NUMERICAL METHODS FOR THE SOLUTION OF EQUATIONS

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This book contains methods for the numerical solution of nonlinear equations in one variable. The methods are divided into three classes:

1. Bracketing Methods, e.g. Bisection,
2. Fixed Point Methods, e.g. Newton's method and
3. Hybrid Methods.

Interval methods will not be presented. The efficiency, $E = p/d$, and efficiency index, $I = p^{1/d}$, of each scheme is given (where p is the order of the method and d is the information usage). The book also updates the excellent bibliography in Traub and in Jarratt (over 300 item in the bibliography).

For example, I will include the list of fixed points method in the following tables.

Algorithm	p	d	E	I	f'	f''	...	$f^{(k)}$
Fixed Point (Picard)	1	1	1	1				
Steffensen	2	2	1	1.414				
Wegstein	1.618	1	1.618	1.618				
Popovski [235]	1.839	1	1.839	1.839				
Newton	2	2	1	1.414	1			
Secant	1.618	1	1.618	1.618				
Muller	1.839	1	1.839	1.839				
Hansen & Patrick	3	3	1	1.442	1	1		
Traub [279], *p.*233	1.839	1	1.839	1.839				
Weerakoon & Fernando	3	3	1	1.442	2	0		
Homeier [137]	3	3	1	1.442	2	0		
Popovski [230]	3	3	1	1.442	1	1		
Halley	3	3	1	1.442	1	1		
Laguerre	3	3	1	1.442	1	1		
Chebyshev	3	3	1	1.442	1	1		
Cauchy [49]	3	3	1	1.442	1	1		
Euler	3	3	1	1.442	1	1		
Ostrowski	3	3	1	1.442	1	1		
Popovski [226]	3	3	1	1.442	1	1		
Milovanovich et al	3	3	1	1.442	1	1		
Dordjevic	2	2	1	1.414	1			
Varyukhin et al	$n+2$	$n+2$	1	$-$	1	1	...	1 $(k=n+1)$
Jarrat & Nudds	1.839	1	1.839	1.839				
Popovski [236]	1.839	1	1.839	1.839				
(2) *Traub* [279], *pp.*$233-234$	1.839	1	1.839	1.839				
Ostrowski	2.414	2	1.207	1.554	1			
Popovski [236]	2.414	2	1.207	1.554	1			
(3) *Popovski* [241]	2.414	2	1.207	1.554	1			
(10) *Popovski* [241]	4.562	3	1.520	1.66	1			
Jarratt [150]	2.732	2	1.366	1.653	1			
(3) *Traub*	2.732	2	1.366	1.653	1			
Jarratt [149]	3	3	1	1.442	2			
(2) *Jarratt* [149]	4	4	1	1.414	3			
Jarratt [149]	5	4	1.25	1.495	3			
Traub [279], *p.*165	3	3	1	1.442	2			
Mikeldze	5	5	1	1.38	4			
Traub [279], *pp.*$200-204$	4	4	1	1.414				
Homeier [138]	3	3	1	1.442	1			
Sharma [260]	3	3	1	1.442	1			
Ostrowski	4	3	1.333	1.587	1			

Table 2: Comparison of Fixed Point Type Methods

Algorithm	p	d	E	I	f'	f''	...	$f^{(k)}$
Sharma & Goyal [262]	4	3	1.333	1.587				
Jarratt [152]	4	3	1.333	1.587	2			
Sharma [261]	6	4	1.5	1.565	2			
Sharma [261]	5	4	1.25	1.495	2			
(4) *Jarratt* [148]	4	3	1.333	1.587	2			
Werner [294]	2.414	2	1.207	1.553	1			
Werner [294]	†	m	$-$	$-$	$m-1$			
Werner [295]	4	3	1.333	1.587	1	1		
Werner [295]	$k+2$	$k+1$	$-$	$-$	1	1	...	1
Werner [297]	3	3	1	1.442	1			
(4) *Werner* [297]	4	3	1.333	1.587	1	1		
King [168]	4	3	1.333	1.587	1			
Ostrowski [217]	4	3	1.333	1.587				
Grau et al [108]	6	4	1.5	1.565				
King [165]	5	4	1.25	1.495	2			
Popovski [237]	7.464	4	1.866	1.653	2			
Neta [206]	6	4	1.866	1.653	2			
Popovski [240]	7	4	1.75	1.626	1			
Neta [208]	10.815	4	2.704	1.813	1			
Werner [296]	$2k$	$k+2$	$-$	$-$	1			
Werner [296]	‡	$k+2$	$-$	$-$				
Werner [296]	$2k+1$	$k+2$	$-$	$-$				
Werner [296]	$2k+2$	$k+2$	$-$	$-$				
Popovski [242]	6	4	1.5	1.565	1	1		
Neta [207]	14	5	2.8	1.695	1			
Neta [207]	16	5	3.2	1.741	1			
Chambers [51]	2.414	2	1.207	1.554	1			
Chambers [51]	2	2	1	1.414				
Chambers [51]	2.732	2	1.366	1.653				
Murakami [203]	5	4	1.25	1.495	1			
Popovski [227]	3	3	1	1.442	1	1		
Popovski [231]	3	3	1	1.442	1	1		
Amat et al	3	3	1	1.442	1	1		
Kiss / Lika	4	4	1	1.414	1	1	1	
Altman [8]	4	4	1	1.414	1	1	1	
Nourein	5	5	1	1.38	1	1	...	1 $(k=4)$
(9) *Popovski* [243]	3	3	1	1.442	1	1		
(21) *Neta* [209]	3	3	1	1.442	1	1		

Table 3: Comparison of Fixed Point Type Methods

† $p = k + \sqrt{k^2+1}$

‡ $p = \frac{m}{2} + \sqrt{\frac{m^2}{4}+1}$

Algorithm	p	d	E	I	f'	f''	$\ldots$	$f^{(k)}$
$Jain\ (implicit)$	5	–	–	–	2			
$Jain\ (semi-expl.)$	4	–	–	–	2			
$Jain\ (implicit)$	3	–	–	–	1			

Table 4: Comparison of Fixed Point Type Methods - Continued