Comparison of Orbit Propagators in the Research and Development Goddard Trajectory Determination System (R & D GTDS). Part I: Simulated Data

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August 20, 1995

Abstract

This paper evaluates the performance of various orbit propagation theories for artificial earth satellites in different orbital regimes. Specifically, R&D GTDS's Cowell (numerical technique), DSST (semianalytical technique), SGP, SGP4, and Brouwer-Lyddane (analytic techniques) orbit propagators are compared for decaying circular (~200 km perigee height), low altitude circular (590 km perigee height), high altitude circular (1340 km perigee height), Molniya,

and geosynchronous orbits. All test cases implement a one orbital period differential correction fit to simulated data derived from a Cowell truth trajectory. These fits are followed by a one orbital period predict with the DC solve-for vector. Trajectory comparisons are made with the Cowell "truth" trajectory over both the fit and predict spans. Computation time and RMS errors are used as comparison metrics. The Unix-based version of R&D GTDS (NPS SUN Sparc 10) is the test platform used in this analysis.

Nomenclature

a	Semimajor Axis
AOG	DSST Averaged Orbit Generator
BL	Brouwer-Lydanne Analytic Propagator
B^*	SGP4 Drag Parameter
C_D	Drag Coefficient
C_R	SRP Parameter
DC	Differential Correction
DSST	Draper Semianalytic Satellite Theory
e	Eccentricity
E	Eccentric Anomaly
f	True Anomaly
F	Eccentric Longitude (E + ω + Ω)
GEM	Goddard Earth Model
GPS	Global Positioning System
HST	Hubble Space Telescope
i	Inclination
ITER	Number of DC Iterations
JGM	Joint Gravitational Model
kg	Kilograms
km	Kilometers
L	True Longitude (f + ω + Ω)
m	Meters
M	Mean Anomaly
min	Minutes
MSIS	Mass Spectrometer Incoherent Scatter
n,m	Degree, Order of Gravity Field
$\dot{n}/2$	SGP Drag Parameter
N_{pq}	BL Drag Parameter
OD	Orbit Determination
opt	Optimized
ORB1	GTDS Output Ephemeris File
osc	Osculating Elements
$R\&D\,GTDS$	Draper Laboratory's Version of the Goddard Trajectory Determination System

RMS	Root Mean Square
SATCAT	Satellite Catalog
SCN	Space Control Network
sec	Seconds
SGP	Analytic Theory Based on the Work of Kozai[13]
SGP4	Analytic Theory Based on the Work of Lane and Cranford[22]
SPG	DSST Short Periodic Generator
SRP	Solar Radiation Pressure
SSN	Space Surveillance Network
TOPEX	Topographical Ocean Experiment Satellite
WGS	World Geodetic Survey
λ	Mean Longitude (M + ω + Ω)
θ	Greenwich Hour Angle
ω	Argument of Perigee
Ω	Longitude of Ascending Node

Introduction

R&D GTDS is Draper Laboratory's research-based orbit determination testbed[1]. This analysis tool evolved from its R&D counterpart at the Goddard Space Flight Center. Dr. Paul Cefola, Program Manager at Draper Laboratory and Lecturer at the Massachusetts Institute of Technology, has overseen the development and expansion of this testbed (by a team of scientists at Draper Laboratory and a continuing string of graduate students at MIT) over the past twenty years (see Fonte[7]). In its current form, R&D GTDS is capable of performing

- early orbit determination
  - Gauss,
  - Double-R iteration,
  - range and angles methods,
  - range-only method,
- orbit propagation
  - numerical,
  - analytical,
  - seminanalytical techniques,
- estimation
  - batch weighted least squares,
  - filtering algorithms,

- error analysis
  - error categorization for the OD process,
- data simulation
  - simulated tracking data from specified ground stations.

This paper will present a comparative study of the various orbit propagation techniques (Cowell, DSST, SGP, SGP4, and BL) imbedded in R&D GTDS. These orbit propagators represent a standard blend of special and general perturbation techniques. With special perturbation techniques (Cowell), a direct numerical integration of the equations of motion is implemented. This entails multiple force evaluations to "step" from a given set of initial conditions to a final solution. If the initial conditions are changed or altered, new evaluations must be made at each of the various time steps. Therefore, each solution is unique to a given set of initial conditions. In special perturbation methods, consideration must be given to the computation time, physical accuracy, truncation error, and round-off error associated with selection of an appropriate time step for integration. In general, high accuracy is obtained at the cost of computational efficiency (these techniques have typically been limited to "special case" processing in Space Command's operational environment due to their inherent computational burden; this includes, but is not limited to, scenarios requiring high accuracy or a limited number of satellites).

General perturbation techniques (SGP, SGP4, BL), on the contrary, do not use multiple time steps to transfer from a set of initial conditions to a final solution. Rather, general perturbation techniques implement an analytical integration of the equations of motion to transform a set of initial conditions directly to a final solution. However, due to the complex nature of the equations representing the physical models, exactly integrable expressions are difficult to obtain. For this reason, simplifications, approximations, and truncations are made to the equations of motion to obtain expressions which are integrable. These simplifications, however, greatly reduce the accuracy attainable with general perturbation techniques. In summary, computational efficiency is gained at the cost of accuracy (these techniques have typically been used for the bulk of Space Command's operational missions due to the computational efficiency advantages).

A more recent approach to perturbation analysis is semianalytic techniques. These techniques combine the primary advantageous aspect of special perturbation methods (high accuracy) with that of general perturbation techniques (computational efficiency). For one such semianalytic technique, the Draper Semianalytic Satellite Theory (DSST), the equations of motion are separated into two distinct categories (see McClain[3,4], Danielson et al[2]). One category contains the secular and long-period perturbative contributions to a satellite's motion; the portion of the software which propagates this motion is referred to as the averaged orbit generator (AOG), and is based upon "mean" elements. The other category contains the short-period perturbative contributions to a satellite's motion, which are modeled in the short periodic generator (SPG; DSST represents the state of the art in short periodic models). The separation of the short-periodic contributions from the secular and long-period contributions is accomplished via the generalized method of averaging (see Morrison[9]). With the short-periodic contributions separated (which, due to their high frequency nature, are step-size constraining), the averaged equations of motion can be integrated numerically with large step sizes (typically on the order of a day), while the short periodics can be recovered analytically at output times (with the use of Fourier analysis and interpolation schemes). The specific force models included in the AOG and SPG can be chosen by the user at run time, creating a theory that is highly accurate, efficient, and flexible. It is of the authors' opinion that these techniques are optimally suited for operational processing (as well as for a multitude of other applications, such as long-term mission planning).

With special, general, and semianalytic perturbation techniques defined, it is now possible to describe the specific force models available in each of the propagators analyzed in this study:

Cowell * (user has ability to select force models) (see GTDS Mathematical Specification[1])

- Geopotential
  - models ranging from GEM1 to GEMT3, JGM1, JGM2, WGS72, and WGS84
  - maximum degree and order of 50 (n,m $\leq$ 50) †
- Atmospheric Drag
  - Harris-Priester (static atmosphere; tabulated values of density versus altitude)
  - Jacchia-Roberts (dynamic atmosphere; solar flux and geomagnetic indices)
  - Jacchia 70
  - Jacchia 64
  - MSIS

*It should be noted that the same force models are available for both Cowell and DSST. Geopotential models, atmosheric data, solar lunar and planetary ephemerides, timing coefficients, and Newcomb operator data exist in a binary data base attached to R&D GTDS. As new models or raw inputs become available, the binary data bases are updated and easily attached to R&D GTDS.

†50 by 50 software is available in a library linkable to R&D GTDS. The current code at Naval Postgraduate School has a maximum degree and order of 21

- Third-Body (Lunar-Solar) Point Mass (JPL solar, lunar, and planetary ephemerides; extendable to multi-body)
- Solar Radiation Pressure (cylindrical shadow model)

DSST ‡ (user has ability to select force models) (see McClain[3,4], and Danielson et al [2])

- Geopotential
  - models ranging from GEM1 to GEMT3, JGM1, JGM2, WGS72 and WGS84
  - maximum degree and order of 50 (n,m $\leq$ 50) §
  - First Order, AOG
    - recursive, closed form zonal model
    - recursive tesseral resonance model
  - First Order, SPG
    - recursive, closed form zonal short periodic model (Fourier series in L)
    - recursive, closed form tesseral m-daily short periodic model (Fourier series in θ)
    - recursive tesseral linear combination short periodic model (Fourier series in λ and θ)
  - Second Order, AOG
    - J_2^2 to first order in e
  - Second Order, SPG
    - J_2^2 to zeroth order in e
  - Second Order Coupling Terms, AOG
    - J_2 / drag coupling
  - Second Order Coupling Terms, SPG
    - recursive, closed form J_2 / tesseral m-daily model
    - Drag / m-daily

‡It should be noted that the same force models are available for both Cowell and DSST. Geopotential models, atmosheric data, solar lunar and planetary ephemerides, timing coefficients, and Newcomb operator data exist in a binary data base attached to R&D GTDS. As new models or raw inputs become available, the binary data bases are updated and easily attached to R&D GTDS.

§50 by 50 software is available in a library linkable to R&D GTDS. The current code at Naval Postgraduate School has a maximum degree and order of 21

- Atmospheric Drag, AOG and SPG
  - same drag models as described for Cowell
  - numerical averaging (quadrature)
  - SPG Fourier series expressed in λ
  - some optional second order drag effects
- Third-Body Point Mass (JPL solar, lunar, and planetary ephemerides; extendable to multi-body)
  - first order AOG
    * recursive, closed form lunar-solar model
  - first order SPG
    * recursive, closed form lunar-solar model (Fourier series in F)
    * weak time dependence corrections
  - double averaged formulation
- Solar Radiation Pressure, AOG and SPG
  - cylindrical shadow model
  - numerical averaging
  - SPG Fourier series expressed in λ

SGP (no force model flexibility other than drag parameter adjustment) (see Hoots[5], Fieger[6], and Herriges[10])

- gravitational model based on a simplification of the work of Kozai[13]
  - J_2 secular effects
  - J_2 and J_3 long-period effects
  - truncated J_2 short periodics to zeroth order in e
  - secular drag effects in a, e, and M (drag model based on Taylor series in mean anomaly ($\dot{n}/2$); drag effect modeled on eccentricity such that perigee height remains constant).

SGP4 (no force model flexibility other than drag parameter adjustment) (see Hoots[5], Fieger[6], Herriges[10], Lane and Hoots[11], and Hujsak and Hoots[12])

- gravitational model based on a simplification of the work of Brouwer[14]
  - J_2, J_4, and J_2^2 secular effects
  - J_2 and J_3 long-period effects

– truncated J_2 short periodics to zeroth order in e

– tesseral terms (2,2), (3,2), (5,2), (4,4), and (5,4) for orbits with periods $\geq$ 225 minutes; designed for geosynchronous and Molniya orbits

– first order lunar-solar point mass terms for orbits with periods $\geq$ 225 minutes

– drag model based on power density function $((q_o - s)^4$, B*), secular effects only

R&D GTDS BL (no force model flexibility) (see GTDS Mathematical Specification[1])

- Lyddane modified Brouwer theory to obtain algorithms applicable for singularities ($i = 0$ or $e = 0$)

  – J_2, J_4, and J_2^2 secular effects

  – J_2, J_3, J_4, and J_5 long-period effects

  – J_2 short periodics

  – drag model based on Brouwer drag coefficients, $\mathrm{N}_{p,q}$ ("free parameters" which can be solved-for in the DC to better model drag); correct mean anomaly only

R&D GTDS OD Options

- Coordinate frames

  – mean Earth equator and equinox of 1950.0

  – FK4 derived true of date (0h0m0s zulu)

  – mean Earth equator and equinox of 2000.0 (J2000; available through externally interfaced program)

  – FK5 derived true of date (true of time of interest; available through externally interfaced program)

- Time standards

  – implements UTC (broadcast time) as standard time

  – other time standards available via conversion utilities

Test Protocol

The comparisons in this analysis are based against Cowell truth trajectories; one Cowell truth trajectory was established for each orbital regime. For the low altitude circular, high altitude circular, Molniya and geosynchronous test cases, the osculating elements (and appropriate perturbation parameters) used to generate the truth trajectories were derived from Cowell fits to an SGP4 ephemeris derived from actual two-card element sets (SATCAT). For the decay case, the osculating elements were taken from Dyar[17]. This specific information pertinent to the truth trajectories is as follows:

Epoch Date	24 February 82
Epoch Time (hhmmss.ssss)	00:00:00.0000
a (osc)	6628.45287 km
e (osc)	0.008921
i (osc)	64.83828°
Ω (osc)	224.50715°
ω (osc)	271.80047°
M (osc)	115.250818°
Period	89.5113 min
Perigee Height	191.1834 km
Gravity Model	21x21 GEM10B
Atmos. Dens.	Jacchia-Roberts Drag (C_D = 2.0)
Third Body	Lunar - Solar Point Mass
SRP	No
Integrator	12^{th} Order Summed Cowell/ Adams Predict-Partially Correct
Step Size	60 sec
Input, Output and Integration Frame	Mean Earth Equator and Equinox of 1950.0
Spacecraft Area, Mass	1 m^2, 100 kg
ORB1 Output Frequency	Every 450 sec

Table 1: Decaying Orbit

Epoch Date	4 December 94
Epoch Time (hhmmss.ssss)	21:10:59.5456
a (osc)	6980.108566
e (osc)	0.00170791
i (osc)	28.327265°
Ω (osc)	218.492078°
ω (osc)	40.611761°
M (osc)	319.915316°
Period	96.7270 min
Perigee Height	590.05 km
Gravity Model	21x21 GEM10B
Atmos. Dens.	Jacchia-Roberts (Schatten) Drag (C_D = 2.59856)
Third Body	Lunar - Solar Point Mass
SRP	C_R = 8.68846
Integrator	12^{th} Order Summed Cowell/ Adams Predict-Partially Correct
Step Size	60 sec
Input, Output and Integration Frame	Mean Earth Equator and Equinox of 1950.0
Spacecraft Area, Mass	1 m^2, 100 kg
ORB1 Output Frequency	Every 400 sec

Table 2: Low Altitude, Circular Orbit (SATCAT # 20580, HST)

Epoch Date	4 December 94
Epoch Time (hhmmss.ssss)	11:30:01.5434
a (osc)	7721.500966
e (osc)	0.000482180
i (osc)	65.830824°
Ω (osc)	296.770687°
ω (osc)	19.611311°
M (osc)	340.277743°
Period	112.5425 min
Perigee Height	1339.6408 km
Gravity Model	21x21 GEM10B
Atmos. Dens.	Jacchia-Roberts (Schatten) Drag (C_D = 1.8661152)
Third Body	Lunar - Solar Point Mass
SRP	C_R = -5.4211956
Integrator	12^{th} Order Summed Cowell/ Adams Predict-Partially Correct
Step Size	60 sec
Input, Output and Integration Frame	Mean Earth Equator and Equinox of 1950.0
Spacecraft Area, Mass	1 m^2, 100 kg
ORB1 Output Frequency	Every 460 sec

Table 3: High Altitude, Circular Orbit (SATCAT # 22076, TOPEX)

Epoch Date	29 August 94
Epoch Time (hhmmss.ssss)	06:25:23.8964
a (osc)	26586.72673
e (osc)	0.731526
i (osc)	63.958664°
Ω (osc)	47.836839°
ω (osc)	289.061990°
M (osc)	9.509759°
Period	$\sim$12 hours
Perigee Height	759.70787 km
Gravity Model	21x21 GEM10B
Atmos. Dens.	Jacchia-Roberts (Schatten) Drag ($C_D = 160.9959772$)
Third Body	Lunar - Solar Point Mass
SRP	$C_R = 4.5991303$
Integrator	12^{th} Order Summed Cowell/ Adams Predict-Partially Correct
Step Size	60 sec
Input, Output and Integration Frame	Mean Earth Equator and Equinox of 1950.0
Spacecraft Area, Mass	1 m^2, 100 kg
ORB1 Output Frequency	Every 450 sec

Table 4: Molniya Orbit (SATCAT # 21897)

Epoch Date	31 August 94
Epoch Time (hhmmss.ssss)	16:19:28.4739
a (osc)	42549.449791
e (osc)	0.000487508
i (osc)	11.162079°
Ω (osc)	47.617617°
ω (osc)	56.938285°
M (osc)	74.862562°
Period	$\sim$24 hours
Perigee Height	36150.5696 km
Gravity Model	21x21 GEM10B
Atmos. Dens.	No
Third Body	Lunar - Solar Point Mass
SRP	$C_R = 5.2927536$
Integrator	12^{th} Order Summed Cowell/ Adams Predict-Partially Correct
Step Size	60 sec
Input, Output and Integration Frame	Mean Earth Equator and Equinox of 1950.0
Spacecraft Area, Mass	1 m^2, 100 kg
ORB1 Output Frequency	Every 450 sec

Table 5: Geosynchronous Orbit (SATCAT # 10516)

Note: Even though the orbits used in this analysis can be tied to a specific object number in the SATCAT, a standard area of 1 m^2 and mass of 100 kg were implemented throughout the test cases (which provides a "rule-of-thumb" area to mass ratio). In addition, all test cases except the decaying orbit have epochs in 1994; the decay case, which was chosen to demonstrate the specific effects of atmospheric drag, used a "noisy" epoch in 1982 (geomagnetic indices were disturbed in this time period).

The Jacchia-Roberts (Schatten) file represents a particular version of a Jacchia-Roberts file generated by Ken Schatten (Goddard Space Flight Center) and David Carter[18] (Draper Laboratory). Schatten uses monthly values of solar flux and geomagnetic activity to generate a "smooth" (yet still dynamic) atmosphere. Carter applied interpolation techniques to reduce Schatten's monthly values to "smooth" daily values, which are required by R&D GTDS. Schatten's files performed as well as any other atmospheric model during the last peak of the solar cycle, and for this reason, have been included in the binary data base of R&D GTDS (Cefola[19]). More details concerning these procedures can be found in the work of Sabol[15].

All tests in support of this analysis implemented one-orbital period DC fits to the appropriate Cowell truth model. The "observations" used in these fits were simulated data in the form of GTDS ORB1 files (evenly spaced, time tagged values of position and velocity; very much like GPS navigation solutions) based on the Cowell trajectories. One orbital period predictions with the DC solve-for vectors were then generated and compared with the Cowell truth trajectories. RMS errors over the fit (DC RMS) and predict (R&D GTDS Ephemeris Comparison Program RMS) spans were used as accuracy metrics; timing comparisons were based on a call to an internal clock routine at the initiation and termination of the program.

Results of Testing

For each of the five classes of satellite orbits (decaying, low altitude circular, high altitude circular, Molniya and geosynchronous) several orbit propagators/orbit propagator configurations were analyzed. In general, the following protocol was used:

- One Cowell DC with force models matching those in the truth ephemeris; this run ensures the DC can reconstruct the truth ephemeris from perturbed initial conditions (i.e., test if the DC is functioning properly).

- Cowell DCs with force models truncated as compared to the truth ephemeris. These runs analyze the "pure" impact of truncating force models (i.e., reducing the geopotential from 21 x 21 to 4 x 4, etc.).

- One DSST DC with force models configured to balance computational efficiency and accuracy; these configurations are based on the work of Fonte and Sabol[16].

- Various analytic DCs (BL, SGP and SGP4) with and without drag solve-fors. It should be noted solving for drag parameters in orbital regimes where drag doesn't have a large effect was intentional; in these cases, the drag parameter simply represents a "free-parameter" in the DC attempting to absorb errors stemming from truncations in force models for perturbations other than drag.

In addition, several different element sets were used as initial conditions for the DC in each case. With the exception of the decay case, results quoted in this paper are based on using the two-card element set from which the truth trajectories were generated. This methodolgy provided a standard set of initial conditions from which to analyze results. It should be noted this may provide an unfair advantage to SGP and SGP4 (i.e., Cowell uses osculating elements and therefore, the DC may have to execute extra iterations in order to converge). This protocol must be considered when analyzing timing results (the initial conditions for the decaying orbit DCs were taken from osculating truth output ephemeris).

Furthermore, three sets of timing statistics given for each case: time to perform the DC only, time to perform the DC and subsequent ephemeris generation, and DC time per iteration. The total time to perform the DC is representative of processing times for operational catalog maintenance; ephemeris generation times provide insight to the speed of pure orbit propagation with the various theories. DC time per iteration provides an alternative comparison metric for theories which require a different number of iterations to arrive at a final solution. However, processing is based upon arriving at a final solution, so both time per DC iteration and total DC time are meaningful metrics.

Results can be now given for each of the various orbit classes.

DECAYING ORBIT:

Configuration Notes: In this case, Cowell$_{21}$ has force models matching those in the truth (21x21 GEM10B geopotential, Jacchia-Roberts drag, lunar solar point mass effects, solve-for drag parameter in DC). Cowell$_4$ is identical to Cowell$_{21}$, with the exception of a reduced geopotential (i.e., 4x4). Cowell$_4^*$ differs from Cowell$_4$ only in that the exact drag term from the truth ephemeris has been hardwired into the run (it is not being solved for by the DC). The annotation of N_{pq} or "drag" after a theory indicates DC solve-for parameters used in the run.

The results given in Table 6 clearly show the dominance of atmospheric and gravitational effects. For example, reducing the Cowell geopotential configuration from 21x21 to 4x4 results in ~265 m predict error over one revolution (refer to Cowell$_4^*$). The Cowell$_4$ results highlight the DC "observability" problems associated to coupling an extremely short fit span with theories having severely truncated force models; the Cowell$_4$ DC, which attempted to solve for a drag term, produced an error ~100 m larger than Cowell$_4^*$ (~375 m). Inspection of

Theory	DC RMS (m)	ITER	Predict RMS (m)	CPU (sec)	DC CPU	CPU per ITER
BL	229.40839	6	5432.4	1.88	1.37	.23
BL (N_{pq})	236.62321	5	2094.90	2.75	1.32	.26
Cowell_{21}	2.60343(E-9)	5	2.2922(E-8)	9.17	2.77	.55
Cowell_4	22.761151	4	374.11	6.96	2.03	.51
Cowell_4^*	22.292897	4	264.14	7.74	2.34	.59
SGP4	138.36233	4	8897.6	1.96	1.45	.36
SGP4 (drag)	138.63008	5	503.23	2.56	1.66	.33
SGP	638.31315	4	7858.4	2.46	1.40	.35
SGP (drag)	1182.5092	4	12598.0	2.47	1.14	.29
DSST (opt)	17.416619	6	65.497	6.83	5.17	.86

Table 6: Decay Orbit 90 Minutes Fit and 90 Minutes Predict

the solved-for drag term in the Cowell_4 DC indicated the DC could not exactly recover the drag term used in the generation of the truth ephemeris (remember, the Cowell_4^* DC used the exact drag term used in the generation of the truth orbit in order to determine the "pure" geopotential error). The analytic theories also experience these observability problems, which are further compounded by their lack of tesseral terms and simplistic drag models. Operationally, these problems are countered by increased fit spans in an attempt to "observe" more of the effect which has been neglected (for decay cases, in which it may not be practical or possible to increase the fit span; increased amounts of observational data can also be used). In this manner, the solve-for parameters in the DC absorb some of the error stemming from truncated force models (the parameters, therefore, become less physical; even though the fit may be improved, predictions with DC solve-for vector become worse). The optimized DSST results indicate properly configured perturbation theories can still produce excellent results with the short fit spans; however, the timing statistics suggest that further refinement of the optimized DSST could decrease the amount of time required for the run–but at the expense of accuracy (users can configure DSST for specific requirements).

It is worth stressing atmospheric effects are the limiting factor in the decay case. Further testing with Cowell and DSST was undertaken in an attempt to more realistically represent real world atmospheric conditions. These DCs used a static drag model (Harris-Priester) to fit the dynamic (Jacchia-Roberts) truth model. This protocol takes into account that Jacchia-Roberts atmospheric parameters (F10.7 solar flux values and geomagnetic indices) are very difficult to predict or may not be obtained in a timely manner. Therefore, fitting the noisy Jacchia-Roberts model with the smooth Harris-Priester model (hottest H-P table) emulates a lack of perfect knowledge of the atmospheric parameters. The results of this testing are given in Table 7.

Theory	DC RMS (m)	ITER	Predict RMS (m)	CPU (sec)	DC CPU	CPU per ITER
Cowell_{21}	1.190631	4	183.30	7.53	2.06	.52
DSST (opt)	16.308181	6	149.86	5.07	3.74	.62

Table 7: Decaying Orbit (Real World) 90 Minutes Fit and 90 Minutes Predict

These results indicate the optimized DSST performs as well as Cowell_{21} with a significant savings in computational time.

LOW ALTITUDE CIRCULAR ORBIT:

Configuration Notes: In this case, Cowell_{21} has force models matching those in the truth (21x21 GEM10B geopotential, Jacchia-Roberts (Schatten) drag, lunar solar point mass effects, SRP, solve-for drag and SRP parameters in DC). Cowell_4 is identical to Cowell_{21}, with the exception of a reduced geopotential (i.e., 4x4). Cowell_4^* differs from Cowell_4 only in that the exact drag and SRP terms from the truth ephemeris have been hardwired into the run (they are not being solved for by the DC). The annotation of N_{pq} or "drag" after a theory indicates DC solve-for parameters used in the run.

In this analysis, reducing the Cowell geopotential configuration from 21x21 to 4x4 results in $\sim$42 m predict error over one revolution (refer to Cowell_4^*). Again, DC "observability" problems associated to coupling a short fit span with theories having severely truncated force models is evident in the Cowell_4 DC; this DC, which attempted to solve for both a drag and SRP term, produced an error $\sim$160 m larger than Cowell_4^* ($\sim$200 m). Inspection of the solved-for drag and SRP terms in the Cowell_4 DC indicated the DC could not exactly recover the original terms used in the generation of the truth ephemeris (remember, the Cowell_4^* DC used the exact drag and SRP terms used in the generation of the truth orbit in order to determine the "pure" geopotential error). In addition, this orbit is at an altitude for which SRP has a very minimal effect, which also limits the ability of the DC to properly "observe" the perturbing effect (in fact, additional testing indicates a solution for the SRP term is not required at this altitude except for scenarios requiring the utmost accuracy). As with the decay

Theory	DC RMS (m)	ITER	Predict RMS (m)	CPU (sec)	DC CPU	CPU per ITER
BL	50.287918	4	197.18	1.84	1.10	.27
BL (N_{pq})	49.534240	4	231.34	1.88	1.06	.26
Cowell_{21}	2.72860(E-8)	5	0.19385(E-3)	4.25	3.00	.60
Cowell_{4}	14.536095	5	202.50	3.77	2.49	.50
Cowell_{4}^{*}	15.768104	5	41.655	3.68	2.88	.58
SGP4	54.361123	3	196.34	2.15	1.03	.34
SGP4 (drag)	53.842927	4	669.41	2.35	1.57	.39
SGP	210.20938	3	933.95	1.95	1.06	.35
SGP (drag)	688.68465	4	14701.0	2.07	1.16	.29
DSST (opt)	34.889745	4	47.274	2.99	2.03	.51

Table 8: HST Orbit 100 Minutes Fit and 100 Minutes Predict

Theory	DC RMS (m)	ITER	Predict RMS (m)	CPU (sec)	DC CPU	CPU per ITER
Cowell_{21}	7.12470788(E-4)	10	0.35502	5.60	4.87	.49
Cowell_{4}	14.595436	5	242.54	3.75	2.67	.53
DSST (opt)	34.862539	4	48.372	2.71	1.68	.42

Table 9: HST Orbit (Real World) 100 Minutes Fit and 100 Minutes Predict

case, the performance of the analytic theories is limited by the truncation of the geopotential and simplistic atmospheric models; however, this orbit class (as well as the high altitude circular case) represents an orbital regime in which the best performance from the analytic theories can be expected . Furthermore, observability problems can also be noticed with the analytic theories in that solving for drag terms in the DC produces larger errors than for cases without drag parameter solutions. The optimized DSST DC again performed very well in terms of accuracy, but run times were not as optimistic as expected. To more realistically assess the run time for optimized DSST, and also to simulate operational atmospheric difficulties, further testing was accomplished to fit the Harris-Priester model (default H-P table) to the Jacchia-Roberts (Cowell) truth. These results are given in Table 9.

These results more accurately reflect the blend of efficiency and accuracy that can be obtained with DSST. Further results for this orbit class can be found in Fonte[8,23].

HIGH ALTITUDE CIRCULAR ORBIT:

Configuration Notes: In this case, Cowell_{21} has force models matching those in the truth (21x21 GEM10B geopotential, Jacchia-Roberts (Schatten) drag, lunar solar point mass effects, SRP, solve-for drag and SRP parameters in DC). Cowell^*_{21} has 21x21 GEM10B geopotential and lunar solar point mass effects only. Cowell_4 is identical to Cowell_{21}, with the exception of a reduced geopotential (i.e., 4x4). Cowell^*_4 differs from Cowell_4 only in that the exact drag and SRP terms from the truth ephemeris have been hardwired into the run (they are not being solved for by the DC). The annotation of N_{pq} or "drag" after a theory indicates DC solve-for parameters used in the run.

For the orbit chosen to fulfill the high altitude circular test case, reducing the Cowell geopotential configuration from 21x21 to 4x4 results in ~105 m predict error over one revolution (refer to Cowell^*_4). It may seem peculiar the error using a 4x4 geopotential model is larger for this orbit than for the lower altitude, HST orbit. This discrepancy can be attributed to the larger inclination for the TOPEX orbit (subsequent testing with HST at an inclination similar to the TOPEX orbit produces a predict error ~ 170m.) As with the decay and low altitude test cases, DC "observability" problems are once again present; specifically, an error ~150 m larger than Cowell^*_4 is obtained when the DC attempts to solve for drag and SRP terms in the Cowell_4 case (predict error ~250 m). In addition, this orbit is at an altitude for which drag has a small effect, which limits the ability of the DC to properly "observe" the perturbing effect (in fact, additional testing proves a solution for the drag term is not necessary at this altitude for *most* applications; it must be noted, however, that in some analytic theories, the drag solve-for can be used as a "free parameter" in the DC to absorb errors stemming from truncated force models for perturbations other than atmospheric drag). The Cowell^*_{21} case indicates the combined effects on drag and SRP are less than 4 meters for this particular case. As mentioned

Theory	DC RMS (m)	ITER	Predict RMS (m)	CPU (sec)	DC CPU	CPU per ITER
BL	24.620212	5	185.25	2.05	1.03	.21
BL (N_{pq})	23.177123	5	524.85	1.92	0.98	.20
Cowell$_{21}$	0.6683(E-9)	5	0.43849(E-3)	4.61	3.11	.62
Cowell$^*_{21}$	0.2995392	5	3.7135	3.74	2.59	.52
Cowell$_4$	7.9674481	5	250.17	3.98	2.69	.54
Cowell$^*_4$	7.8043229	6	105.07	4.62	3.32	.55
SGP4	23.568135	3	182.27	2.28	0.95	.32
SGP4 (drag)	19.894640	7	358.55	2.17	1.41	.20
SGP	556.90745	3	2994.5	1.85	1.02	.34
SGP (drag)	773.92103	3	6828.3	2.02	1.10	.37
DSST (opt)	19.431369	4	31.769	2.23	1.41	.35

Table 10: TOPEX Orbit 115 Minutes Fit and 115 Minutes Predict

previously, the high altitude circular case represents an orbital regime in which the best performance from the analytic theories can be expected; however, the errors from the analytic theories are still 6 times as large as from optimized DSST (with optimized DSST running at speeds comparable to SGP4). Furthermore, the use of drag solve-for terms in the analytic theories did not help to absorb errors from truncations in perturbation models other than atmospheric drag.

MOLNIYA ORBIT:

Configuration Notes: In this case, Cowell$_{21}$ has force models matching those in the truth (21x21 GEM10B geopotential, Jacchia-Roberts (Schatten) drag, lunar solar point mass effects, SRP, solve-for drag and SRP parameters in DC). Cowell$_4$ is identical to Cowell$_{21}$, with the exception of a reduced geopotential (i.e., 4x4). Cowell$^*_4$ differs from Cowell$_4$ only in that the exact drag and SRP terms from the truth ephemeris have been hardwired into the run (they are not being solved for by the DC). The annotation of N_{pq} or "drag" after a theory indicates DC solve-for parameters used in the run.

In the Molniya case, geopotential terms beyond the 4x4 configuration of Cowell$^*_4$ (up to the 21x21 limit in the truth ephemeris) introduce almost 65 m worth of "pure" error over the twelve hour predict. The majority of this error can be attributed to neglected resonance terms (the Molniya orbit is resonant at the even orders; the Cowell$^*_4$ DC captured only the 2^{nd} and 4^{th} resonant orders). If these results are compared to those of Cowell$_4$, it is clear this case

Theory	DC RMS (m)	ITER	Predict RMS (m)	CPU (sec)	DC CPU	CPU per ITER
BL	N.A.		.			
BL (N_{pq})	N.A.		.			
Cowell_{21}	0.4356(E-8)	6	0.40299(E-4)	16.05	11.94	1.99
Cowell_4	0.119446(E-2)	28	577.20	43.15	38.81	1.39
Cowell_4^*	0.111818(E-2)	29	64.329	39.45	39.34	1.36
SGP4	1906.6052	15	6988.8	11.19	9.80	0.65
SGP4 (drag)	div. DC		.			
SGP	1942.0303	14	6392.6	9.18	8.21	.59
SGP (drag)	1721.1822	18	105170.	11.74	10.35	.58
DSST (opt)	34.803726	8	174.37	11.87	10.87	1.36

Table 11: Molniya Orbit 12 Hour Fit and 12 Hour Predict

also exhibits observability problems. Specifically, a poor solution for the drag parameter was obtained by the Cowell_4 DC. The force models for BL proved inadequate for this orbit in that the satellite impacted the earth during the DC processing. Therefore, neither accuracy nor timing metrics could be derived (in fairness, the BL theory was neither developed nor intended for use with this orbit type). Even though SGP may not be practical for this orbit type, its performance was still analyzed (very poor performance); however, since SGP4's geopotential model had been modified to include some 12 hour resonance terms (a subset of terms from the second and fourth orders), it was tested (see Hujsak and Hoots [12]). If an attempt was made to solve for the drag term, the DC diverged. SGP4 results for which the drag term is not solved-for are extremely poor. On the contrary, the optimized DSST performed extremely well in terms of accuracy. In order to provide a timing assessment that was more realistic, further testing was undertaken to again fit a Harris-Priester drag model to the Jacchia-Roberts truth:

These results provide a more realistic assessment of the blend of accuracy and computational efficiency which can be obtained from optimized DSST for these orbits.

It is worth stating the Molniya represents the most challenging orbit analyzed in this study. This is due to the wide range of perturbing effects experienced by this orbit. Molniya orbits are characterized by low perigee heights, which introduce substantial atmospheric and geopotential effects (not to mention that most Molniya orbits maintain repeat groundtrack constructs, which significantly

Theory	DC RMS (m)	ITER	Predict RMS (m)	CPU (sec)	DC CPU	CPU per ITER
$Cowell_{21}$	2.78328397(E-4)	6	0.12155	14.90	12.37	2.06
$Cowell_4$	1.19451595(E-3)	28	583.21	41.20	38.53	1.38
DSST (opt)	34.804297	10	174.64	13.72	12.36	1.24

Table 12: Molniya Orbit (Real World) 12 Hour Fit and 12 Hour Predict

contribute to resonance); in addition, the apogee heights of these orbits lead to significant third body and solar radiation pressure effects. This wide range of perturbative effects must be considered in relation to the high quality of the observational data available for these orbits. For these reasons, only a perturbation theory with high quality force models should be considered to determine the orbits of these objects. Further results for this orbit class can be found in Fonte[21].

GEOSYNCHRONOUS ORBIT:

Configuration Notes: In this case, $Cowell_{21}$ has force models matching those in the truth (21x21 GEM10B geopotential, lunar solar point mass effects, SRP, solve-for SRP parameter in DC). $Cowell_4$ is identical to $Cowell_{21}$, with the exception of a reduced geopotential (i.e., 4x4). The annotation of N_{pq} or "drag" after a theory indicates DC solve-for parameters used in the run. As with the other cases (except decay), DSST uses a two-card element set guess; DSST* uses the osculating elements from which the truth trajectory was generated (resulting from a Cowell fit to an SGP4 ephemeris derived from the two card element set).

For the geosynchronous orbit, sub-meter differences arise between the Cowell 21x21 and 4x4 geopotential configurations. The impact of the tesseral terms (mainly resonance) can be clearly seen by comparing the $Cowell_4$ results to those of the analytic theories. Mathematically, the 4x4 Cowell configuration completely captures resonant terms through the fourth order; the analytic theories (with the exception of SGP4) do not model tesseral terms (SGP4 models a subset of the tesseral terms at the second and fourth order (see *Introduction*). This lack of tesseral resonance modelling, as well as crude third body models, limit the analytic theories to kilometer level accuracy. The optimized DSST DC, which contains a 4x4 geopotential configuration like $Cowell_4$, provides an approximate 15 m predict error. This small predict error can be attributed to truncations in the third body short periodic model and the neglect of solar radiation pressure short periodics (truncations made to enhance DSST's speed).

Theory	DC RMS (m)	ITER	Predict RMS (m)	CPU (sec)	DC CPU	CPU per ITER
BL	680.91647	4	2152.8	6.31	4.96	1.24
BL (N_{pq})	682.19405	4	19531.0	7.07	4.73	1.18
Cowell_{21}	0.2114(E-8)	5	0.63256(E-3)	21.18	16.20	3.24
Cowell_{4}	0.231096(E-2)	5	0.2085	16.18	12.92	2.58
SGP4	909.57138	3	1245.9	8.24	7.07	2.36
SGP4 (drag)	673.00834	3	9427.0	8.63	6.77	2.26
SGP	701.47090	3	2020.7	8.40	6.01	2.00
SGP (drag)	701.63614	3	2188.7	7.76	5.86	1.95
DSST*(opt)	3.733008	7	11.219	11.45	9.36	1.34
DSST (opt)	4.6368485	9	10.778	13.09	11.15	1.24

Table 13: Geosynchronous Orbit 1 Day Fit and 1 Day Predict

The optimized DSST configuration also contains a weak time dependent formulation, which accounts for the movement of third bodies over the averaging interval. A time independent formulation assumes the third bodies do not move over the course of an averaging interval (typically one orbital period, which for the geosynchronous case equals one day) (see Green[20]). Obviously, this assumption is reasonable for low altitude objects; however, this assumption breaks down for objects at much higher altitudes. Previous studies indicate not modelling weak time dependence adds about a 200 m predict error over the course of three days and increased numbers of iterations for DC convergence (see Fonte and Sabol[16]).

In Table 13, results for two separate DSST DCs are listed. An inspection of the number of iterations for the DSST case emphasizes the two-card element set used by the Space Command theories is based upon a different set of "mean" elements than those used by the Draper Semianalytic Satellite Theory. DSST* uses initial conditions which are slightly more appropriate than the two card element set; however, Draper "mean" elements still would provide the best initial conditions for the DC.

In general, the orbit propagators can be ranked in terms of accuracy:

Theory	Decay	HST	TOPEX	Molniya	Geosynchronous	Average
$Cowell_{21}$	1	1	1	1	1	1
DSST (opt)	2	2	2	2	3	2.2
$Cowell_4$	3	3	3	3	2	2.8
SGP4	4	4	4	5	4	4.2
BL	5	5	5	6	6	5.4
SGP	6	6	6	4	5	5.4

Table 14: Accuracy Rankings

In consideration of these accuracy rankings, it should be noted the analytic theories performed very poorly for the Molniya and geosynchronous cases; predict accuracies were on the order of kilometers.

Theory	Decay	HST	TOPEX	Molniya	Geosynchronous
$Cowell_{21}$ vs DSST (opt)	32.67%	51.61%	51.63%	7.92%	45.94%
DSST (opt) was	faster	faster	faster	faster	faster
SGP4 vs DSST (opt)	98.05%	26.05%	2.19%	22.61%	38.96%
DSST (opt) was ¶	slower	slower	faster	slower	slower

Table 15: Total CPU Timing Comparisons

In consideration of the timing metrics, remember the optimized DSST results are for particular configurations of DSST developed by Fonte and Sabol[16]. Their rule of thumb in the development of these configurations was to keep DSST 5-10 times more accurate than SGP4. If speed is truly an issue, these DSST configurations can be further tailored to decrease DSST's run time; the user is free to configure the theory at run time to meet personal speed vs accuracy requirements.

As a final note, it should be mentioned the timing statistics aren't exactly fair. As was discussed previously, a two-card element set guess was used as initial conditions for the various differential correction runs. SGP4 and SGP have been tailored to use the "mean" elements provided on the two-card element set; DSST and Cowell DCs, which *can* use the two card element sets as initial conditions, would run more efficiently with their own tailored initial conditions (i.e., Draper "mean" elements for DSST and osculating elements for Cowell). In addition,

¶These time metrics are based on the short fit span chosen. Other studies[16] indicate that opt DSST performs more favorably with longer fit spans.

limited mismatching of force models was done in this analysis for Cowell_{21} (with the exception of using the Harris-Priester model for the drag perturbed cases); in essence, the Cowell_{21} DC were simply attempting to recover appropriate initial conditions. Operational scenarios would be less optimistic.

Conclusion

This paper evaluated the performance of various orbit propagation theories for artificial earth satellites in different orbital regimes. Specifically, R&D GTDS's Cowell (numerical), DSST (semianalytical), SGP, SGP4, and BL (analytic) orbit propagators were compared for decaying circular ($\sim$ 200 km perigee height), low altitude circular (590 km perigee height), high altitude circular (1340 km perigee height), Molniya, and geosynchronous orbits. (R&D GTDS was chosen because it has all these theories available in it.) Computation time and RMS errors were used as comparison metrics on a SUN Sparc 10.

It should be noted this study was theoretical in nature (i.e., attempting to understand the limitation of various orbit propagation theories with "GPS-like", equally spaced observational data). Without question, special perturbation techniques with rigorous force models proved the most accurate; however, this accuracy comes at the cost of computational efficiency (an issue whose importance decreases as computing horsepower increases and becomes commonly available). As expected, the analytic theories (general perturbation techniques) performed poorly in terms of accuracy (due to the severely truncated force models), but were efficient. DSST, which represents a hybrid of special and general perturbation techniques, provided accuracies approaching those of special perturbation techniques at speeds comparable to the analytic theories.

Clearly, improving the orbit determination process also includes equal consideration of the observational data (to include station coordinates, biases, standard deviations, solar and geomagnetic activity, refraction, coordinate frames, and timing issues). In addition, propagating an element set in a manner consistent with which it was determined is also of paramount importance. With the widespread technological advances of computing platforms, orbit propagation theories, and timing mechanisms, as well as strong observational data (laser, GPS, radar, transponder, etc), the time is ripe to re-analyze current operational orbit determination practices. Specifically, an analysis should be undertaken to trade the long term cost implication of upgrading current orbit determination techniques to modern hardware, software, and astrodynamics capabilities. The authors recommend that this study be performed by a wide range of organizations to tap a broad source of knowledge and expertise.

Acknowledgement

The authors would like to thank Paul Cefola, Ron Proulx, and Wayne McClain of the Charles Stark Draper Laboratory for their comments throughout this effort. In addition, LtCol Salvatore Alfano (PL/VTA), Maj Dave Vallado (PL/VTA), Paul Schumacher (Naval Space Command), Steve Knowles (Naval Space Command), and the Naval Postgraduate School are thanked for their support. The authors would like to thank Draper Laboratory for providing the version of R&D GTDS used at NPS.

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