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PATROLLING A CHANNEL REVISITED

by

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January 1976

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The crossover barrier analysis on pp. 106-107 of "Search and Screening" (OEG56) is approximate. The calculations in this report are more exact but not essentially different.

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Patrolling a Channel Revisited

1. Introduction:

In this problem, targets pass down a channel at a fixed speed u known to a searcher who patrols at a fixed speed v and attempts to maximize the probability of coming within $W/2$ of the typical target at some time in its passage down the channel. Target tracks are assumed to be uniformly distributed across the channel. In the most general form of the problem, the searcher would pick any closed curve and patrol around it indefinitely at fixed speed v (we are ignoring questions of endurance), and the question would be "What curve maximizes the detection probability"? This general problem has not been and probably will not be solved because

a) As we shall see later, it hardly makes any difference how the searcher patrols.

b) Many searchers couldn't or wouldn't patrol in any fixed but complicated manner.

c) The problem is extremely difficult mathematically. The reason for this is that it is fundamentally geometrical in nature, and geometrical coverage-type problems, while simple in concept, are usually very difficult computationally.

A natural reaction to a, b, c is to invent a simple class of curves and optimize within the class. This is the approach taken here, with the class of curves being those with the shape of a bow-tie. This class includes the tactic of simply patrolling back and forth across the channel.

In section 2, the problem is defined more precisely and results are presented. In section 3, the method of analysis is discussed, with the details being in the appendix.

2. Results:

If the width of the channel is less than W, detection is certain. Accordingly, we let the channel width be $L + W$, where $L \geq 0$. We also restrict the searcher to back and forth motions of width L; that is, the cookie cutter detection radius is assumed to barely graze the sides of the channel. This assumption prohibits the searcher from having a detection probability of 1.0 as long as $L > 0$, and is therefore a bad one in situations where a very large detection probability is possible (in which case the searcher should "overshoot" a little). The assumption leaves the searcher with a single parameter for description of his path:

$$\alpha = \text{angle at which searcher proceeds across the channel}$$

The situation is depicted in the two pages from reference 1 (pp. 3 and 4 of this report); the bow tie at the top of the page shows the type of track we are talking about, with the back and forth search shown lower being regarded as a special case where $\alpha = 0$. The speed of the searcher in the direction of the target's motion is $v \sin \alpha$. In reference 1, the only values of α examined were α_0, where $v \sin \alpha_0 = u$, and $\alpha = 0$. In this report, we will examine all values of α such that $0 \leq \alpha \leq \alpha_0$. The reasons for the upper limit on α are that the coverage geometry becomes complicated when the searcher moves faster than the target along the target's line of motion, and that the restriction is apparently not an important one, anyway.

It is sufficient to give only the dimensionless ratios $\lambda = L/W$ and $r = v/u$ to determine the optimal α and the resulting detection probability. The former is shown in figure 1 and the latter in figure 2. These two figures constitute the principal results of this report. The hyperbola in Figure 1 marks the maximum value of $\sin \alpha$ considered; the limitation is not important

7.1.4 Barrier When Target Speed Is Close to Observer's Speed

So far it has been assumed that v considerably exceeds u; indeed, when $v \leq u$, the crossover type of barrier is kinematically impossible. This is no obstacle when the observer is airborne and the target is a ship, but when both observer and target are units of the same type (both ships or both aircraft), the situation excluded heretofore becomes important. Although many plans of barring a channel can be devised for this case, attention will be confined here to the very simple case in which the observer moves back and forth across the channel on a straight path perpendicular to its (parallel) banks: such a patrol is always possible and its design evidently does not involve the speed ratio u/v.

This back-and-forth barrier will be compared with the symmetrical crossover (when $u < v$), and since only a rough comparison is sought here, the definite range law will be assumed (range $= R$ in each case). A more accurate detection law is not likely to alter the comparison appreciably.

The two diagrams on Figure 6 show the geographic as well as the relative tracks for the two types of patrol.

In each relative track a half cycle has been selected and the area swept shaded. The probability of detection for each case has been taken as the ratio of the shaded area to the total area in the channel between the two dashed lines marking off the half cycle. It is convenient to introduce two new variables to describe the probability of detection, $r = v/u$ and $\lambda = L/W$. For the case of the crossover patrol, the probability $P_{\times}$ of detection is given by

$$P_{\times} = \min\left[1, \left(1 + \frac{r\sqrt{r^2-1}}{r+1}\right)\frac{1}{\lambda+1}\right].$$

For the back-and-forth patrol the probability P of detection is given by

$$P_{\leftrightarrow} = \begin{cases} 1 - \left(\lambda - \dfrac{\sqrt{r^2+1}-1}{2}\right)^2 \Big/ \lambda(\lambda+1) & \text{for } r \leq 2\sqrt{\lambda(\lambda+1)} \\ 1 & \text{for } r > 2\sqrt{\lambda(\lambda+1)}. \end{cases}$$

In Figure 7 the values of P for the two cases are plotted as functions of r with λ kept fixed for a given curve. In comparing crossover patrols with back-and-forth patrols, curves bearing the same value of λ

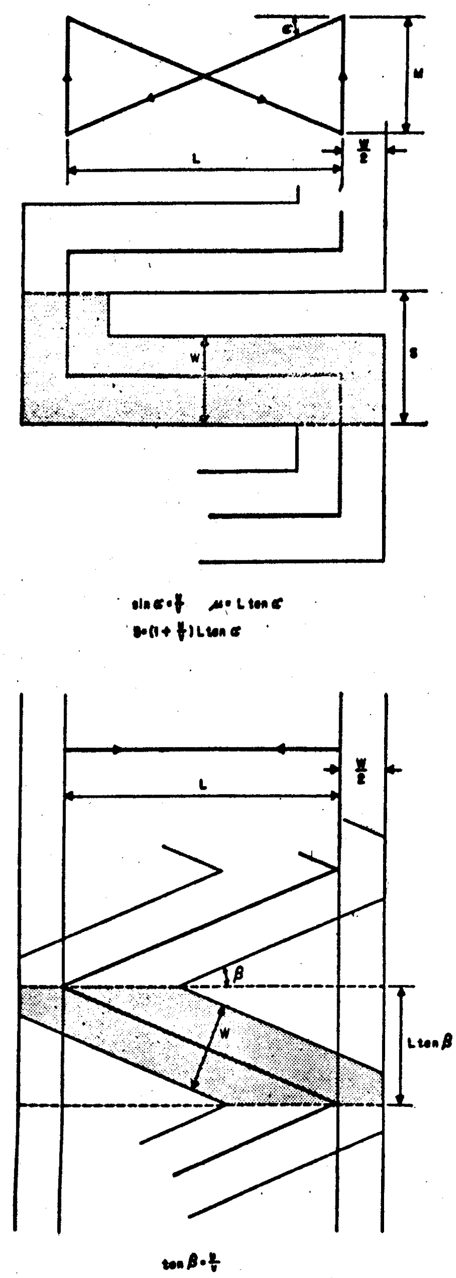

FIGURE 6. A comparison of barriers.

should be compared. The solid curve passes through the points of intersection of the curves being compared and marks the boundary between the regions

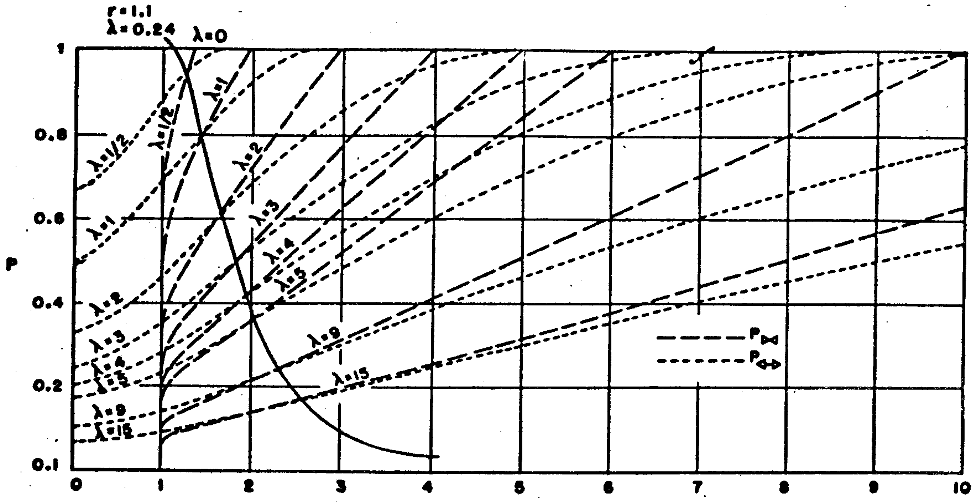

FIGURE 7. The comparative effectiveness of back-and-forth and crossover plans.

where back-and-forth is preferable and where crossover is preferable.

In order to facilitate the selection of the preferable type of patrol, Figure 8 is included. This curve shows the relation between λ and r for the points of intersection of curves in Figure 7.

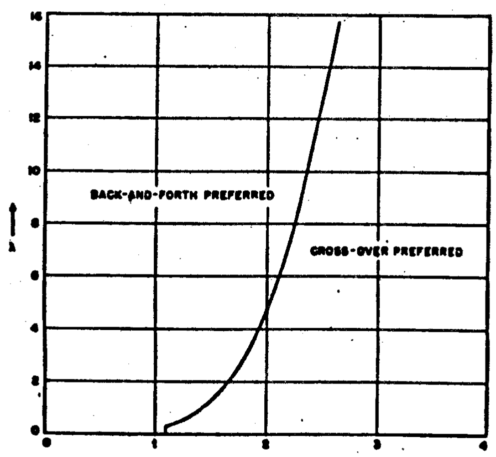

FIGURE 8. Regions of effectiveness of back-and-forth and crossover plans.

An example will illustrate the use of the curves. Suppose a ship making 12 knots is trying to prevent undetected penetration of a barrier by a 6-knot submerged submarine. Assume further that the channel being guarded is 8 miles wide and that the sonar search width W is 2 miles. Then $L = 7 - 2 = 5$, $\lambda = 5/2 = 2.5$, $r = 12/6 = 2$. Entering Figure 8 with these values for r and λ one discovers that a crossover patrol is preferred.

7.2 CIRCULAR BARRIERS

7.2.1 Constant Radial Flux of Hostile Craft

In the case where enemy surface craft or submarines are attempting to leave a point of the ocean, such as an island or exposed harbor, and in the case in which they are attempting to approach such points or to close positions at which our forces are conducting landing operations, the vector velocity pattern is a radial one: it is "centrifugal" (directed away from the central point) in the former case and "centripetal" (directed toward the center) in the latter. But in each case it can be regarded as *constant in time*: over long periods, the density of outgoing or incoming craft is not expected to vary. This sets the present situation in strong contrast with that considered in Section 7.3, in which the unit to be detected is, to be

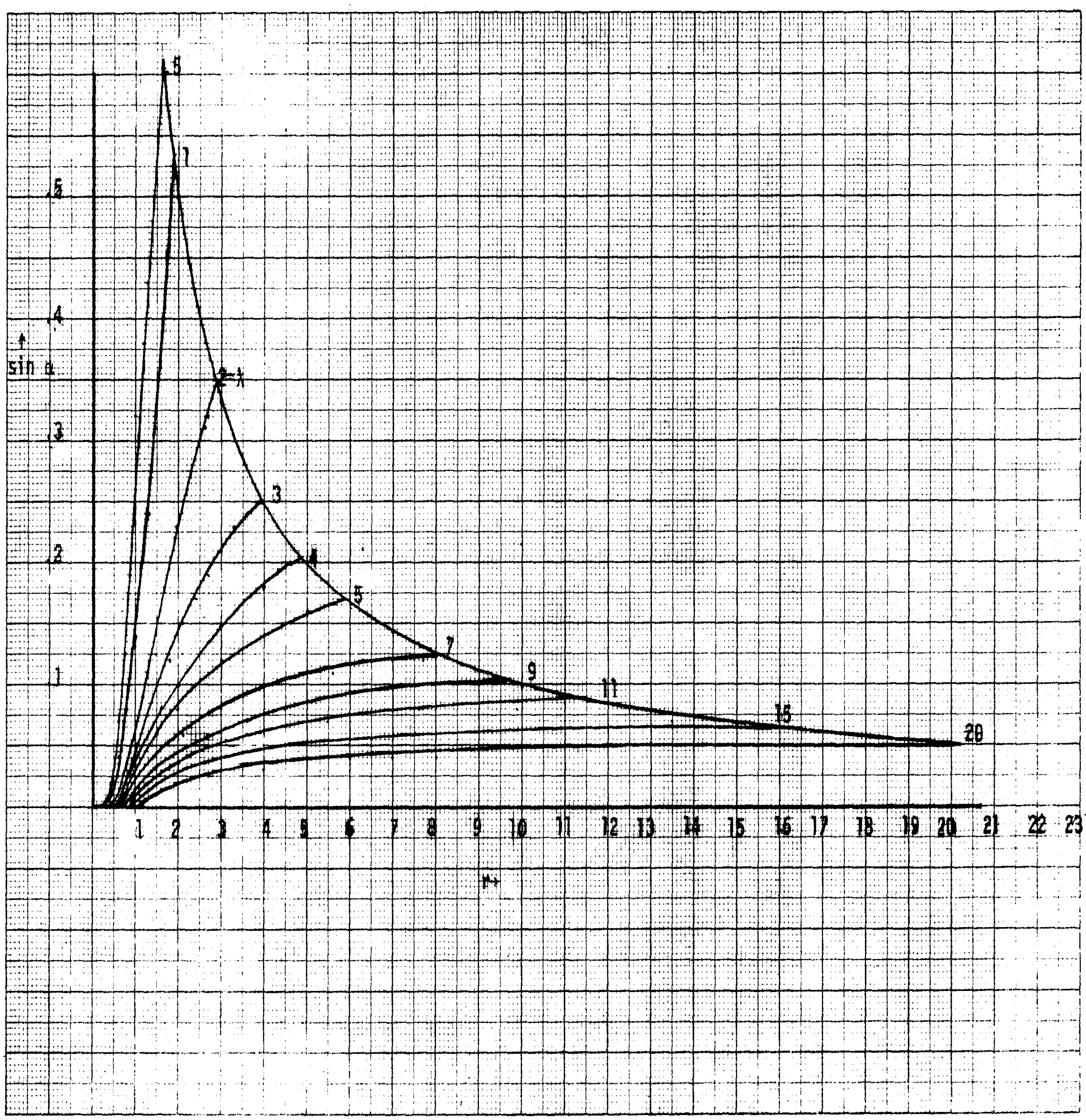

Figure 1 The optimal value of sin α as a function of r for various λ

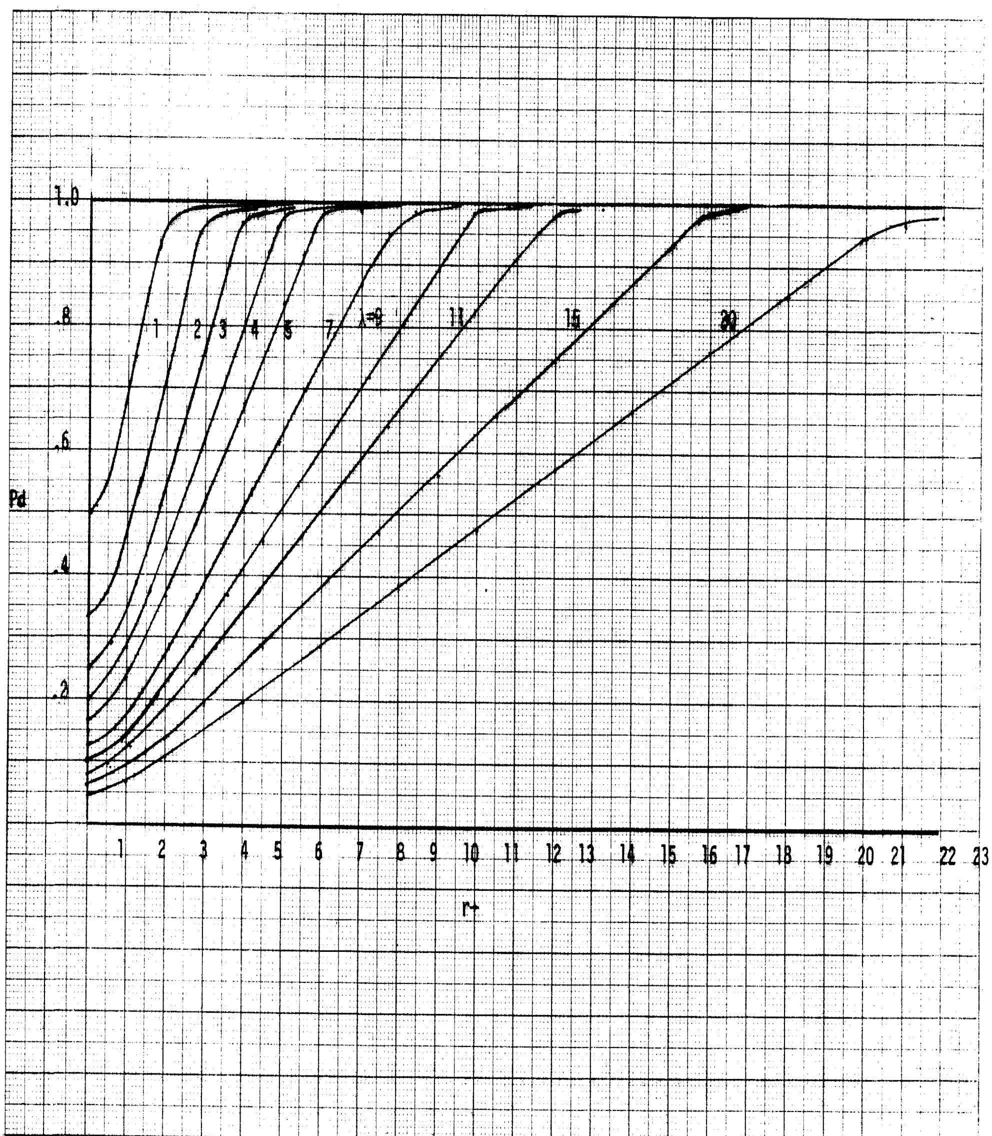

Figure 2 Probability of detection vs. r for various λ

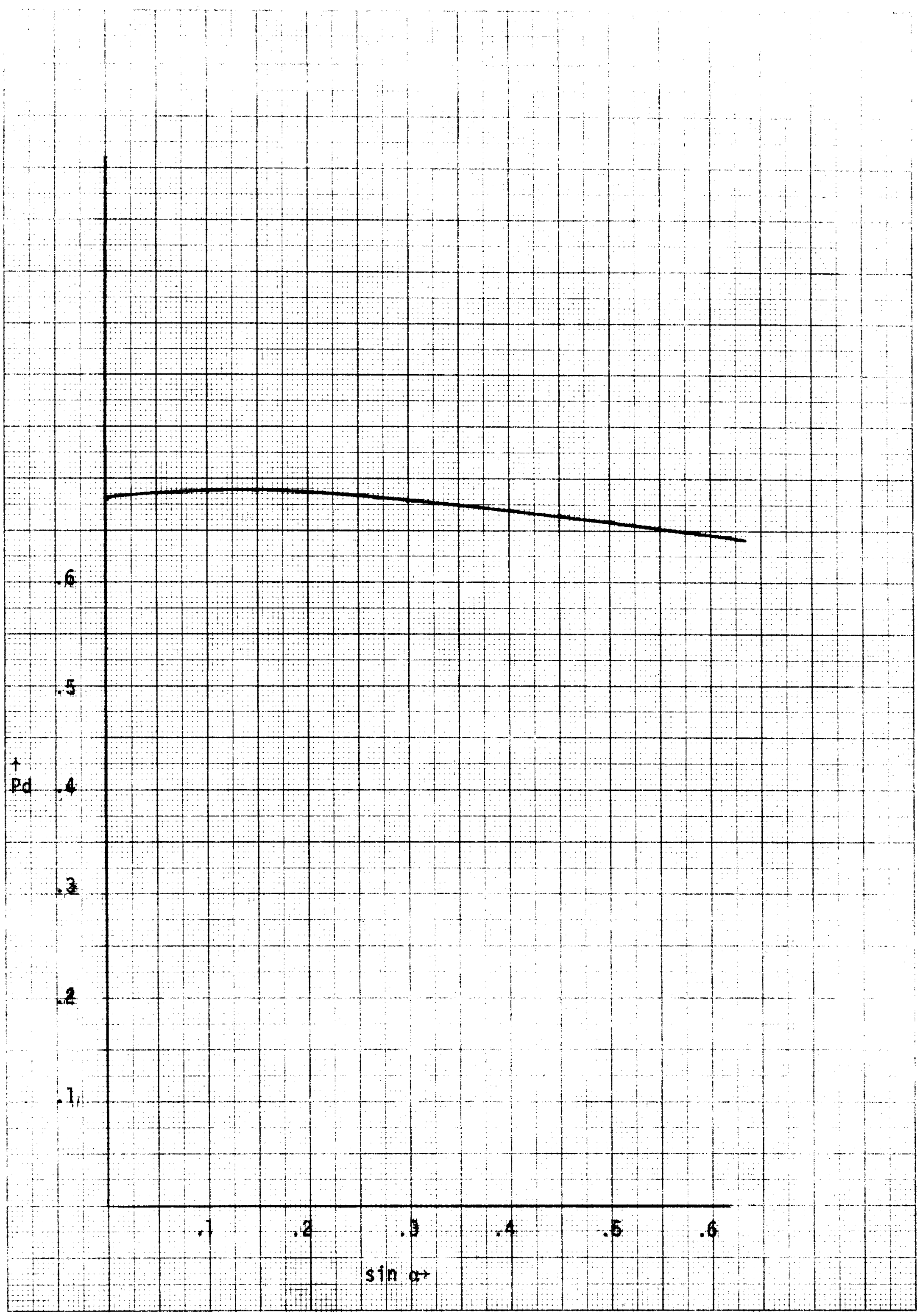

Figure 3 probability of detection vs. sin α for $r = 1$ and $\lambda = 1$

because the probability of detection is nearly 1. on the hyperbola anyway. Figure 2 can be compared to the upper figure on p. 4 of this report. Figure 3 illustrates the insensitivity of the probability of detection to α .

The curves shown in Figure 2 can be approximated by $p_d \cong \min (1, \sqrt{1 + r^2} / (1+\lambda))$, with the approximation being a bit high and most accurate for small r .

3. Analysis:

The detection probability does not change if one simply imagines that a "drift" u is added to the velocity of the searcher, and that the searcher is actually looking for a stationary target. In this case, the detection probability is simply the fraction of the channel covered, and it is sufficient to compute the fraction of some repeating element that is covered. These are the shaded areas on page 3 of this report; note the approximation of making a round corner into a square one. In this "relative space", our searcher proceeds up the channel at some angle β for the horizontal distance L , then proceeds straight up the channel for a distance $2D$ then back across the channel at angle β , etc. β and D are related to α by:

$$\tan \beta = \frac{1}{r \cos \alpha} - \tan \alpha$$

$$D = L (1 + 1/r) \tan \alpha$$

The area swept out by the searcher has the general appearance shown in figure 4, and the problem of computing the detection probability for a given α is now "merely" one of finding the ratio of shaded to unshaded area (however glance at figures 5 and 6, which show several possible shapes for the shaded part). This is discussed further in the appendix. The results given in the previous section were obtained by exploring the interval $[0, \min (1, 1/r)]$ for $\sin \alpha$ in steps of size .01.

Appendix:

Without loss of generality, we assume $W = 2$. In general, the area covered by the searcher in relative space appears as shown in Figure 4. The searcher moves across the channel at angle β, then moves forward a distance $2D$, then moves back across the channel again, etc. Both β and D are determined by the angle α, but we ignore this fact for the moment, assuming only that $\beta \geq 0$ and $D \geq 0$. A repeating element of the pattern is shown, covering all but the crosshatched part of the area $A \equiv (L+2)(2D+L \text{ ton } \beta)$. If the area of either crosshatched part is A_u then $p_d \equiv$ fraction of area covered $= 1- 2A_u/A$. We need to find an expression for A_u as a function of β, D, and L. Again without loss of generality, we look at the lower right hand corner. The precise shape of the lower right hand corner depends on the vertical order of the points a, b, and c shown in Figure 4. Whether point a lies below point c depends on L and β, but not D. We therefore let (L, β) determine whether Case 1 or Case 2 holds, and then further subdivide according to the magnitude of D. Figure 5 applies when a lies to the left of c (Case 1), the analytical criterion for which is $L \geq 1- \sin\beta$. The quantity $D_o = \frac{\cos\beta}{1+\sin\beta}$ plays a prominent role in subdividing according to D. In case 1.1 $(D \geq D_o)$ and case 1.2 $(\cos\beta - L\tan\beta \leq D \leq D_o)$, A_1 is $F_1(\beta)$ (see figure 7 for the function $F_1(\cdot)$). In case 1.3 $(D \leq \cos\beta - L \text{ ton } \beta)$, $\cos\gamma = L\tan\beta + D$, and A_1 is $F_2(\gamma)$ (see figure 7 for the function $F_2(\cdot)$). In Case 1.1, A_2 is $L(D- D_o + \frac{1}{2}L\tan\beta)$. In Case 1.2, A_2 is $\frac{1}{2}(D- D_o + L\tan\beta)^2/\tan\beta$. In Case 1.3, $A_2 = 0$. Note that both A_1 and A_2 are continuous in D across the two boundarys. In all cases, $A_u = A_1 + A_2$.

Figure 6 applies when $L \leq 1- \sin\beta$ (case 2), in which case point a lies to the right of point c. Let $\sin\gamma = 1- L$. If $D \geq \cos\gamma - L\tan\beta$, then Case 2.1 holds, $A_1 = F_2(\gamma)$, and $A_2 = L(L\tan\beta + D- \cos\gamma)$. If $D \leq \cos\gamma - L\tan\beta$, let $\cos\gamma = L \tan\beta + D$, $A_1 = F_2(\delta)$, and $A_2 = 0$. In either case, $A_u = A_1 + A_2$, and A_u is again continuous in D across the boundary.

These calculations are summarized in the flow diagram shown in Figure 8.

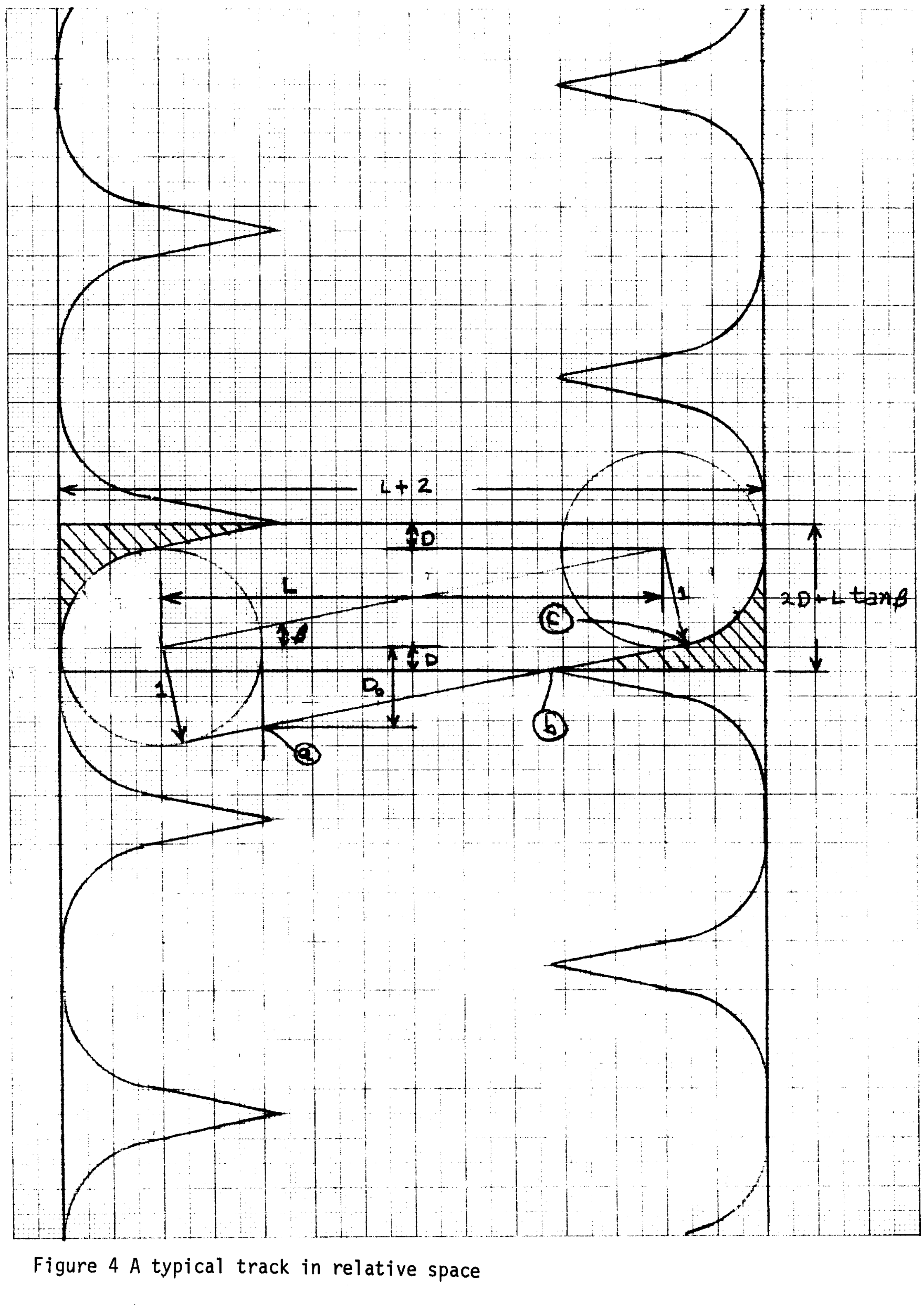

Figure 4 A typical track in relative space

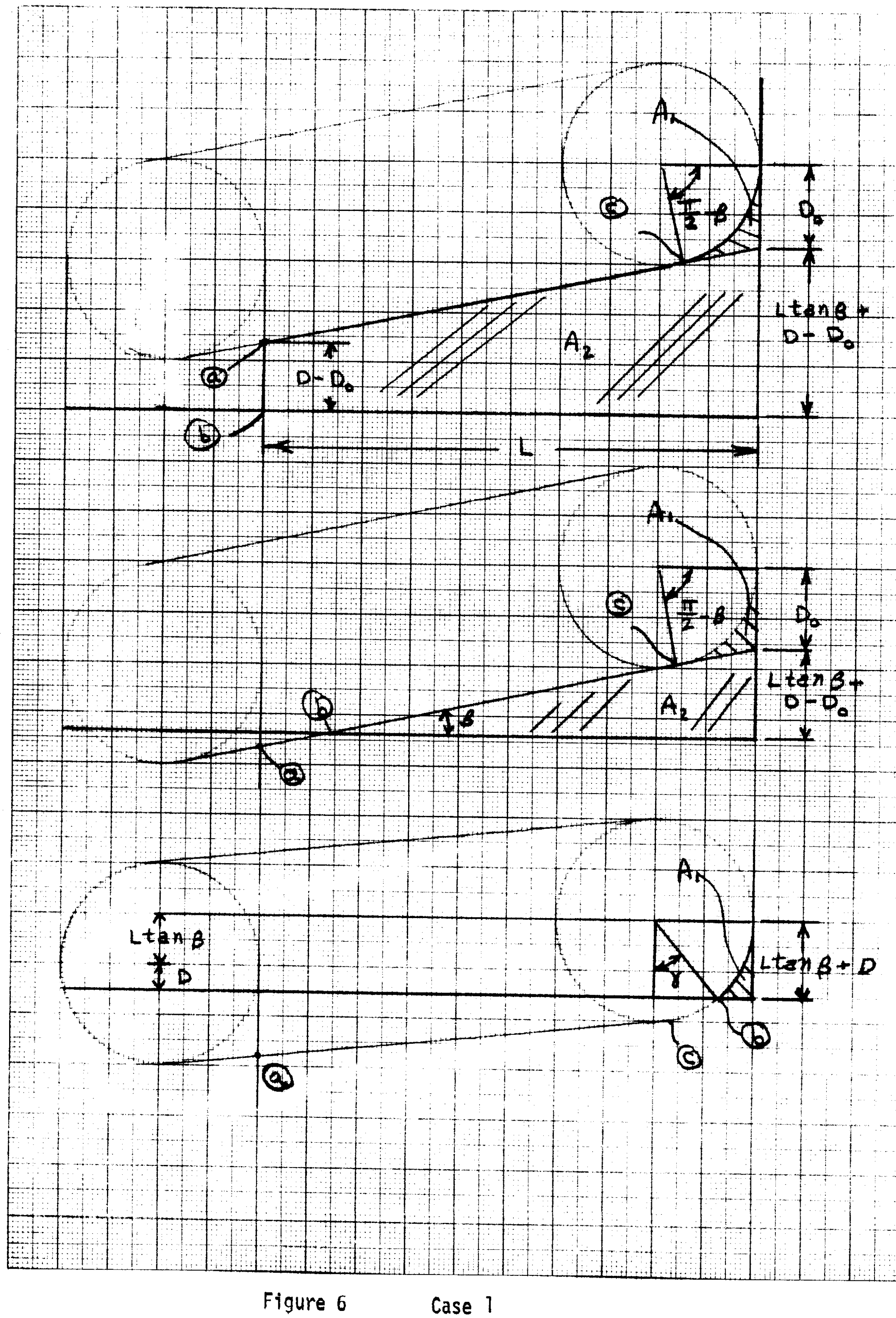

Figure 6 Case 1

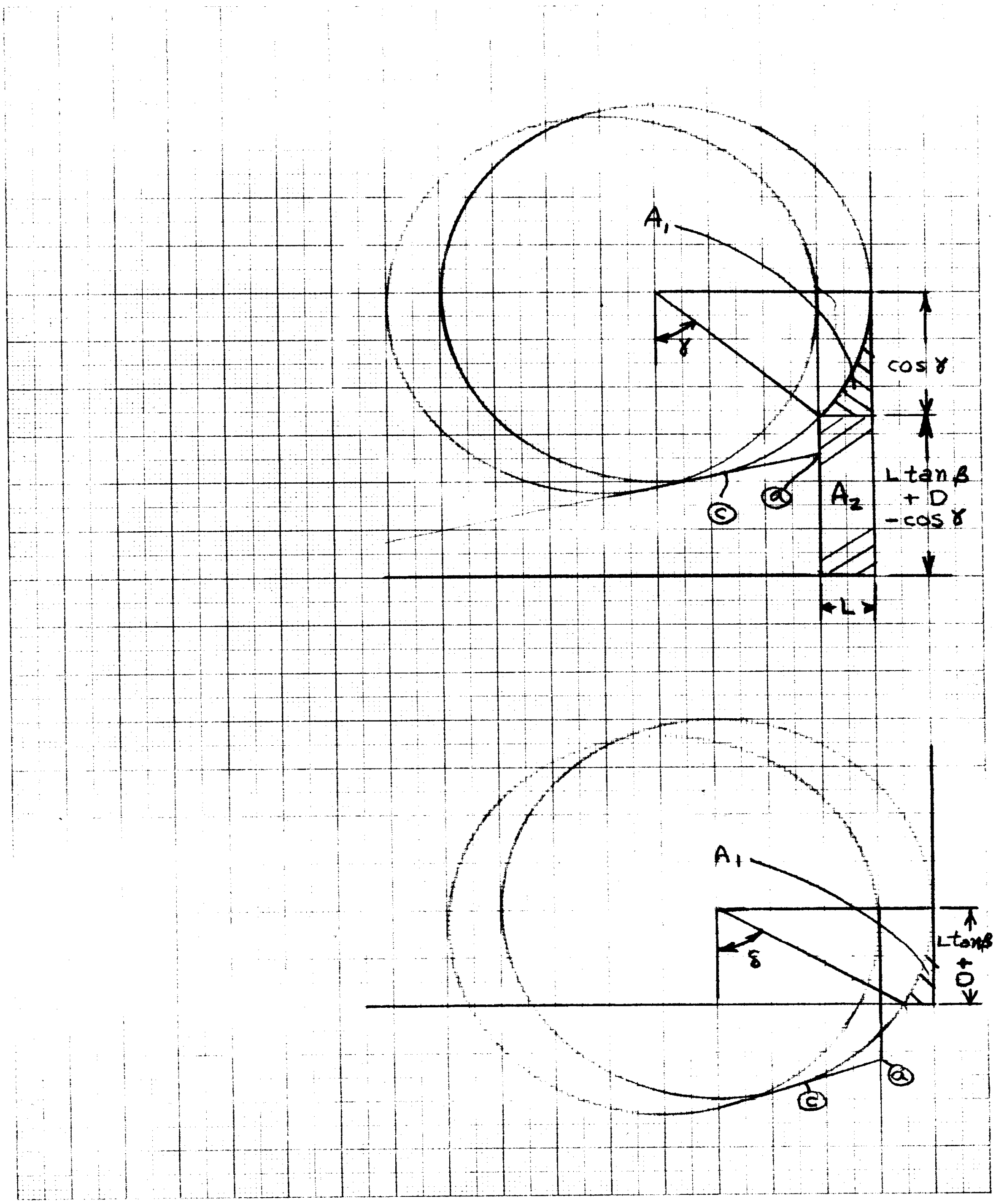

Figure 6 Case 2

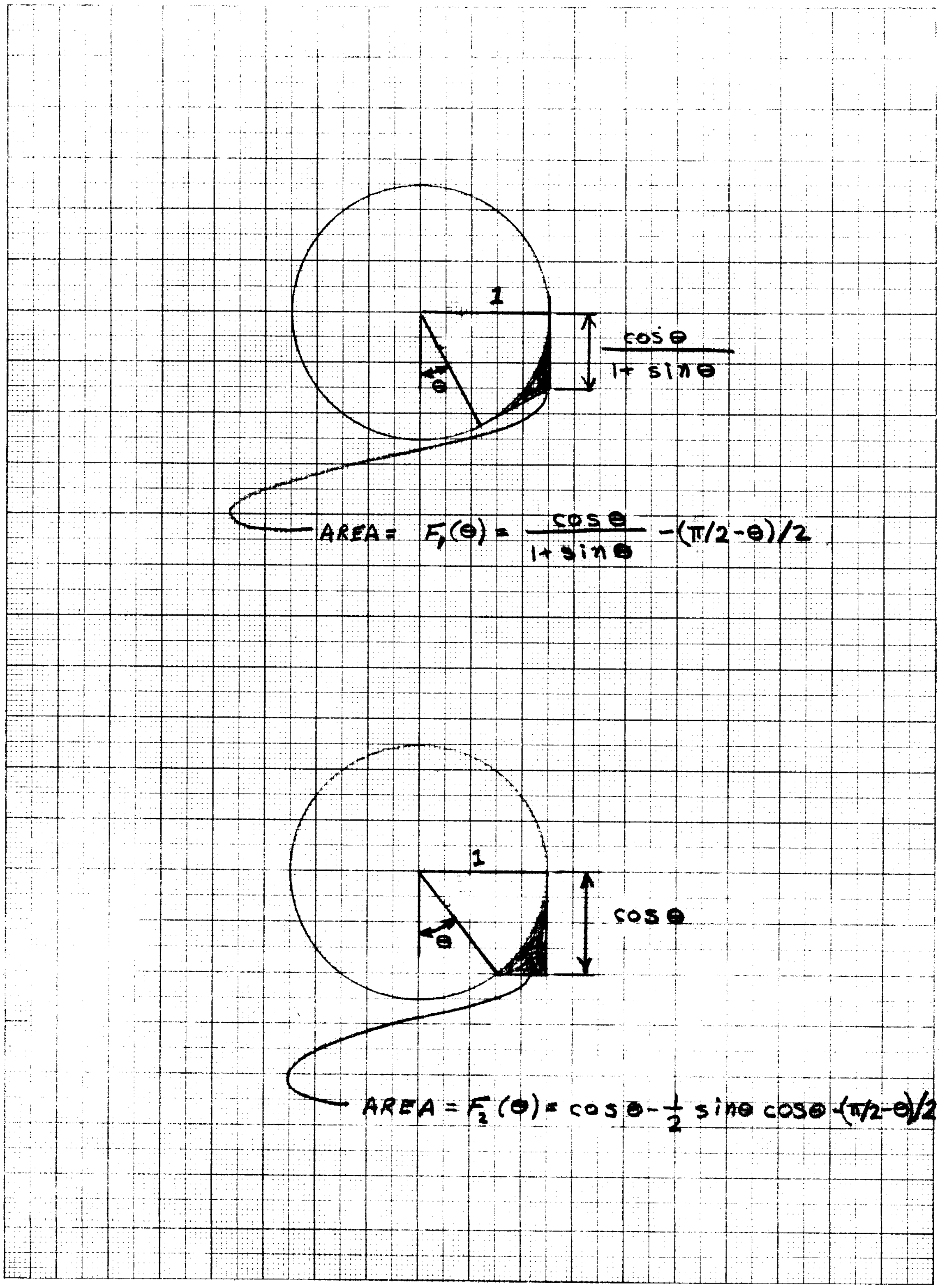

Figure 7 Two geometry problems

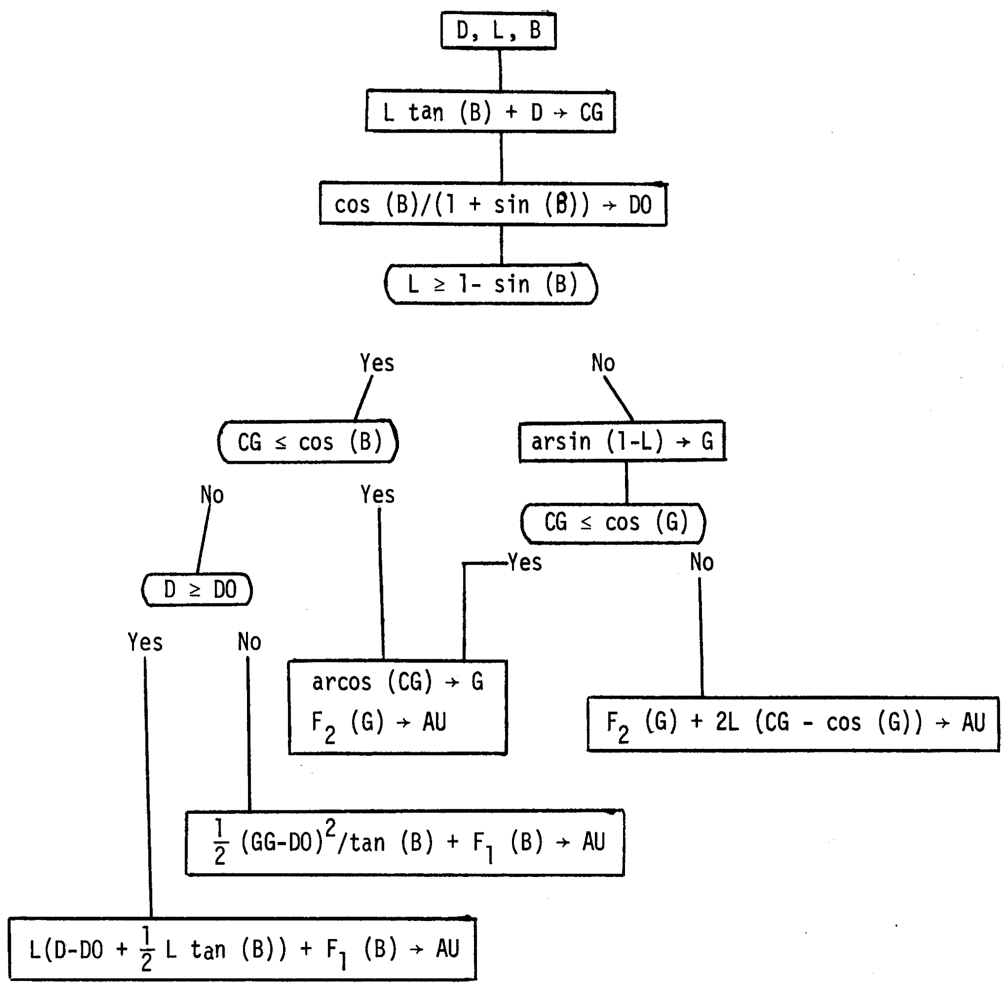

Figure 8 Flow diagram for computation of uncovered area

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