quad z_0(t) \geq 0 \\
q(0) &= \hat{q}
\end{align*}
\]
Results

Optimal allocation when $k_0 = 0$

Optimal allocation when $k_0 = 0.4$

Budget

Number of periods since disruption
Comparing allocations when budget = 2

Optimal allocation when $k_0 = 0$

Optimal allocation when $k_0 = 0.4$

Water
Railroad
Highway
General
Conclusions

• Static model
  – Optimal allocation as function of initial inoperability, effectiveness of allocation, and importance of transportation infrastructure to firms
  – Solution approach based on effectiveness of allocation to $z_0$
  – Different tradeoffs in model illustrated with numerical example

• Dynamic model
  – If effectiveness of resources is constant or decreases with time, optimal to allocate immediately
  – If effectiveness of resources increases with time, may be desirable to wait to allocate
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