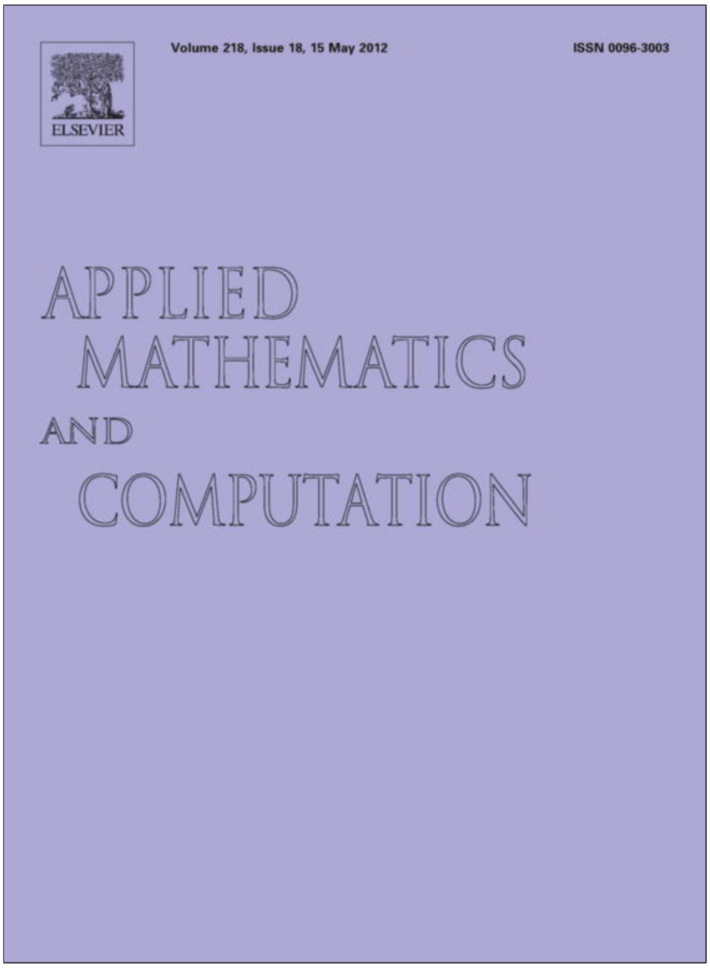
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A note on the modified super-Halley method

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ABSTRACT

There are many methods for solving nonlinear algebraic equations. Some of these methods are just rediscovered old ones. In this note we show that the modified super Halley scheme is the same as one of Jarratt's methods.

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1. Introduction

There is a vast literature for the numerical solution of nonlinear equations. The methods are classified by their order of convergence, p, and the number, d, of function- (and derivative) evaluation per step.

Halley's method [1] is one of the oldest methods of third order.

$$x_{n+1} = x_n - \frac{1}{1 - \frac{1}{2}L_f} u_n, \tag{1}$$

where

$$u_n = \frac{f_n}{f'_n}, \tag{2}$$

$$L_f = \frac{f_n}{f'_n}\frac{f''_n}{f'_n}, \tag{3}$$

and $f_n = f(x_n)$ and similarly for the derivative.

This method, according to Traub [2] is the most rediscovered after Newton's. Halley's method has been rediscovered through various means, see e.g. [2–14]. Petković et al. [15] have found several methods that were just rediscovered old ones.

This method was also modified to reach fourth order. Gutiérrez and Hernández [16] have developed super Halley's method given by

$$x_{n+1} = x_n - u_n\left[1 + \frac{1}{2}\frac{L_f}{1 - L_f}\right], \tag{4}$$

where u_n and L_f are as before.

Another fourth order method was developed by Chun and Ham [17] and is given by

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$$\begin{aligned} y_n &= x_n - \frac{2}{3}u_n, \\ x_{n+1} &= x_n - \left[1 + \frac{1}{2}\frac{\widehat{L}_f}{1-\widehat{L}_f}\right]u_n, \end{aligned} \tag{5}$$

where

$$\widehat{L}_f = \frac{f_n}{(f_n')^2}\frac{f'(y_n)-f_n'}{y_n-x_n}. \tag{6}$$

This is called modified super Halley since it removes the need for the second derivative in super Halley's method (4).

We will show that the super Halley method is just a rediscovered Jarratt's scheme [18] given by

$$\begin{aligned} y_n &= x_n - \frac{2}{3}u_n, \\ x_{n+1} &= x_n - \frac{1}{2}u_n - \frac{1}{2}\frac{u_n}{1+\frac{3}{2}\left(\frac{f'(y_n)}{f_n'}-1\right)}. \end{aligned} \tag{7}$$

Theorem 1.1. *The modified super Halley scheme (5) can be rearranged to match Jarratt's method (7).*

Proof. Note that y_n is the same for both methods and

$$\widehat{L}_f = \frac{f_n}{(f_n')^2}\frac{f'(y_n)-f_n'}{y_n-x_n} = \frac{u_n}{f_n'}\frac{f'(y_n)-f_n'}{y_n-x_n} = \frac{u_n}{y_n-x_n}\frac{f'(y_n)-f_n'}{f_n'} = -\frac{3}{2}\frac{f'(y_n)-f_n'}{f_n'}$$

and so

$$\begin{aligned} x_{n+1} &= x_n - u_n - \frac{1}{2}\frac{\widehat{L}_f}{1-\widehat{L}_f}u_n = x_n - u_n - \frac{1}{2}\frac{(-\frac{3}{2})(f'(y_n)-f_n')}{f_n'\left(1+\frac{3}{2}\frac{f'(y_n)-f_n'}{f_n'}\right)}u_n = x_n - u_n + \frac{3}{2}u_n\frac{f'(y_n)-f_n'}{2f_n'+3(f'(y_n)-f_n')} \\ &= x_n - \frac{1}{2}u_n - \frac{1}{2}u_n + \frac{3}{2}u_n\frac{f'(y_n)-f_n'}{3f'(y_n)-f_n'} = x_n - \frac{1}{2}u_n + \frac{1}{2}u_n\left(3\frac{f'(y_n)-f_n'}{3f'(y_n)-f_n'} - 1\right), \end{aligned}$$

$$x_{n+1} = x_n - \frac{1}{2}u_n + \frac{1}{2}u_n\frac{-2f_n'}{3f'(y_n)-f_n'} = x_n - \frac{1}{2}u_n - u_n\frac{f_n'}{3f'(y_n)-f_n'} = x_n - \frac{1}{2}u_n - \frac{1}{2}\frac{u_n}{\frac{3}{2}\frac{f'(y_n)}{f_n'}-\frac{1}{2}}$$

which is exactly Jarratt's method (7). □

This is a good example where the ordering of the calculations is important since we found that the Jarratt's form performed significantly better than the modified super Halley form, see Neta et al. [19].

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