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Corrigendum

Corrigendum to “On a family of Laguerre methods to find multiple roots of nonlinear equations”

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ABSTRACT

This is a note to correct the typographical errors in our paper. Dr. Xiaojian Zhou has emailed us “…we have some confusions on the results of $S(z)$ (or $S(u)$). For example, for Osada’s method, you have $S(z) = z^3[(m-1)z + 2m]$ as shown in Theorem 2.5, but we have $S(z) = \frac{z^3[(m-1)z+2m]}{2mz+m-1}$ where there is a denominator. We have checked all the conjugacy maps $S(z)$ and we present the corrected theorems.

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1. Introduction

The methods discussed here are all special cases of Laguerre’s method

$$x_{n+1} = x_n - \frac{\lambda \frac{f(x_n)}{f'(x_n)}}{1 + sgn(\lambda - m)\sqrt{\left(\frac{\lambda-m}{m}\right)\left[(\lambda-1) - \lambda\frac{f(x_n)f''(x_n)}{f'}(x_n)^2\right]}}, \tag{1}$$

where $\lambda\,(\neq 0, m)$ is a real parameter. When $f(x)$ is a polynomial of degree n, this method with $\lambda = n$ is the ordinary Laguerre method for multiple roots, see Bodewig [1]. Some special cases are:

- Euler–Cauchy for $\lambda = 2m$

$$x_{n+1} = x_n - \frac{2m\frac{f(x_n)}{f'(x_n)}}{1 + \sqrt{(2m-1) - 2m\frac{f(x_n)f''(x_n)}{f'(x_n)^2}}}. \tag{2}$$

- Halley for $\lambda \to 0$ after rationalization

$$x_{n+1} = x_n - \frac{\frac{f(x_n)}{f'(x_n)}}{\frac{m+1}{2m} - \frac{f(x_n)f''(x_n)}{2f'(x_n)^2}}. \tag{3}$$

- Ostrowski for $\lambda \to \infty$

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$$x_{n+1} = x_n - \frac{\sqrt{m}\frac{f(x_n)}{f'(x_n)}}{\sqrt{1 - \frac{f(x_n)f''(x_n)}{f'(x_n)^2}}}. \tag{4}$$

Two other cubically convergent methods that sometimes mistaken as members of Laguerre's family are: Euler–Chebyshev [2] given by

$$x_{n+1} = x_n - \left(\frac{m(3-m)}{2} + \frac{m^2}{2}\frac{f(x_n)}{f'(x_n)}\frac{f''(x_n)}{f'(x_n)}\right)\frac{f(x_n)}{f'(x_n)}, \tag{5}$$

and Osada's method [4] given by

$$x_{n+1} = x_n - \frac{1}{2}m(m+1)\frac{f(x_n)}{f'(x_n)} + \frac{(m-1)^2}{2}\frac{f'(x_n)}{f''(x_n)}. \tag{6}$$

2. Corresponding conjugacy maps for quadratic polynomials

Given two maps f and g from the Riemann sphere into itself, an analytic conjugacy between the two maps is a diffeomorphism h from the Riemann sphere onto itself such that $h \circ f = g \circ h$. Here we consider only quadratic polynomials raised to mth power.

Here we give only the modified Theorems appearing in [3]. The details of the proofs can be found in maple sheets in the Appendix.

Theorem 2.1 (Euler–Cauchy's method (2)). *For a rational map $R_p(z)$ arising from Euler–Cauchy's method applied to $p(z) = ((z-a)(z-b))^m$, $a \neq b, R_p(z)$ is conjugate via the Möbius transformation given by $M(z) = \frac{z-a}{z-b}$ to*

$$S(z) = z\frac{1 + sgn(z^2-1)}{-1 + sgn(z^2-1)}.$$

Proof. Let $p(z) = ((z-a)(z-b))^m$, $a \neq b$ and let M be the Möbius transformation given by $M(z) = \frac{z-a}{z-b}$ with its inverse $M^{-1}(u) = \frac{ub-a}{u-1}$, which may be considered as a map from $\mathbb{C} \cup \{\infty\}$. We then have

$$S(u) = M \circ R_p \circ M^{-1}(u) = M \circ R_p\left(\frac{ub-a}{u-1}\right) = u\frac{1 + sgn(u^2-1)}{-1 + sgn(u^2-1)}. \qquad \square$$

Theorem 2.2 (Ostrowski's method (4)). *For a rational map $R_p(z)$ arising from Ostrowski's method applied to $p(z) = ((z-a)(z-b))^m$, $a \neq b, R_p(z)$ is conjugate via the Möbius transformation given by $M(z) = \frac{z-a}{z-b}$ to*

$$S(z) = z\frac{\left[sgn(z+1)\sqrt{z^2+1} - 1\right]}{sgn(z+1)\sqrt{z^2+1} - z}.$$

Proof. Let $p(z) = ((z-a)(z-b))^m$, $a \neq b$ and let M be the Möbius transformation given by $M(z) = \frac{z-a}{z-b}$ with its inverse $M^{-1}(u) = \frac{ub-a}{u-1}$, which may be considered as a map from $\mathbb{C} \cup \{\infty\}$. We then have

$$S(u) = M \circ R_p \circ M^{-1}(u) = M \circ R_p\left(\frac{ub-a}{u-1}\right) = u\frac{\left[sgn(u+1)\sqrt{u^2+1} - 1\right]}{sgn(u+1)\sqrt{u^2+1} - u}. \qquad \square$$

Theorem 2.3 (Euler–Chebyshev's method (5)). *For a rational map $R_p(z)$ arising from Euler–Chebyshev's method applied to $p(z) = ((z-a)(z-b))^m$, $a \neq b, R_p(z)$ is conjugate via the Möbius transformation given by $M(z) = \frac{z-a}{z-b}$ to*

$$S(z) = z^3\frac{z+2}{2z+1}.$$

Proof. Let $p(z) = ((z-a)(z-b))^m$, $a \neq b$ and let M be the Möbius transformation given by $M(z) = \frac{z-a}{z-b}$ with its inverse $M^{-1}(u) = \frac{ub-a}{u-1}$, which may be considered as a map from $\mathbb{C} \cup \{\infty\}$. We then have

$$S(u) = M \circ R_p \circ M^{-1}(u) = M \circ R_p\left(\frac{ub-a}{u-1}\right) = u^3\frac{u+2}{2u+1}. \qquad \square$$

Theorem 2.4 (Osada's method (6)). *For a rational map $R_p(z)$ arising from Osada's method applied to $p(z) = ((z-a)(z-b))^m,\ a \neq b, R_p(z)$ is conjugate via the Möbius transformation given by $M(z) = \frac{z-a}{z-b}$ to*

$$S(z) = z^3 \frac{(m-1)z + 2m}{2mz + m - 1}.$$

Proof. Let $p(z) = ((z-a)(z-b))^m,\ a \neq b$ and let M be the Möbius transformation given by $M(z) = \frac{z-a}{z-b}$ with its inverse $M^{-1}(u) = \frac{ub-a}{u-1}$, which may be considered as a map from $\mathbb{C} \cup \{\infty\}$. We then have

$$S(u) = M \circ R_p \circ M^{-1}(u) = M \circ R_p\left(\frac{ub-a}{u-1}\right) = u^3 \frac{(m-1)u + 2m}{2mu + m - 1}. \qquad \square$$

These corrections to the maps did not affect any of the numerical results or the conclusions.

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Appendix

- Euler–Cauchy's method

```
restart:
```
```
a:=1:b:=-1:
f:=x-> (x^2-1)^m:
fd:= unapply(diff(f(x),x),x):
fdd:= unapply(diff(f(x),x,x),x):
EulerCauchy3:=x-> x-2*m*f(x)/fd(x)/(1+sqrt((2*m-1)-2*m*f(x)*fdd(x)/fd(x)^2)):
x:= (u*b-a)/(u-1): RMinv:=simplify(EulerCauchy3(x)):
R:= u-> simplify((RMinv-a)/(RMinv-b)): simplify(R(u)):
```

- Ostrowski's method

```
restart:
```
```
a:=1:b:=-1:
f:= x-> (x^2-1)^m:
fd:= unapply(diff(f(x),x),x):
fdd:= unapply(diff(f(x),x,x),x):
Ostrowski3:=x-> x-sqrt(m)*f(x)/fd(x)/sqrt(1-f(x)*fdd(x)/fd(x)^2):
x:= (u*b-a)/(u-1): RMinv:=simplify(Ostrowski3(x)):
R:= u-> simplify((RMinv-a)/(RMinv-b)): simplify(R(u)):
```

- Euler-Chebyshev's method

```
restart:
```
```
a:=1:b:=-1:
f:= x-> (x^2-1)^m:
fd:= unapply(diff(f(x),x),x):
fdd:= unapply(diff(f(x),x,x),x):
EulerChebyshev3:=x-> x-m*(3-m)/2*f(x)/fd(x)-m^2/2*(f(x)/fd(x))^2*fdd(x)/fd(x):
x:= (u*b-a)/(u-1): RMinv:=simplify(EulerChebyshev3(x)):
R:= u-> simplify((RMinv-a)/(RMinv-b)): simplify(R(u));
```

- Osada's method

```
restart:
a:=1:b:=-1:
f:= x-> (x^2-1)^m:
fd:= unapply(diff(f(x),x),x):
fdd:= unapply(diff(f(x),x,x),x):
Osadam3:=x-> x-m*(m+1)/2*f(x)/fd(x)+(m-1)^2/2*fd(x)/fdd(x):
x:= (u*b-a)/(u-1): RMinv:=simplify(Osadam3(x)):
R:= u-> simplify((RMinv-a)/(RMinv-b)): simplify(R(u)):
```

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