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THE EFFECTS OF LIQUID PROPELLANT MOTION
ON THE
ATTITUDE STABILITY OF SPIN STABILIZED SPACECRAFT

by

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The Effects of Liquid Propellant Motion on the Attitude Stability of Spin Stabilized Spacecraft

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of the requirements for the degree of

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ABSTRACT

An analysis of the effects of liquid motion on the attitude stability of spin stabilized spacecraft is presented. The effects of varying the fuel load and the asymmetry of the platform are emphasized. The energy sink stability criteria are derived and applied to a marginally stable spacecraft. The stability predictions based on the energy sink stability criteria are compared to the results of a computer simulation. Based on this comparison the limitations of the energy sink stability criteria are identified.

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I. INTRODUCTION

A. BACKGROUND

Historically, the attitude stability conditions of spinning spacecraft have been derived using the energy sink method. A significant shortcoming of this method is that it does not account for the dynamic interaction of liquid motion. For current spacecraft with large amounts of liquid propellant, dynamic interaction can destabilize the spacecraft. As a result, under some circumstances the stability prediction of the energy sink method can be in error.

Despite this shortcoming, the energy sink method continues to be in wide spread use as an analytical technique for determining attitude stability for dual spin spacecraft. This being the case, it is essential that the nature of the deficiency and the conditions under which it occurs be understood.

B. OBJECTIVE

The objective of this study is to investigate and more accurately define the nature of this shortcoming. Specifically, the case of a marginally stable, dual spin spacecraft with a despun platform will be explored. The stability of the spacecraft will be examined as the percent of platform asymmetry and the fuel load are varied. Marginally stable, in terms of the energy sink stability conditions, means the spacecraft has an inertia ratio slightly greater than one. The percent asymmetry is defined as the ratio of the difference of the two transverse moments of inertia to the sum of the two transverse moments of inertia times 100. The fuel load is the amount of fuel on board expressed as a fraction of the total fuel capacity of the spacecraft.

To perform the analysis, a computer simulation developed by Chung [Ref. 1] of a dual spin spacecraft will be used to determine the stability of the various configurations. These results will be compared to the predictions of the energy sink stability criteria as developed by Likins in Reference 2.

C. LITERATURE REVIEW

The energy sink approach was first applied to study the effects of energy dissipation on the stability of a freely spinning body in 1963 by Thomson and Reiter [Ref. 3]. This led to the well known requirement for stability, an object must spin about its axis with the largest principal moment of inertia.

In 1966, Likins [Ref. 2] developed the energy sink stability conditions for asymmetric, dual spin spacecraft assuming the energy transfer between the platform and the rotor was small. His results showed that the average energy dissipation on the platform or on the rotor is proportional to the average of the rate of change of the corresponding inertial spin rate. He went on to point out that the ratio of the spin moment of inertia to the algebraic mean transverse moment of inertia was the critical stability quantity.

In 1972, two papers questioned Likins' conclusions suggesting that the key stability parameter was the ratio of the spin moment of inertia to the geometric mean transverse moment of inertia rather than the algebraic mean transverse moment of inertia. In his 1974 paper [Ref. 4], Spencer's analysis corroborated the importance of the geometric mean transverse moment of inertia.

In 1981, Hubert [Ref. 5] concluded that using core energy instead of total energy in the expression of energy dissipation, made the energy sink prediction applicable to a dual spin satellite with energy dissipating devices on the platform.

In 1983, Cochran and Shu [Ref. 6] used the generalized method of averaging to study both energy dissipation and the energy addition required to maintain the constant spin rate of the rotor. Their conclusions substantiated Hubert's hypothesis concerning core energy assuming that the internal mass motion is sufficiently small.

In Reference 1, Chung studied the application of the energy sink method to the INTELSAT VI satellite. His conclusion was that the energy sink method did not correctly predict stability for all cases. The results of Reference 7, also a study of the INTELSAT VI satellite, indicated that stability increased as the fuel load increased, and that stability decreased as the platform asymmetry increased.

The energy sink method continues to be used throughout the community to determine the stability of spinning spacecraft. In order to have confidence in the predictions of the method, it is necessary to understand its limitations. It is hoped that this study will provide additional insight into the energy sink method's transition zone. In other words, where the boundary is between accurate energy sink predictions and erroneous predictions.

D. ORGANIZATION OF STUDY

Chapter II presents the derivation of the energy sink attitude stability criteria for an asymmetric dual spin spacecraft, a description of the simulation used to model the satellite motion, and a description of the satellite configuration used for the study. Chapter III describes the development of the system parameters for the spacecraft configuration under study, the application of the energy sink stability criteria to determine a stability prediction, and the methodology used to determine what conditions to simulate. Chapter IV presents the results and analysis of both the simulation and the energy sink predictions. Chapter V summarizes the conclusions based on the results presented in Chapter IV.

II. BACKGROUND

The analysis conducted in this study involves the comparison of the energy sink stability predictions with the results of a computer simulation. Both require a satellite configuration in order to define their input. This chapter covers the derivation of the energy sink stability criteria, a description of the simulation used for the study and a description of the satellite configuration.

A. ENERGY SINK DERIVATION

The general Likins' model [Ref. 2] used to derive the stability conditions consists of an asymmetric body P and an axisymmetric body R (Figure 1).

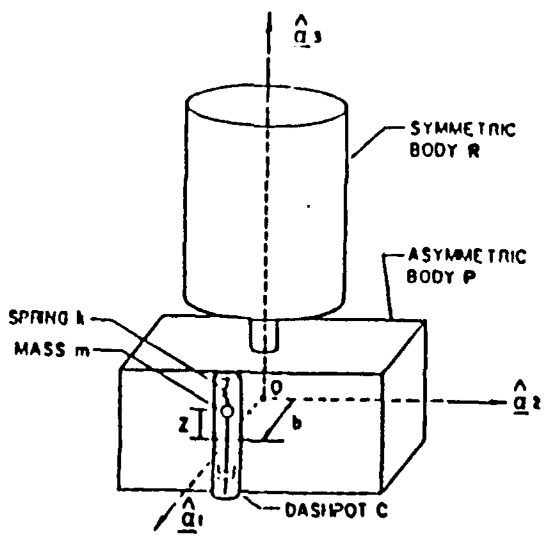


Figure 1. Idealized dual spin system.

With the coordinate system located at the center of mass of the total system, and fixed in the body P such that the center of mass of both bodies lies along the $\hat{a}_3$ axis, the rotational equation of motion governing the system is,

$$\mathbf{M} = d\mathbf{H}/dt = 0 \tag{1}$$

where **M** is the total external moment exerted on the system and **H** is the total angular momentum of the system.

The resulting linearized equations of motion are,

$$M_1 = I_1\dot{w}_1 + (I_p - I_2)w_p w_2 + I_r w_r w_2 = 0 \tag{2}$$

$$M_2 = I_2\dot{w}_2 + (I_1 - I_p)w_p w_1 - I_r w_r w_1 = 0 \tag{3}$$

$$M_3 = I_p\dot{w}_p + I_r\dot{w}_r + I_1 w_1 w_2 - I_2 w_1 w_2 = 0 \tag{4}$$

where I_p is the total principal axial moment of inertia of the system, I_r is the principal axial moment of inertia of the body R, I_1 and I_2 are the principal transverse moments of inertia of the system, w_p is the axial component of the angular velocity of the body P, w_r is the relative rate at which the body R is moving with respect to the body P, and w_1 and w_2 are the transverse components of the total system's angular velocity. These three equations have four unknowns: w_1, w_2, w_p, and w_r. The fourth equation describes the rotor/platform interface,

$$T_m = I_r(\dot{w}_p + \dot{w}_r) \tag{5}$$

where T_m is the resultant moment of all the forces acting by P on R about the rotor axis. Equations (2) and (3) can be rewritten as,

$$\dot{w}_1 + \sigma_1 w_2 = 0 \tag{6}$$

$$\dot{w}_2 - \sigma_2 w_1 = 0 \tag{7}$$

where,

$$\sigma_1 = [(I_p - I_2)w_p + I_r w_r]/I_1 \tag{8}$$

$$\sigma_2 = [(I_p - I_1)w_p + I_r w_r]/I_2 \tag{9}$$

Letting $w_2(0) = w_0$ and $w_1(0) = 0$ yields the following solution to equations (6) and (7),

$$w_1 = -w_0\sqrt{(\sigma_1/\sigma_2)}\sin\sqrt{(\sigma_1\sigma_2)}t \tag{10}$$

$$w_2 = w_0\cos\sqrt{(\sigma_1\sigma_2)}t \tag{11}$$

Applying the Routh-Hurwitz stability criterion to the characteristic equation of equations (6) and (7) shows that for stability,

$$\sigma_1\sigma_2 > 0 \tag{12}$$

The equations for the angular momentum and rotational kinetic energy for the system described above are,

$$h^2 = I_1^2 w_1^2 + I_2^2 w_2^2 + (I_p w_p + I_r w_r)^2 \tag{13}$$

$$2T = I_1 w_1^2 + I_2 w_2^2 + I_p w_p^2 + I_r w_r^2 + 2I_r w_r w_p \tag{14}$$

The nominal angular momentum is given by,

$$h_0 = I_p w_p + I_r w_r \tag{15}$$

and is assumed to be constant. Taking the derivative of equations (13) and (14) with respect to time, and taking into consideration conservation of angular momentum and assuming an energy dissipation mechanism yields,

$$0 = I_1^2 w_1 \dot{w}_1 + I_2^2 w_2 \dot{w}_2 + (I_p w_p + I_r w_r)(I_p \dot{w}_p + I_r \dot{w}_r) \qquad (16)$$

$$\dot{T} = I_1 w_1 \dot{w}_1 + I_2 w_2 \dot{w}_2 + I_p w_p \dot{w}_p + I_r w_r \dot{w}_r +$$

$$+ I_r w_r \dot{w}_p + I_r \dot{w}_r w_p < 0 \qquad (17)$$

Now the solutions in equations (10) and (11) are no longer correct. However, if the effects of the energy dissipation mechanism are felt slowly, a solution of the same form may be assumed but with $w_0 = w_0(t)$, a slowly varying function of time. Substituting this new solution into equations (16) and (17), and averaging over the period $\tau = 2\pi/\sigma_2\sqrt{(\sigma_1/\sigma_2)}$ eliminates all but the secular terms and results in,

$$w_0 \dot{w}_0 = -2\sigma_2 h_0 (I_p \dot{w}_p + I_r \dot{w}_r)/(I_1^2 \sigma_1 + I_2^2 \sigma_2) \qquad (18)$$

$$\dot{T} = w_0 \dot{w}_0 (I_1 \sigma_1 + I_2 \sigma_2)/2\sigma_2 + I_p w_p \dot{w}_p + I_r w_r \dot{w}_r +$$

$$+ I_r w_r \dot{w}_p + I_r \dot{w}_r w_p \qquad (19)$$

Substituting equation (18) into equation (19) and introducing,

$$\sigma_0 = h_0 (I_1 \sigma_1 + I_2 \sigma_2)/(I_1^2 \sigma_1 + I_2^2 \sigma_2) \qquad (20)$$

with a little manipulation produces,

$$\dot{T} = -(I_p - I_r)\dot{w}_p(\sigma_0 - w_p) +$$

$$- I_r(\dot{w}_p + \dot{w}_r)(\sigma_0 - (w_p + w_r)) \qquad (21)$$

$$= P_p + P_r$$

where P_p and P_r are the platform and rotor components of the energy dissipation rate. It is necessary that,

$$P_p < 0$$

$$P_r < 0$$

Now, let

$$\sigma_p = \sigma_1 - w_p \tag{22}$$

$$\sigma_r = \sigma_1 - (w_p + w_r) \tag{23}$$

so that,

$$P_p = -(I_p - I_r)\dot{w}_p\sigma_p \tag{24}$$

$$P_r = -I_r(\dot{w}_p + \dot{w}_r)\sigma_r \tag{25}$$

or alternately,

$$P_p/\sigma_p = -(I_p - I_r)\dot{w}_p \tag{26}$$

$$P_r/\sigma_r = -I_r(\dot{w}_p + \dot{w}_r) \tag{27}$$

Substituting equations (26) and (27) into equation (18) yields,

$$w_1\dot{w}_1 = [2\sigma_1 h_1/(I_1^2\sigma_2 + I_2^2\sigma_1)][(P_p/\sigma_p) + (P_r/\sigma_r)] \tag{28}$$

As a necessary and sufficient condition for stability,

$$w_1\dot{w}_1 < 0 \tag{29}$$

Since $\sigma_1\sigma_2 > 0$ is also necessary for stability and $h_1 > 0$ by convention, it follows that,

$$(P_p/\sigma_p) + (P_r/\sigma_r) < 0 \tag{30}$$

And because P_p and P_r are both negative, at least one of the two of σ_p and σ_r must be positive such that the total effective energy dissipation rate of equation (30) is negative.

In terms of the specific application to this study the equations simplify as follows. For a system with a despun platform and no damping mechanism on the platform, w_p is essentially zero and P_p is zero. Therefore,

$$\sigma_1 = I_r w_r / I_1 \tag{8}$$

$$\sigma_2 = I_r w_r / I_2 \tag{9}$$

and,

$$\sigma_0 = I_r w_r (I_1 \sigma_1 + I_2 \sigma_2) / (I_1^2 \sigma_1 + I_2^2 \sigma_2) \tag{20}$$

$$h_0 = I_r w_r \tag{15}$$

Now,

$$\sigma_p = \sigma_0 \tag{22}$$

$$\sigma_r = \sigma_p - w_r \tag{23}$$

and the stability criteria is,

$$P_r / \sigma_r < 0 \tag{30}$$

Or, since P_r must be negative,

$$\sigma_r > 0 \tag{31}$$

Substituting equations (8) and (9) into equation (20) yields,

$$\sigma_0 = 2 I_r w_r / (I_1 + I_2) \tag{32}$$

and substituting equation (32) into equation (23) gives,

$$\sigma_r = 2 I_r w_r / (I_1 + I_2) - w_r \tag{33}$$

or, from equation (31),

$$w_r [2 I_r / (I_1 + I_2) - 1] > 0 \tag{34}$$

Dividing by w_r and adding one to both sides of equation (34) yields,

$$2 I_r / (I_1 + I_2) > 1 \tag{35}$$

which is the familiar spin axis moment of inertia divided by the algebraic mean transverse moment of inertia developed by

Likins. It is at this point that Spencer asserts that using the geometric mean in the denominator yields more accurate results.

Spencer's conclusion is based on the fact that w^2, the rotor-fixed nutation frequency, actually varies over time as a function of I_t, the transverse moment of inertia. And he states that I_t varies as,

$$I_t(\hat{w}t) = I_1 \sin^2 \hat{w}t + I_2 \cos^2 \hat{w}t \qquad (36)$$

where,

$$\hat{w} = [I_r/\sqrt{(I_1 I_2)}]w_r \qquad (37)$$

from equations (10) and (11). These variations in w^2 must be reflected in $\dot{T}$. Likins neglected this in averaging over the period to obtain equations (18) and (19). Unfortunately, this makes the solution for an asymmetric satellite much more complicated. Spencer does not provide a complete solution. Instead, he presents a simplified example and shows that using the geometric mean for I_t provides a closer approximation to simulation results.

B. SIMULATION DESCRIPTION

The computer simulation used in this study to determine the numerical solution to the equations of motion of the satellite was developed as a part of the work done by Chung in Reference 1. This section provides a description of this simulation.

A dual spin spacecraft can be modelled as two rigid bodies capable of rotating relative to each other about a common axis. This common axis passes through the mass center of the body representing the rotor. The liquid in spherical fuel tanks can be modelled as an axisymmetric spherical pendulum with the hinge point at the center of the tank. The spherical pendula can be mounted on either of the two bodies that constitute the system (Figure 2). The energy dissipation due to the liquid sloshing can be included in the model by introducing viscous damping in the spherical joint of the pendula.

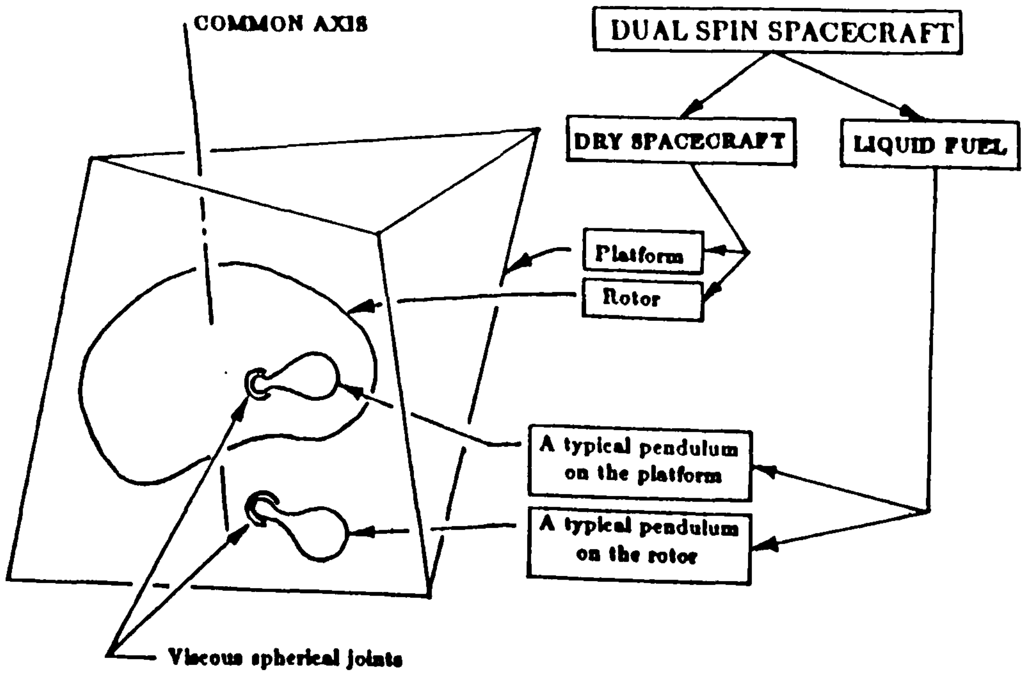


Figure 2. Schematic of a generic dual spin spacecraft.

Chung applied Kane's method to the model described above to determine the equations governing the dynamic behavior of the system. This method entails defining generalized forces called inertia forces and active forces. These forces are expressed as functions of the generalized speeds (rotational velocities) of the various components of the system. The total force acting on the system can be summarized as,

$$F_r + F_r^* = 0 \quad (r = 1, 2, 3, \ldots, N) \tag{38}$$

where N is the total number of generalized speeds, and F_r and F_r^* are the total generalized active and inertia forces in inertial space. The total number of generalized speeds, N, is equal to the number of degrees of freedom of the system. In this case, three for the three components of the angular velocity of the platform, plus one for the relative velocity between the platform and the rotor, plus three times the number of fuel tanks (each pendulum representing a fuel tank has three degrees of freedom).

To obtain the generalized inertia forces of the system, the contributions from the platform, rotor, and pendula are summed. To obtain the generalized active forces of the system, the contributions from all active forces on each part of the system are summed. It is assumed that the resultant of the external forces of the platform and the rotor are zero and that no external forces act on the pendula mounted on either the platform or the rotor.

In addition to the generalized speeds obtained by solving the differential equations developed using Kane's method, several other quantities are useful in understanding the motion of the spacecraft. These include the central angular momentum of the system, the kinetic energy of the system, the energy dissipated through the spherical joints of the pendula, the work done by the motor and the external forces, and the nutation angle of the system. All of these can be expressed in terms of the generalized speeds.

The simulation takes as input the system parameters that characterize the properties of the satellite being simulated. These include the mass, moments of inertia, location and orientation for each component of the system relative to the center of mass, as well as the key properties of the pendula (length and damping coefficient). The initial conditions for all of the generalized speeds and coordinates must also be provided. The output of the simulation is a set of values that characterize the state of the system at a given point in time. These include the nutation angle, the kinetic energy, the total energy, the work done by the rotor and the external forces, the two components of the transverse angular velocity, the platform angular velocity and the rotor angular velocity.

C. SATELLITE CONFIGURATION

The configuration of the satellite used in this study is a derivative of the INTELSAT VI satellite. The satellite consists of a platform and a symmetric rotor. There are four fuel tanks and four oxidizer tanks mounted on the platform in the configuration shown in Figure 3.

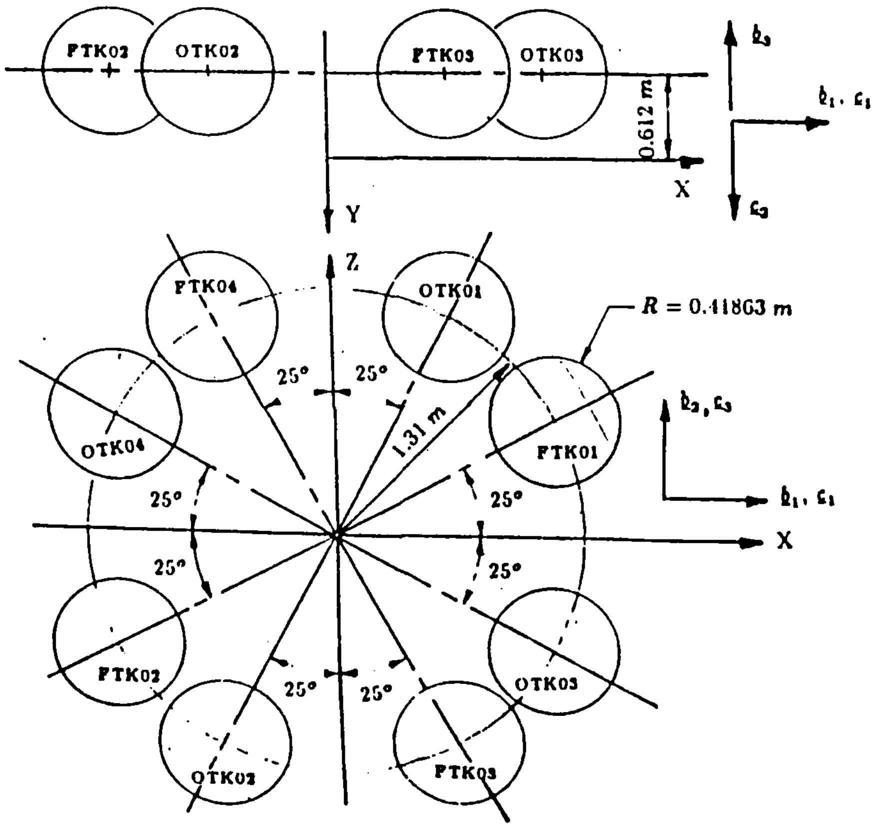


Figure 3. Arrangement of fuel and oxidizer tanks.

The liquid characteristics are as follows.

fuel density = 376.2 kg/m^3

oxidizer density = 1448.3 kg/m^3

fuel kinematic viscosity = 9.73×10^{-7}

oxidizer kinematic viscosity = 2.92×10^{-7}

The mass and inertia properties of the INTELSAT VI satellite [Ref. 1] are shown in Table I.

TABLE I. INTELSAT VI mass and inertia properties.

	PLATFORM (DRY)	ROTOR (DRY)	TOTAL % FUEL 26.2	20	15
MASS(Kg)	1058.9	695.7	2503.1	2326.0	2193.1
I_{xx}($Kg-m^2$)	1587.1	927.0	4469.5	4182.5	3950.6
I_{yy}($Kg-m^2$)	1518.3	1166.0	4491.2	4098.7	3778.3
I_{zz}($Kg-m^2$)	1529.4	973.7	4458.5	4171.5	3939.6
I_{yz}($Kg-m^2$)	0.0	0.0	0.0	0.0	0.0
I_{xz}($Kg-m^2$)	44.4	-6.1	38.3	38.3	38.3
I_{xy}($Kg-m^2$)	0.0	0.0	0.0	0.0	0.0

These parameters are modified using the procedure described in Chapter III, Section A to obtain the configuration used in the study (Tables II-IV).

TABLE II. Study satellite mass and inertia properties - - 26.2% fuel load.

	PLATFORM (DRY)	ROTOR (DRY)	TOTAL
MASS(Kg)	1058.2	695.7	2503.1
I_{xx}(Kg-m^2)	1095.2	1418.9	4469.5
I_{yy}(Kg-m^2)	1518.3	2791.0	6116.2
I_{zz}(Kg-m^2)	1084.2	1418.9	4458.5
I_{yz}(Kg-m^2)	0.0	0.0	0.0
I_{xz}(Kg-m^2)	44.4	-6.1	38.3
I_{xy}(Kg-m^2)	0.0	0.0	0.0

TABLE III. Study satellite mass and inertia properties - -20% fuel load.

	PLATFORM (DRY)	ROTOR (DRY)	TOTAL
MASS(Kg)	1058.8	695.7	2326.0
I_{xx}(Kg-m^2)	1006.0	1467.3	4182.5
I_{yy}(Kg-m^2)	1518.3	2887.9	5820.6
I_{zz}(Kg-m^2)	1035.8	1467.3	4171.5
I_{yz}(Kg-m^2)	0.0	0.0	0.0
I_{xz}(Kg-m^2)	44.4	-6.1	38.3
I_{xy}(Kg-m^2)	0.0	0.0	0.0

TABLE IV. Study satellite mass and inertia properties - - 15% fuel load.

	PLATFORM (DRY)	ROTOR (DRY)	TOTAL
MASS(Kg)	1058.8	695.7	2183.1
I_{xx}(Kg-m^2)	1006.0	1508.1	3950.6
I_{yy}(Kg-m^2)	1518.3	2969.4	5581.7
I_{zz}(Kg-m^2)	995.0	1508.1	3939.6
I_{xy}(Kg-m^2)	0.0	0.0	0.0
I_{xz}(Kg-m^2)	44.4	-6.1	38.3
I_{yz}(Kg-m^2)	0.0	0.0	0.0

III. PROCEDURE

This chapter describes the methodology used over the course of the study. The first section deals with how to determine the spacecraft configuration and system parameters. Section B describes how the energy sink criteria is applied to a specific configuration to determine the stability of that configuration. Section C contains the procedure for running the simulation and obtaining the simulation output.

A. SYSTEM PARAMETERS

In order to investigate the behavior of a marginally stable spacecraft it is first necessary to model a spacecraft with this configuration. To accomplish this, the system parameters of the INTELSAT VI satellite were modified to obtain the appropriate configuration. One of the programs written by Chung in Reference 1 to support the simulation produces as its output a summary of the INTELSAT VI system parameters. To modify the system parameters to achieve a given inertia ratio, $I_s/\bar{I}_t$, where $\bar{I}_t = (I_1 + I_2)/2$, the algebraic mean of the spacecraft transverse moments of inertia must be calculated. This is then multiplied by the inertia ratio to determine the desired axial moment of inertia, I_s. For the case of a despun platform, this equates to the axial moment of inertia of the wet rotor, I_r. However, the simulation takes as its input the dry spacecraft parameters. Subtracting the old wet rotor moment of inertia from the desired moment of inertia yields the amount the dry rotor axial moment of inertia must be increased. It is a general theorem of rigid body mechanics that for a given body, the sum of any two of the principle moments of inertia must be greater than the third. This must be kept in mind to achieve a realistic design. To adhere to this principle, the sum of the dry rotor transverse moments of inertia is subtracted from the new dry rotor axial moment of inertia. The result is divided by two and added to each of the dry rotor transverse moments of inertia. An additional adjustment is made to achieve perfect symmetry on the rotor. The difference between the dry rotor transverse moments of inertia is added to the smaller of the two to make them equal. To balance the system and maintain the same total spacecraft transverse moments of inertia, the corresponding platform transverse moments of inertia are decremented by the same amounts as were added to the rotor transverse moments of inertia.

B. ENERGY SINK PREDICTION

Given numerical values for I_1, I_2, I_p, I_r, w_p, and w_r, developing an energy sink prediction is a reasonably straight forward application of the equations derived in Chapter II.

The best way to implement the energy sink equations is by means of a simple spread sheet using the equations below.

$$h_a = I_p w_p + I_r w_r \tag{15}$$

$$\sigma_1 = [(I_p - I_2)w_p + I_r w_r]/I_1 \tag{8}$$

$$\sigma_2 = [(I_p - I_1)w_p + I_r w_r]/I_2 \tag{9}$$

$$\sigma_n = h_a(I_1\sigma_1 + I_2\sigma_2)/(I_1^2\sigma_1 + I_2^2\sigma_2) \tag{20}$$

$$\sigma_p = \sigma_n - w_p \tag{22}$$

$$\sigma_r = \sigma_n - (w_p + w_r) \tag{23}$$

Recall that the stability criteria were,

$$\sigma_1\sigma_2 > 0 \tag{12}$$

$$(P_p/\sigma_p) + (P_r/\sigma_r) < 0 \tag{30}$$

and that P_p and P_r are always negative. Concentrating on the second relationship, for the system being studied, P_p is zero and P_r is negative since all of the fuel tanks are mounted on the rotor. Therefore, if σ_r is positive the system is inherently stable. However, if σ_r is negative σ_p must be positive and a nutation damper must be installed on the platform to overcome the destabilizing effect of the rotor.

C. SIMULATION PROCEDURE

To determine the stability of the system the parameter of interest is the nutation angle, Φ. If Φ grows without bound, the system is unstable. If Φ damps out to zero, the system is stable. In executing the simulation, an initial transverse rate of 0.2 radians per second is applied to induce an initial nutation angle.

The first step in the study entails finding a satellite configuration with an inertia ratio slightly greater than one for which the simulation results indicate the system is stable. An inertia ratio of 1.01 is arbitrarily chosen for the initial run. If the simulation results indicate the system is unstable the inertia ratio must be increased and the simulation run again. This process is continued until a stable configuration is found upon which the remainder of the study will be based. After determining a stable configuration the platform asymmetry is varied and the satellite motion is simulated for three different fuel loads. The INTELSAT VI satellite uses a liquid propellant rocket for its apogee kick motor and consumes nearly 75% of its fuel during this maneuver. The beginning of life fuel load for the INTELSAT VI satellite upon which the study configuration is based is 26.2%. Therefore, fuel loads of 26.2%, 20%, and 15% were chosen for this study. The initial asymmetry was arbitrarily selected to be 5%. If the results indicate the system is stable the asymmetry is increased by 5% and the simulation run again. This process continues until the simulation results indicate the system is unstable.

IV. RESULTS AND ANALYSIS

This chapter presents the results of the study and a discussion of their significance. The first section presents a summary of the results of the simulation runs. Section B contains a summary of the energy sink predictions for all of the cases simulated.

A. SIMULATION RESULTS

As outlined in chapter III, the first step in the study was to find a configuration for which the system was stable. The first simulation run was for the case of a symmetric system with an inertia ratio of 1.01 and a fuel load of 26.2%. The simulation time was set at 200 seconds. The result is shown in Figure 4.

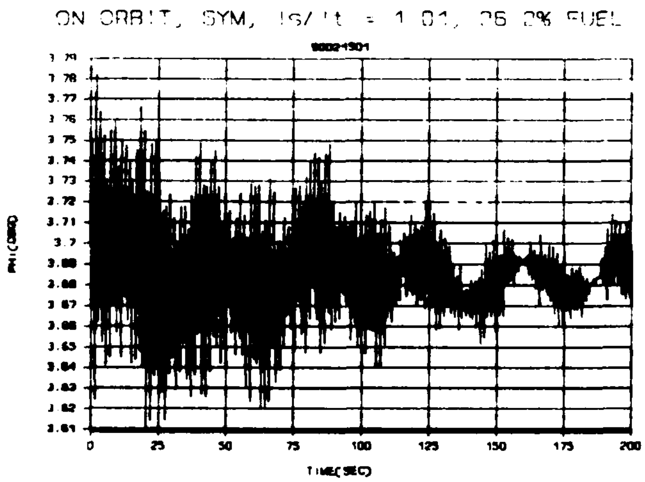


Figure 4. Initial simulation result.

From the graph it is difficult to determine whether the system is stable or unstable because the simulation time was not of sufficient length. Rather than extending the simulation time to arrive at a more conclusive result, the simulation was run again with the damping coefficients increased by a factor of 100. Figure 5 shows that this time the results are much more definitive.

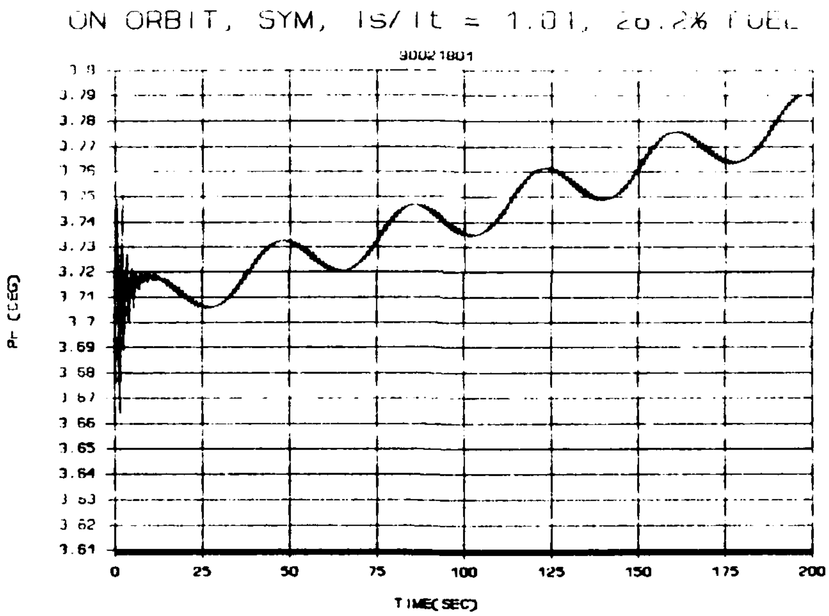


Figure 5. Initial simulation result with increased damping.

The system is obviously unstable. From this point on, all simulation runs were made with the damping coefficients increased in order to minimize the main frame CPU time.

Next the simulation was run with an inertia ratio of 1.1. The result reveals that the system is stable as shown in Figure 6.

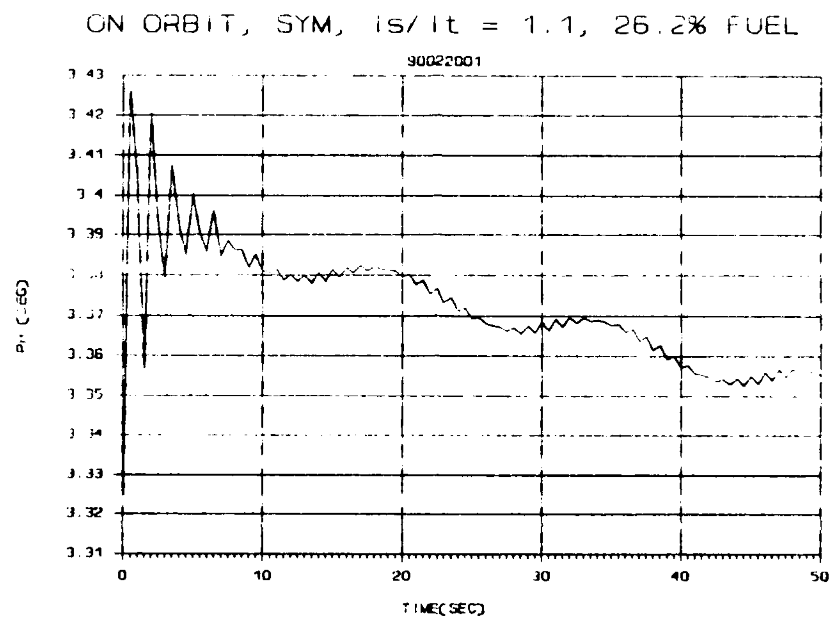


Figure 6. Result for an inertia ratio of 1.1 and a fuel load of 26.2%.

Having achieved stability, the fuel load was changed to 20% and the simulation was run again. The system remained stable as shown in Figure 7 so the fuel load was reduced to 15% and the simulation run again. The result still indicated that the system was stable (Figure 8). Having achieved stability for all three fuel loads, the next step was to begin varying the asymmetry of the platform.

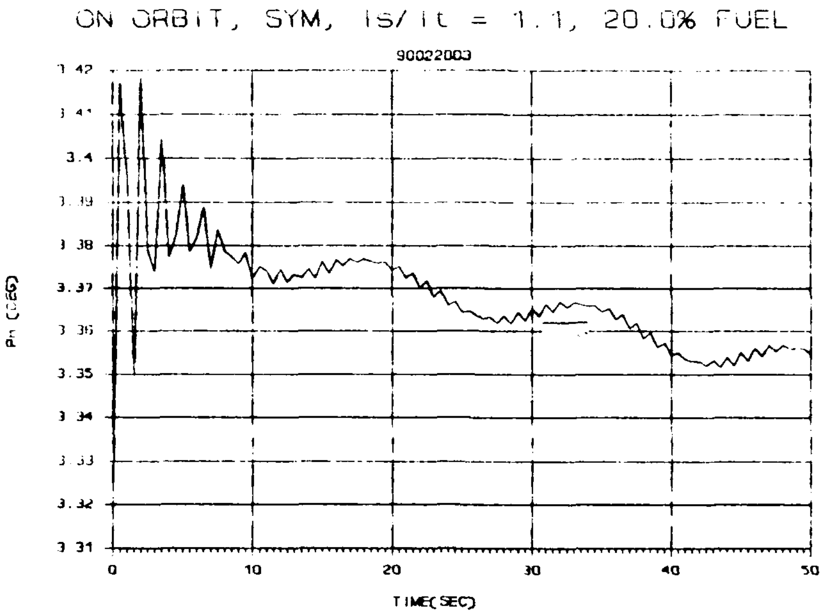


Figure 7. Result for an inertia ratio of 1.1 and a fuel load of 20%.

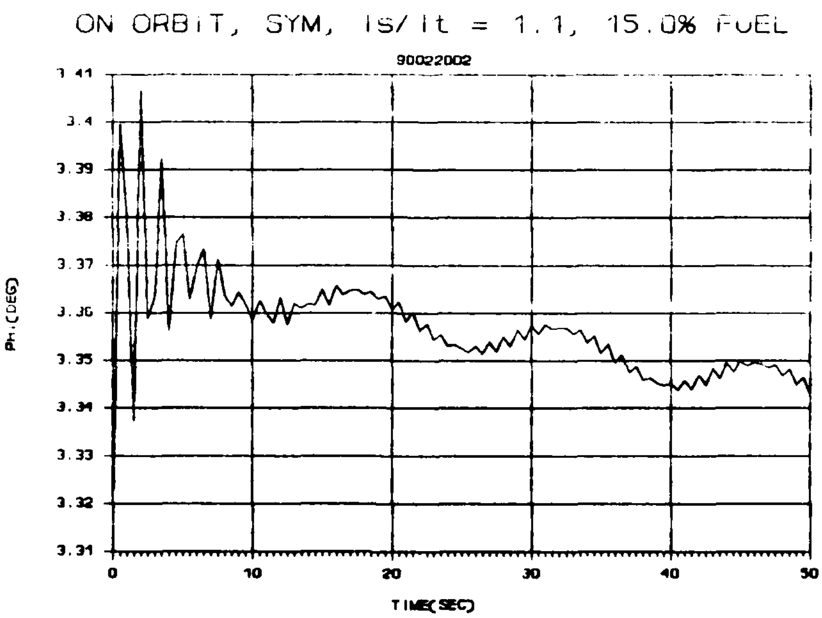


Figure 8. Result for an inertia ratio of 1.1 and a fuel load of 15%.

A summary of the results for all of the asymmetry variations can be seen in Table V. The system remained stable for every case simulated.

TABLE V. Simulation results, $2I_s/(I_1 + I_2)$ = 1.1.

FUEL LOAD	ASYMMETRY 5%	10%	15%	20%	25%	35%	55%
15%	S	S	S	S	S	S	S
20%	S	S	S	S	S	S	S
26.2%	S	S	S	S	S	S	S

Note: U = Unstable, S = Stable

In light of the results reported in Reference 7, the results in Table V were surprising. However, upon re-examining the energy sink equations, it was discovered this is exactly what should have been expected. Starting with a symmetric satellite, such that $I_1 = I_2 = I_t$ the stability criteria is,

$$2I_r/(I_1 + I_2) > 1 \tag{35}$$

To create an asymmetry, let $I_1 = I_t + a$ and $I_2 = I_t - a$. Substituting for I_1 and I_2 in equation (35) reduces to the identical equation indicating the system should remain stable. Using the geometric mean as suggested by Spencer, equation (35) becomes,

$$I_r/(\sqrt{I_1 I_2}) > 1 \qquad (39)$$

Substituting for I_1 and I_2 here yields,

$$I_r/(\sqrt{(I_t^2 - a^2)}) > 1 \qquad (40)$$

which has to be greater than the original inertia ratio indicating that the system is getting more stable as the asymmetry is increased. Regardless of the method used to determine the average transverse moment of inertia, both predict the system should remain stable because $I_r/\bar{I}_t$ is greater than one. This being the case, it was now necessary to find a configuration which was unstable to determine if the system would become stable as the asymmetry was increased.

The simulation was run for symmetric configurations with inertia ratios of 1.03, 1.05, and 1.07. The results are summarized in Table VI. They show that for a symmetric spacecraft the stability cutoff falls between inertia ratios of 1.03 and 1.05.

TABLE VI. Simulation results of varying inertia ratio.

FUEL	INERTIA RATIO				
LOAD	1.01	1.03	1.05	1.07	1.1
15%	-	U	S	S	S
20%	-	U	S	S	S
26.2%	U	U	S	S	S

Note: U = Unstable, S = Stable

Given the results in Table VI, the configuration with an inertia ratio of 1.03 was selected to evaluate the influence of varying the asymmetry.

The summary of these results is provided in Table VII. As expected, the system eventually became stable as the asymmetry was increased.

TABLE VII. Simulation results for varying the platform asymmetry.

FUEL LOAD	ASYMMETRY 0%	25%	40%	55%
15%	U	U	S	S
20%	U	U	S	S
26.2%	U	U	S	S

Note: U = Unstable, S = Stable

What appeared to have happened was that as the asymmetry was increased, the inertia ratio increased enough to become greater than the stability cutoff inertia ratio as equation (40) indicates. To confirm this, a graph of inertia ratio versus asymmetry was created using equation (39) to employ the geometric mean transverse moment of inertia (Figure 9). As expected, it showed that for a symmetric inertia ratio of 1.03, as the platform asymmetry was increased, the inertia ratio increased enough to become greater than the stability cutoff inertia ratio that falls between 1.03 and 1.05. This plot combined with Table VII indicate that the stability cutoff inertia ratio falls between 1.032 and 1.035.

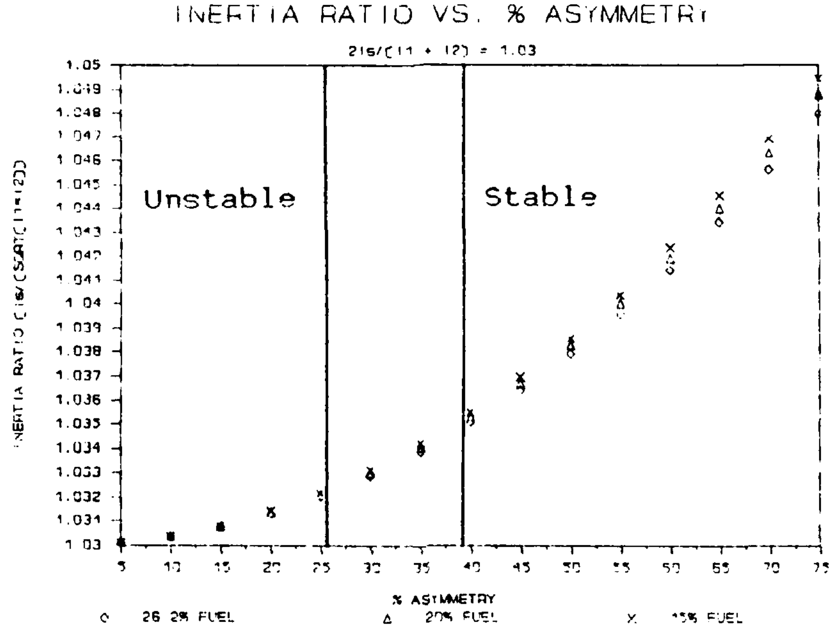


Figure 9. Inertia ratio vs. asymmetry for 26.2%, 20%, and 15% fuel loads.

To try and pin down the stability cutoff, the simulation was run for the symmetric case, for all three fuel loads with inertia ratios of 1.035, 1.04 and 1.045. Adding the results to Table VI shows that the stability cutoff inertia ratio for a symmetric satellite is different than for an asymmetric satellite. The simulation results indicate the stability cutoff for the symmetric satellite is approximately 1.045 (Table VIII).

TABLE VIII. Simulation results for varying inertia ratio of symmetric spacecraft.

FUEL LOAD	INERTIA RATIO 1.01	1.03	1.035	1.04	1.045	1.05	1.07	1.10
15%	-	U	U	U	M	S	S	S
20%	-	U	U	U	M	S	S	S
26.2%	U	U	U	U	M	S	S	S

Note: U = Unstable, S = Stable, M = Marginal

Finally, several simulation runs were made with fuel loads of 50% and 75% to investigate the effect of higher fuel loads. Higher fuel loads make the system more stable. In fact these higher fuel loads lowered the stability cutoff inertia ratio for a symmetric satellite to approximately 1.02 (Table IX).

TABLE IX. Simulation results of varying inertia ratio for higher fuel loads.

FUEL LOAD	INERTIA RATIO 1.01	1.02	1.03	1.05	1.07	1.1
50%	U	M	S	S	S	S
75%	U	M	S	S	S	S

Note: U = Unstable, S = Stable, M = Marginal

The results of varying the asymmetry on a configuration with an inertia ratio of 1.01 and a fuel load of 75% are shown in Table X.

TABLE X. Simulation results of varying asymmetry for a 75% fuel load.

FUEL LOAD	ASYMMETRY 0%	25%	40%
75%	U	U	S

Note; U = Unstable, S = Stable

Here, as with the lower fuel loads the system became stable as the asymmetry was increased. Graphing inertia ratio versus asymmetry for a configuration with a 75% fuel load shows that the stability cutoff inertia ratio in this case is between 1.012 and 1.015 (Figure 10). As with the symmetric cases, the stability cutoff inertia ratio is significantly lower for the higher fuel loads than for the lower fuel loads.

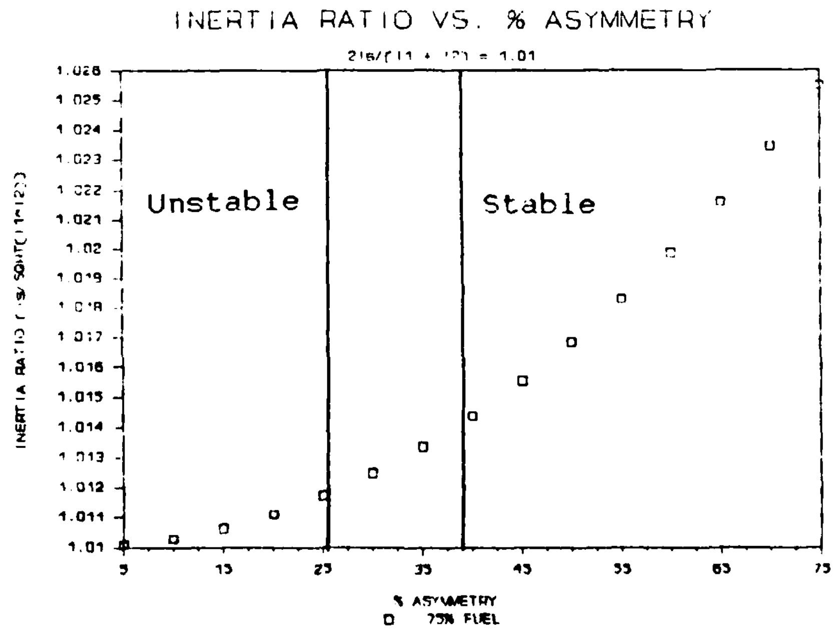


Figure 10. Inertia ratio vs. asymmetry for a 75% fuel load.

B. ENERGY SINK PREDICTIONS

As was shown in Chapter II, the energy sink stability criteria is,

$$w_3\dot{w}_3 < 0 \tag{29}$$

and,

$$(P_p/\sigma_p) + (P_r/\sigma_r) < 0 \tag{30}$$

where one or the other, or both, of σ_p or σ_r must be positive such that equation (30) is true. For the specific case of a dual spin spacecraft with a despun platform, equation (30) reduces to,

$$I_r/\bar{I}_t > 1$$

By definition, all of the energy sink predictions for this study were stable since the initial requirement in defining the configuration was an inertia ratio slightly greater than one. It should be noted that it is irrelevant which method is used to calculate the average transverse moment of inertia. The energy sink predictions were incorrect for every case where the simulation results showed the system was unstable. For symmetric configurations with the lower fuel loads, the energy sink predictions were incorrect when the inertia ratio was less than 1.045. For symmetric configurations with the higher fuel loads, the energy sink predictions were incorrect when the inertia ratio was less than 1.02. For the asymmetric variations on a configuration with a symmetric inertia ratio of 1.03, with the lower fuel loads, the energy sink predictions were incorrect for asymmetries less than 25%, correct for asymmetries greater than 40%, and indeterminate for the transition zone between 25% and 40% asymmetry. Using Spencer's method to determine the inertia ratio, 40% asymmetry equates to an inertia ratio of 1.035, 25% asymmetry equates to an inertia ratio of 1.032.

V. SUMMARY AND CONCLUSIONS

This chapter presents the conclusions based on the results presented in the Chapter IV.

A. SUMMARY

The energy sink stability criteria for a dual spin spacecraft with a despun platform specify that the system must have an inertia ratio greater than one. The results of this study indicate that it is not sufficient for the system's inertia ratio just to be greater than one. It must be greater than one by a predictable amount. Below this stability cutoff inertia ratio the energy sink method predicts stability when the simulation results indicate the system is unstable. Further, the stability cutoff inertia ratio varies inversely with the fuel load. For the case of the lower fuel loads used in this study the stability cutoff inertia ratio is approximately 1.045 for a symmetric spacecraft. For the higher fuel loads it is approximately 1.02 (Figure 11).

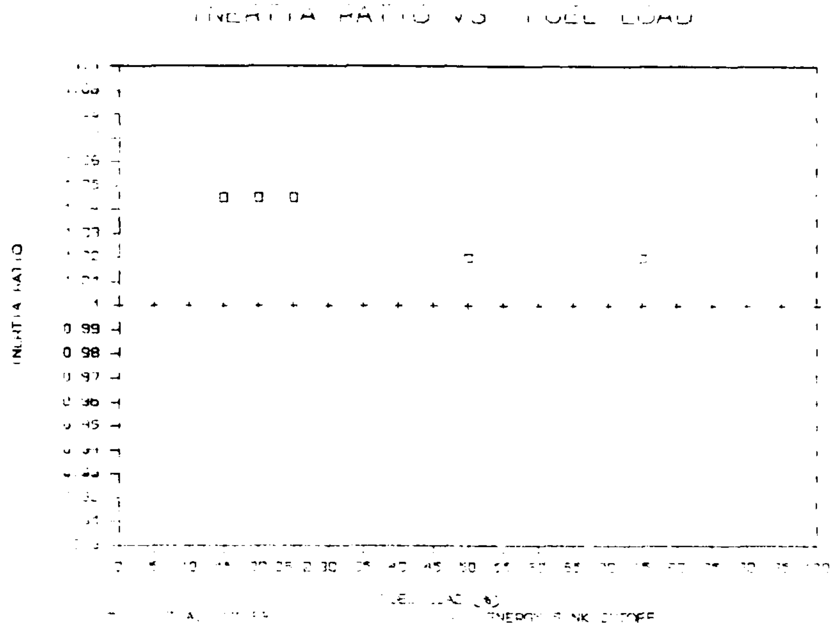


Figure 11. Inertia ratio vs. fuel load showing for stability, inertia ratio must be greater than one.

As shown in the development in Chapter II, Likins' stability criteria can be reduced to show that the key parameter for stability predictions is the ratio of the spin moment of inertia to the algebraic mean transverse moment of inertia. Recall that Spencer claimed that Likins was incorrect and that the geometric mean transverse moment of inertia is the key stability parameter. The results of this study support Spencer, indicating that as the asymmetry is increased the system becomes more stable. This trend is predictable using the geometric mean in computing the inertia ratio. Interestingly, the stability cutoff inertia ratio for the asymmetric variations on a satellite with a symmetric inertia ratio of 1.03 is around 1.035. As the asymmetry is increased, the inertia ratio grows from the symmetric 1.03 until it eventually reaches 1.05 for a 75% platform asymmetry. It exceeds the stability cutoff inertia ratio at 40% asymmetric (Figure 12). It must be pointed out that while Spencer's geometric mean transverse moment of inertia more accurately shows the effects of asymmetry, it is also inaccurate in predicting stability below the stability cutoff inertia ratio.

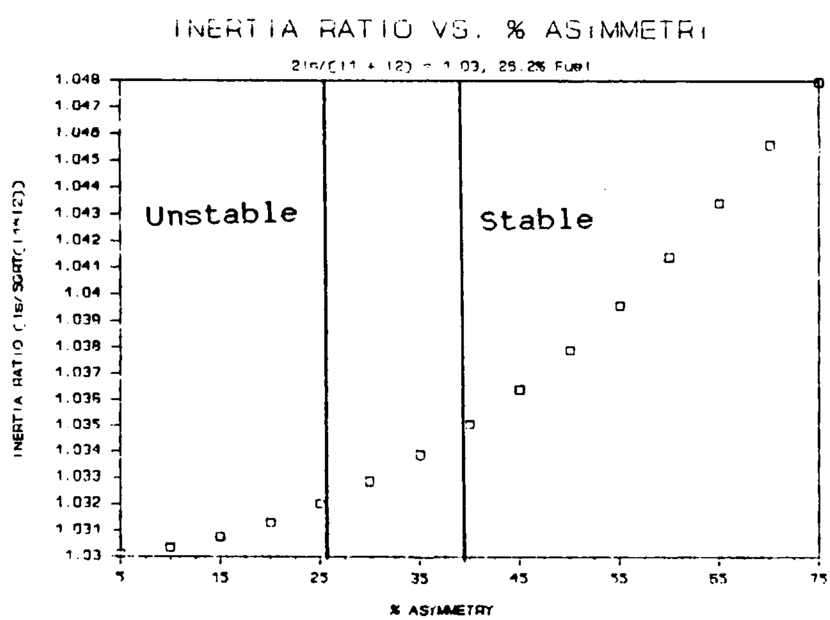


Figure 12. Inertia ratio vs. asymmetry. Calculating inertia ratio using Likins' method vs. Spencer's method.

B. CONCLUSIONS

It is unclear why there is a difference in the stability cutoff for the symmetric and asymmetric configurations. This is perhaps an opportunity for future research. What has been determined or confirmed in this study is that:

(1) the energy sink stability criteria is not valid below a stability cutoff inertia ratio which is not just equal to one, but greater than one.

(2) stability increases as the fuel load is increased.

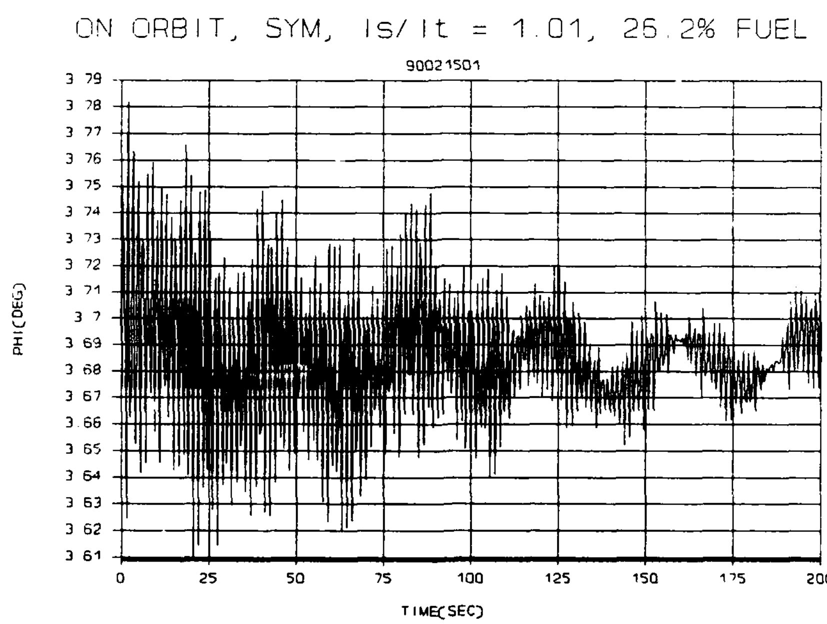
(3) stability increases as the platform asymmetry is increased.

(4) Spencer was correct in asserting that it is more accurate to use the geometric mean transverse moment of inertia than the algebraic mean transverse moment of inertia in computing the inertia ratio.

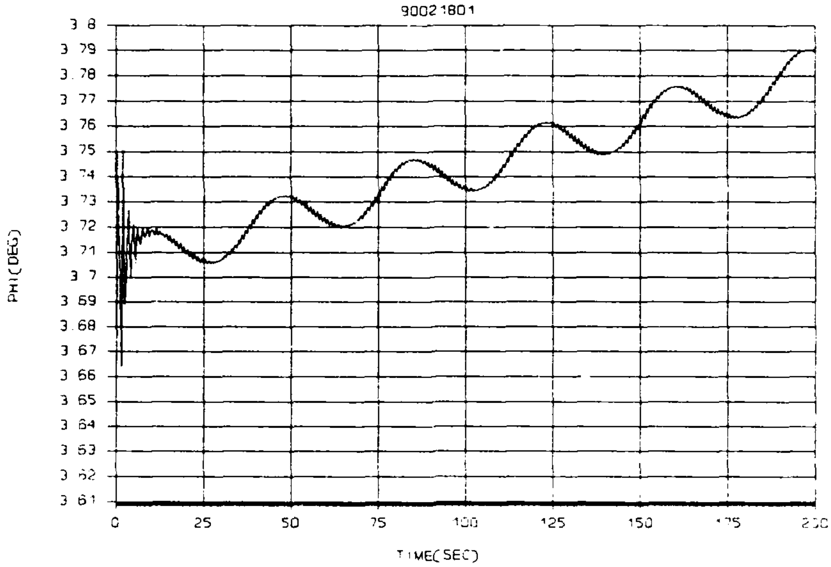
APPENDIX A

SIMULATION DATA

Included here are the graphs of nutation angle versus time for each simulated case. The graphs are ordered in the sequence in which they are presented in the text.



ON ORBIT, SYM, Is/It = 1.01, 26.2% FUEL



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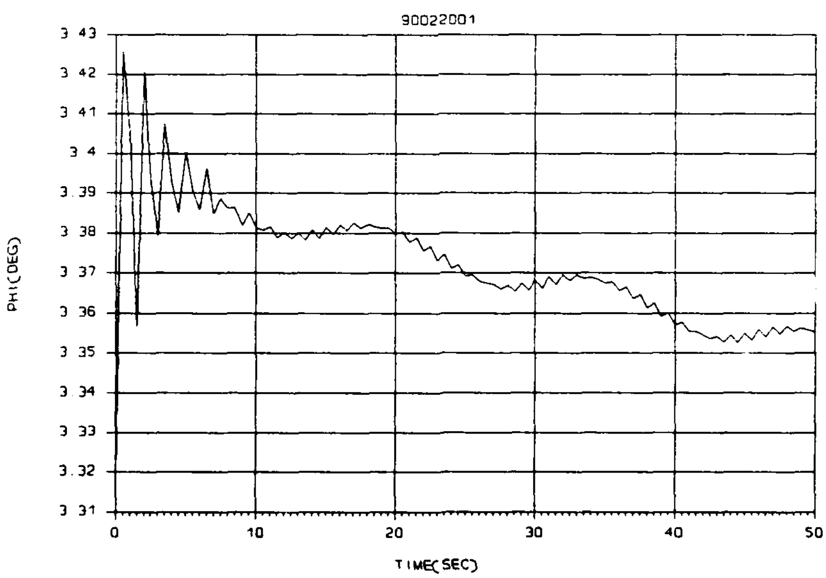

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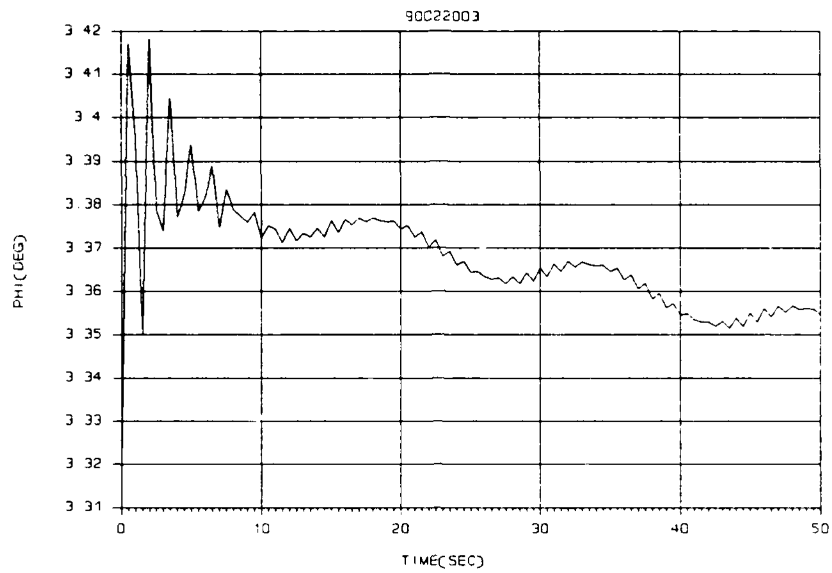

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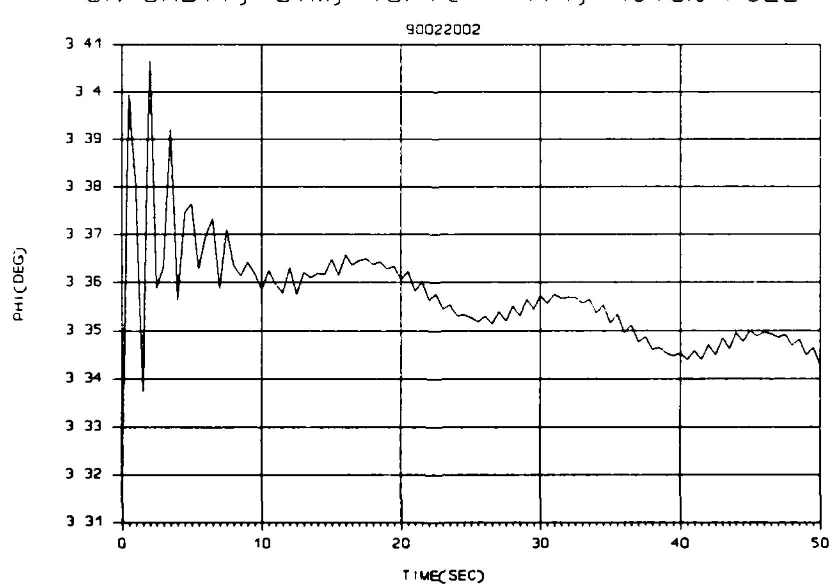

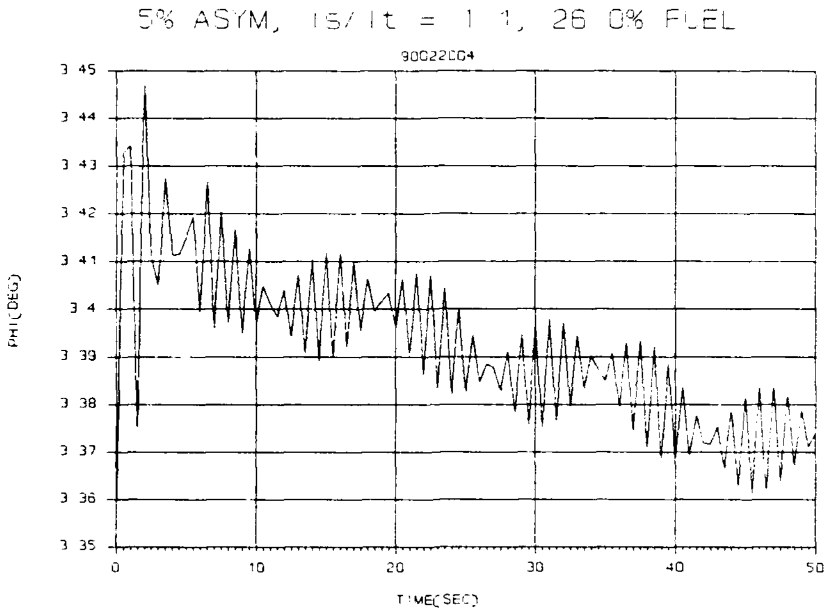
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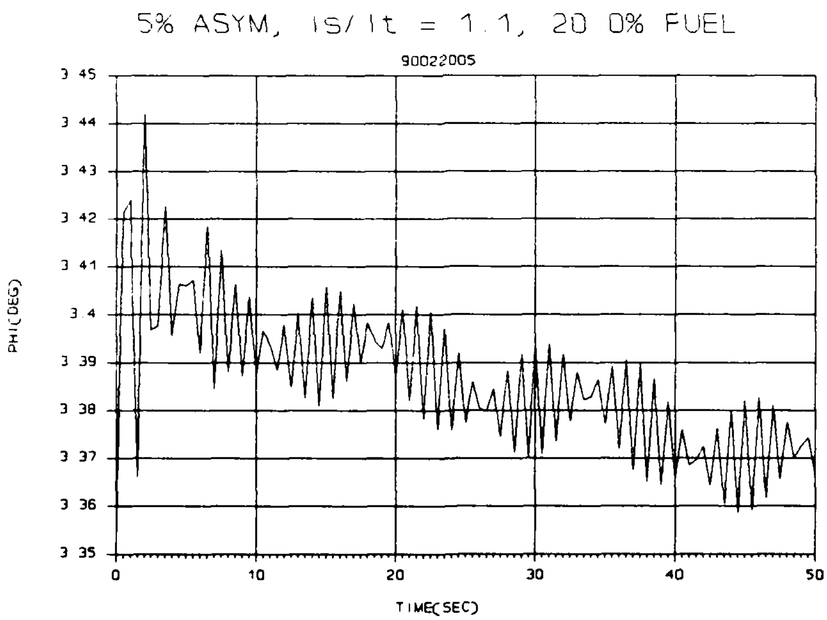
PHI(DEG)
3.45
3.44
3.43
3.42
3.41
3.4
3.39
3.38
3.37
3.36
3.35
0
10
20
30
40
50
TIME(SEC)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90022005
PHI(DEG)
3.45
3.44
3.43
3.42
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3.4
3.39
3.38
3.37
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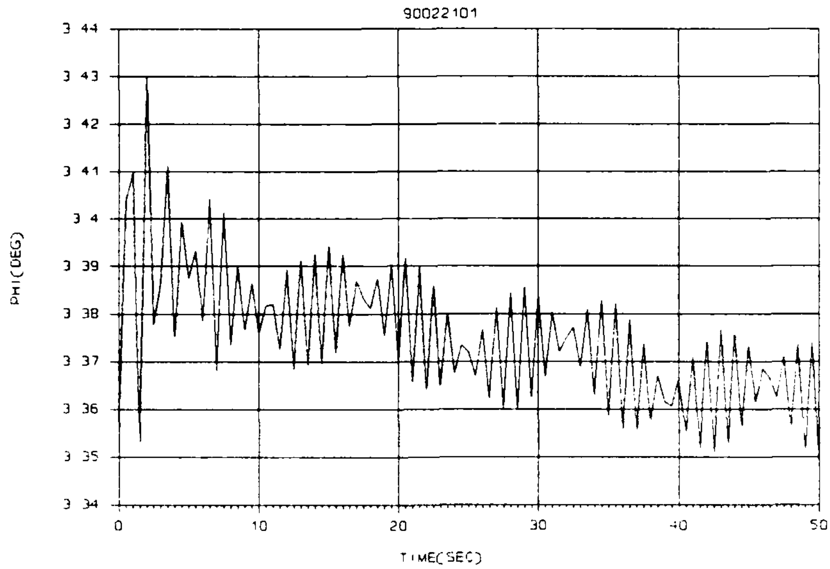
10
20
30
40
50
TIME(SEC)

5% ASYM, Is/It = 1 1, 15% FUEL



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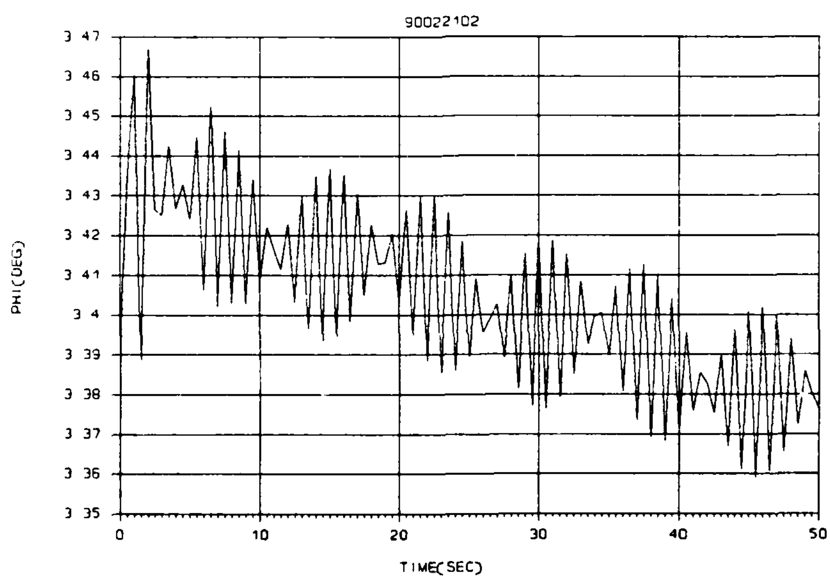

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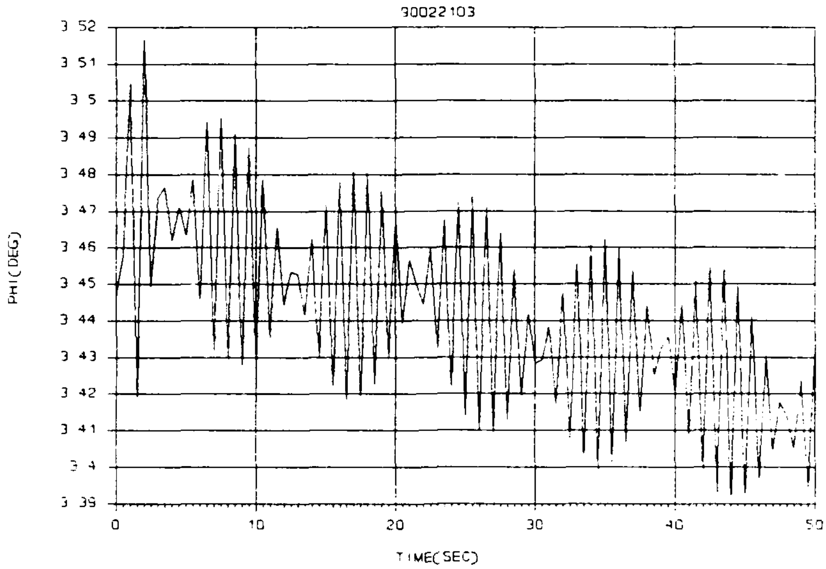

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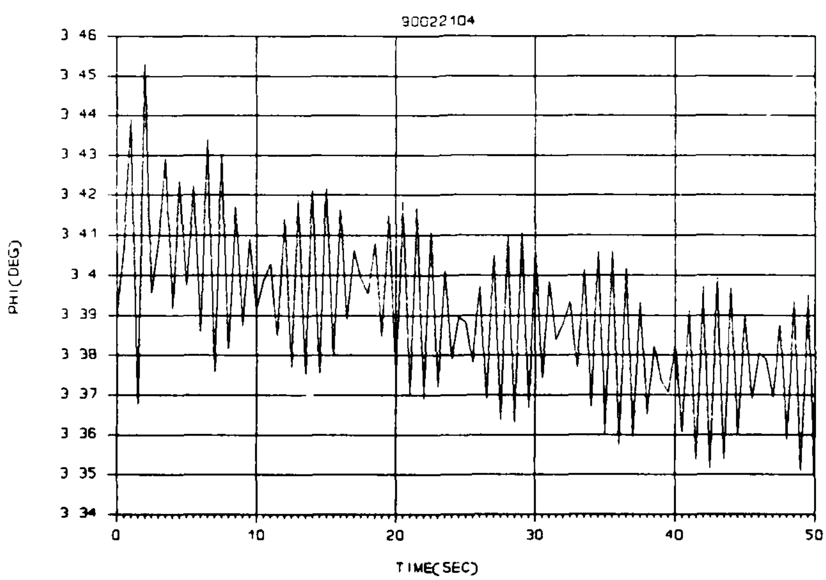

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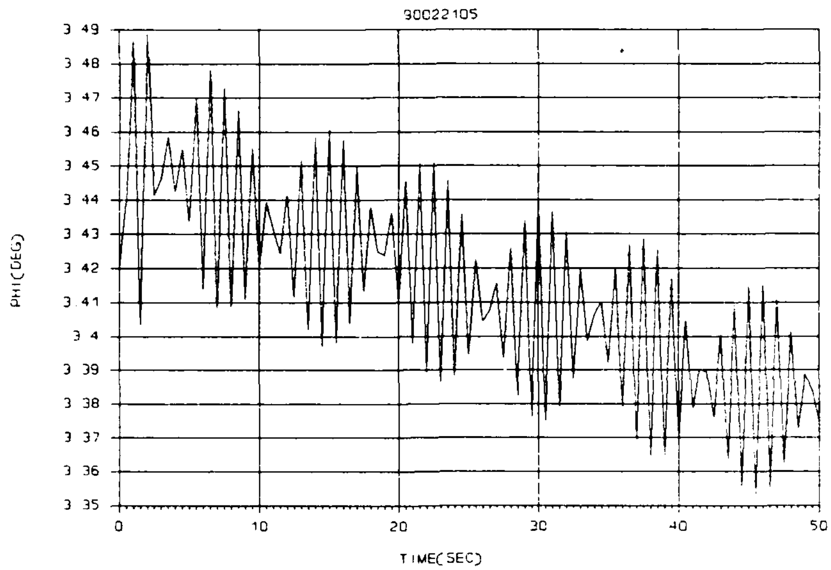

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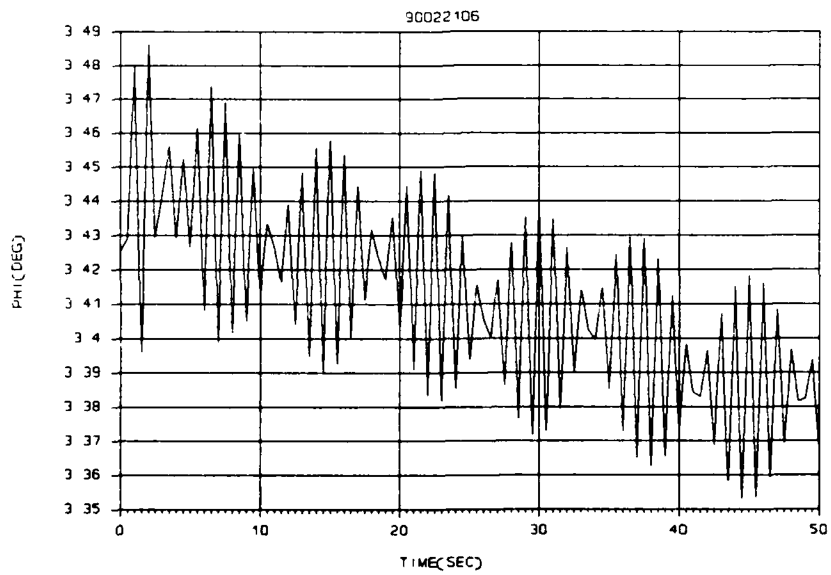

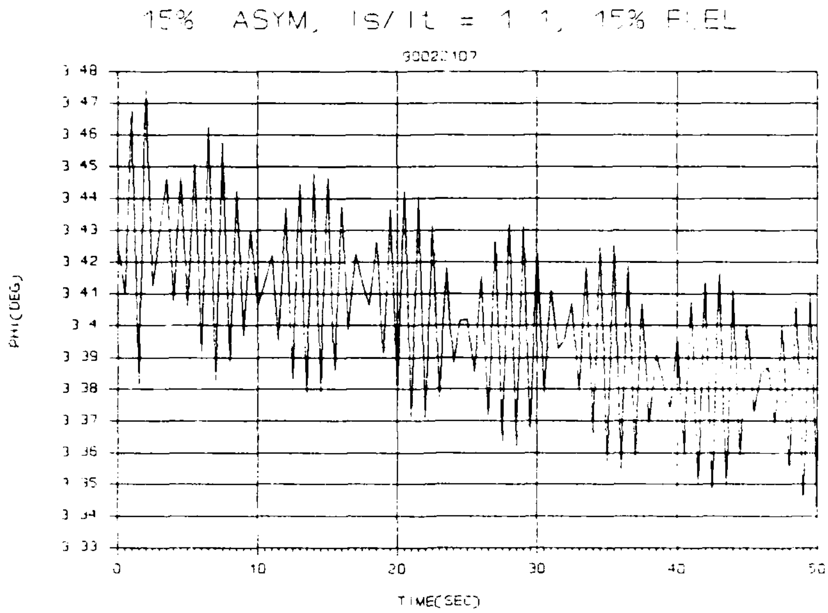

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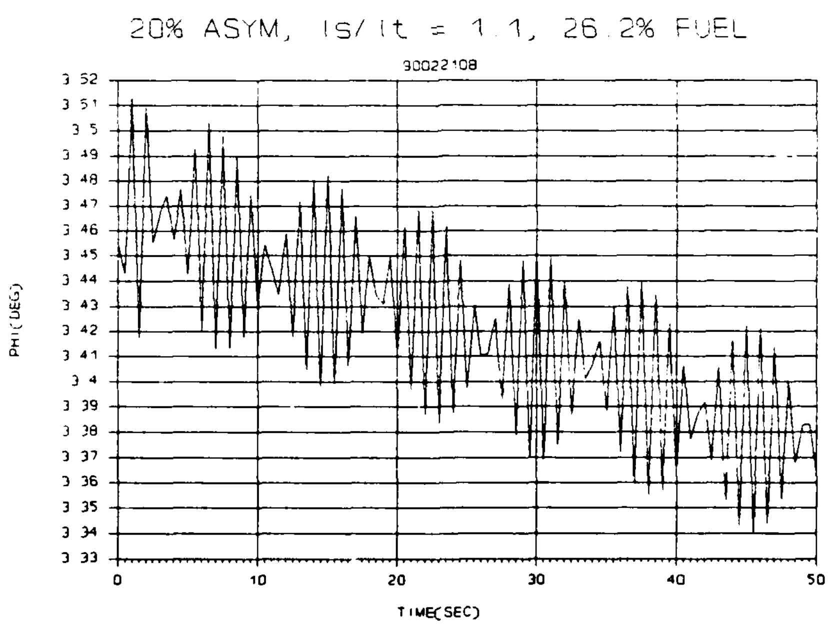

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TIME(SEC)

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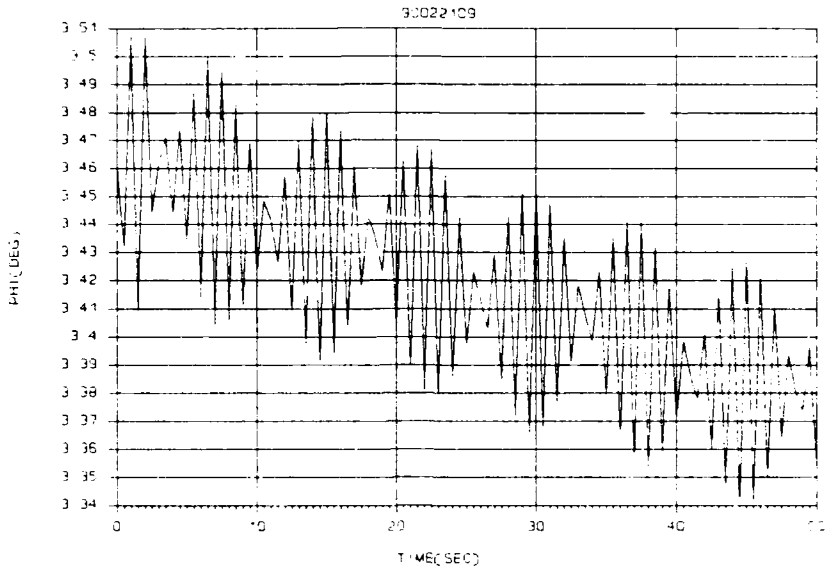

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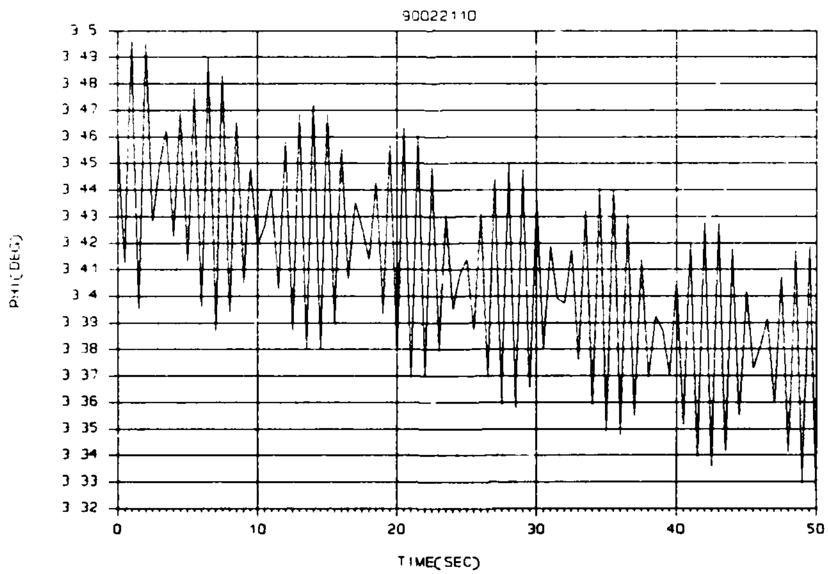

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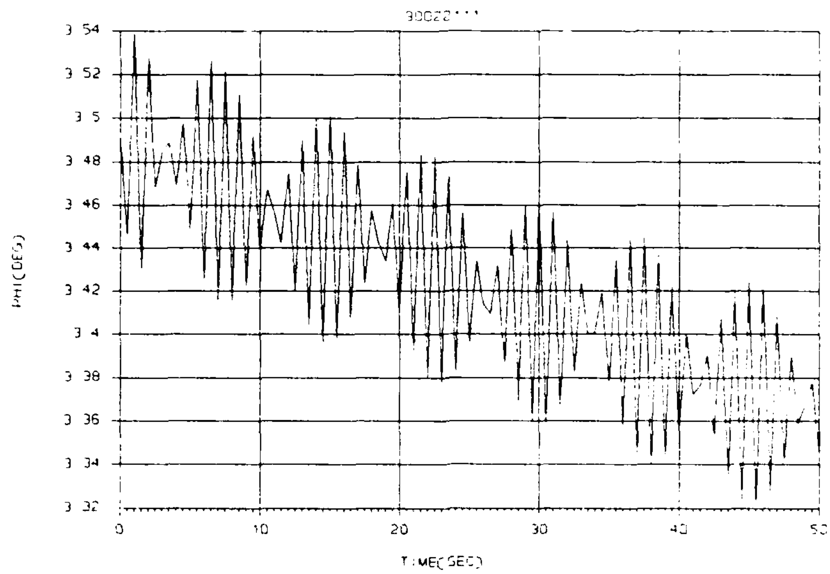

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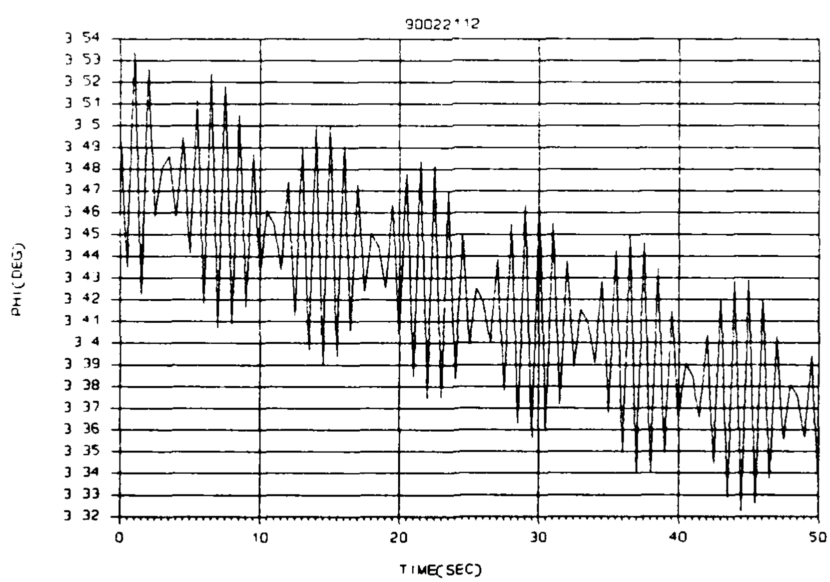

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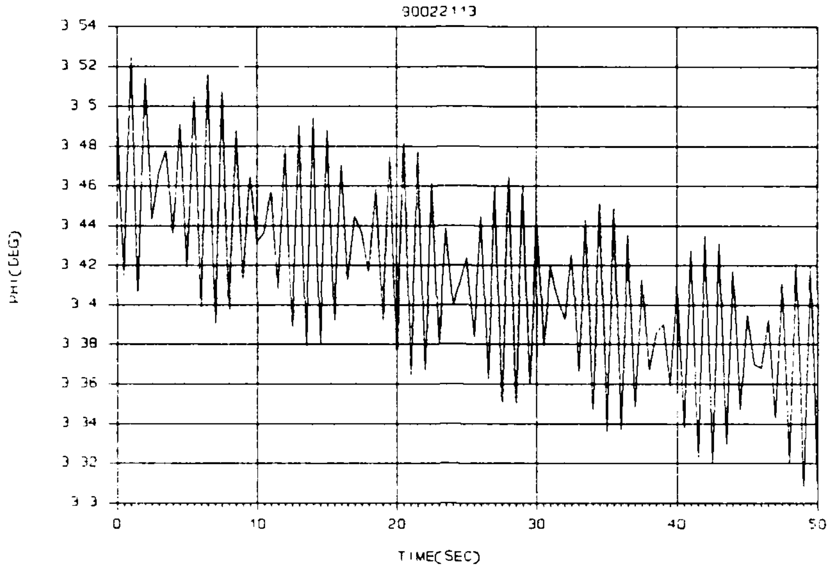

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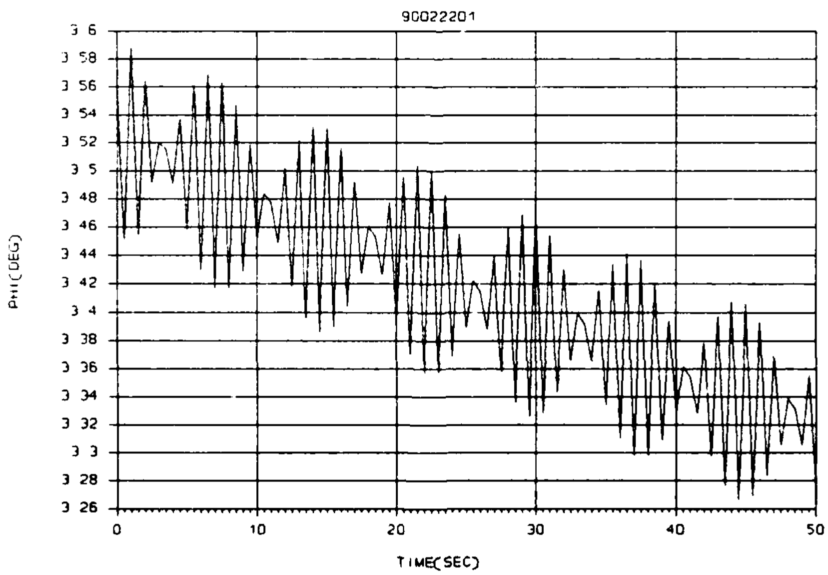

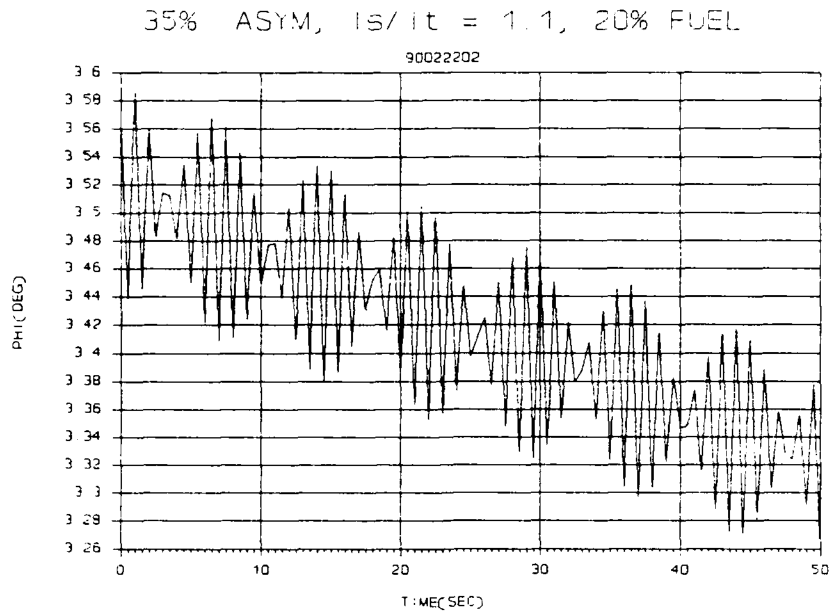
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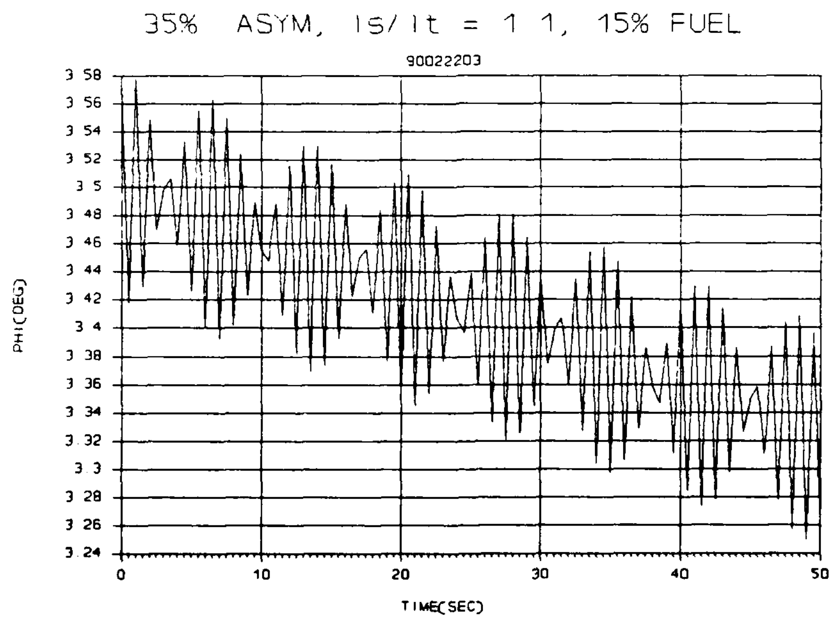
TIME(SEC)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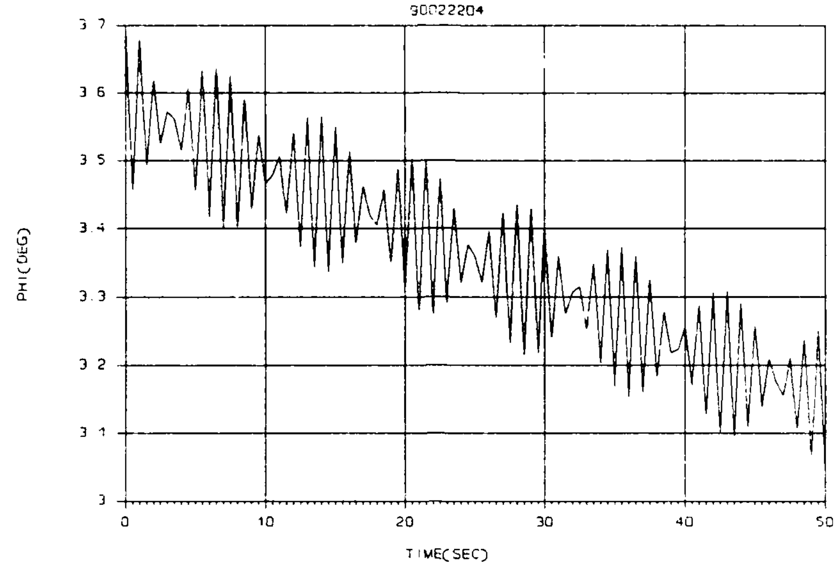
TIME(SEC)

55% ASYM, Is/It = 1 1, 26 2% FUEL



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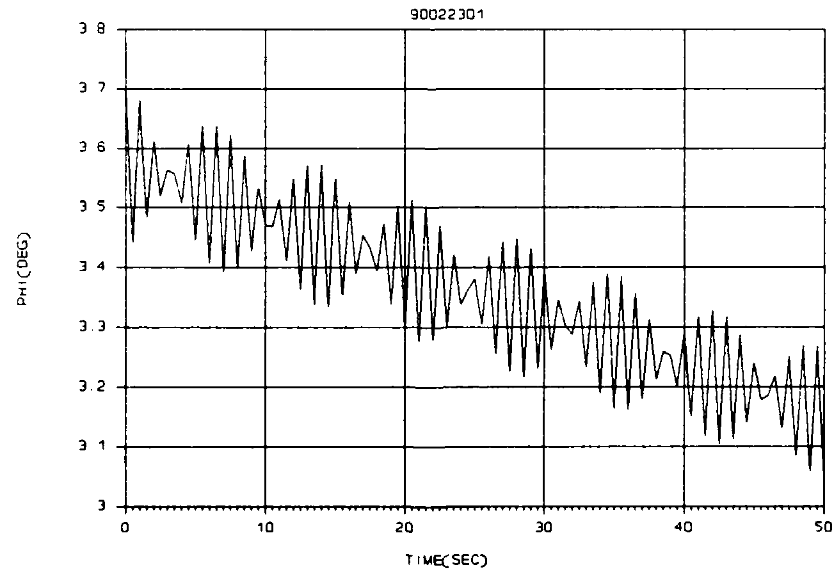
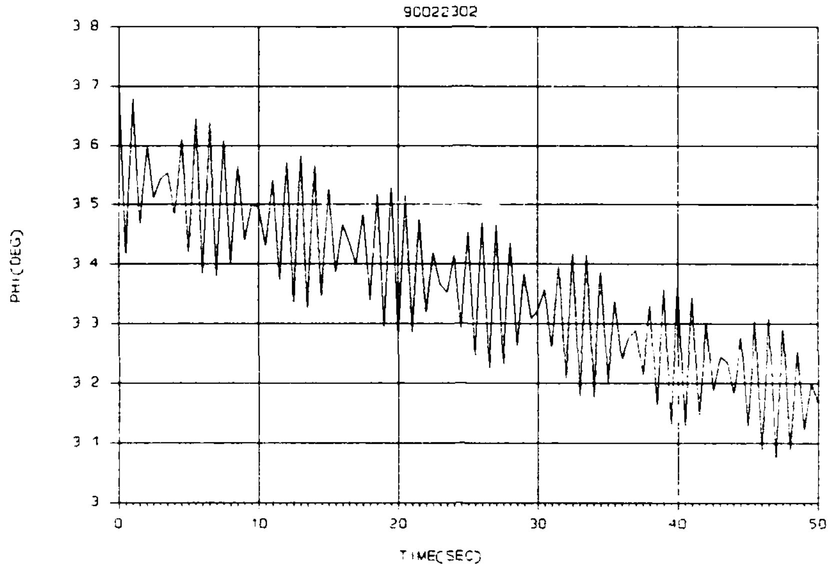
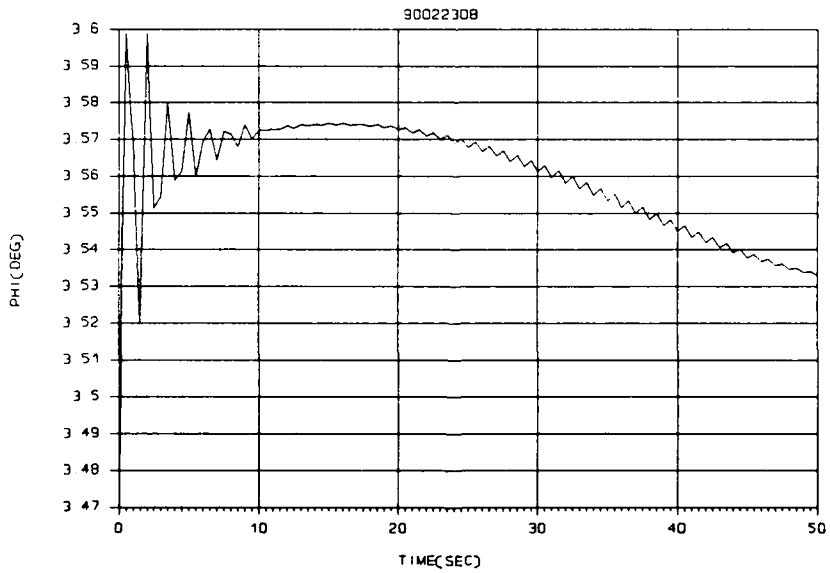


55% ASYM, Is/It = 1.1, 15% FUEL



SYM, Is/It = 1.05, 26.2% FUEL



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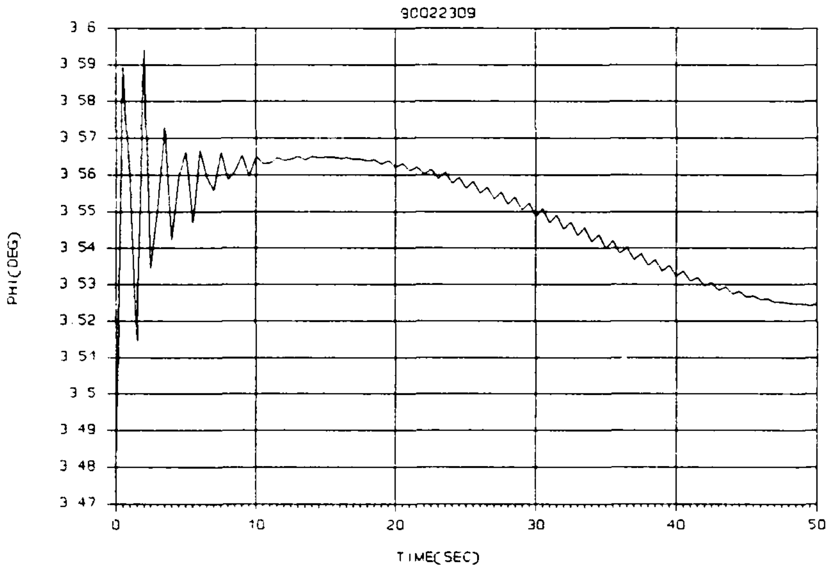
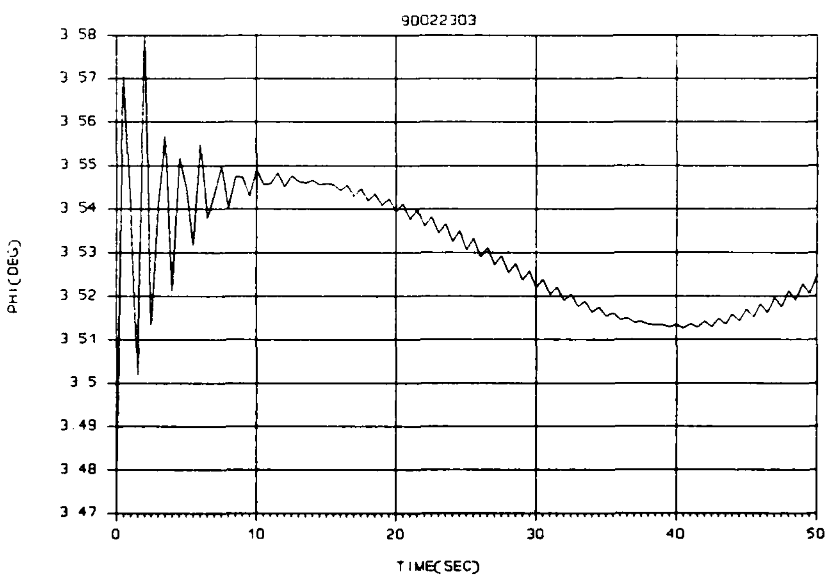
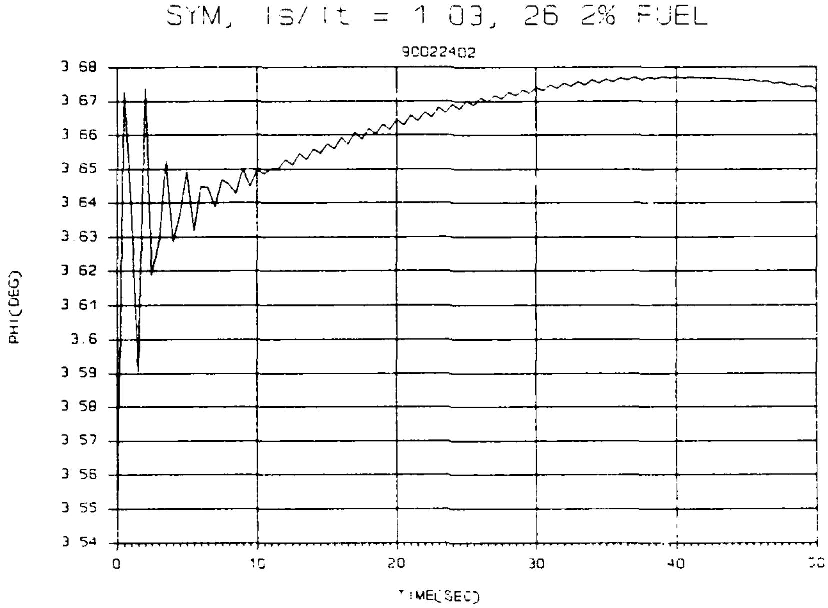
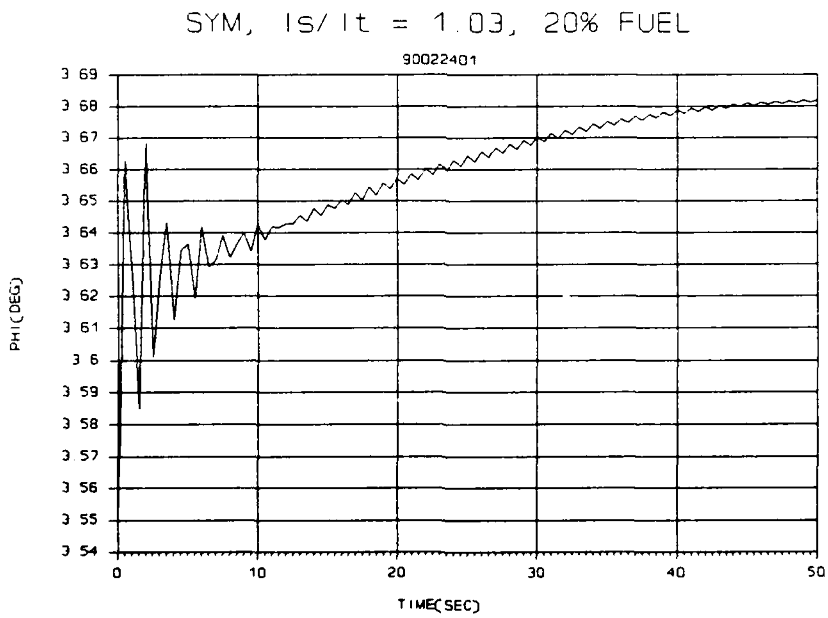


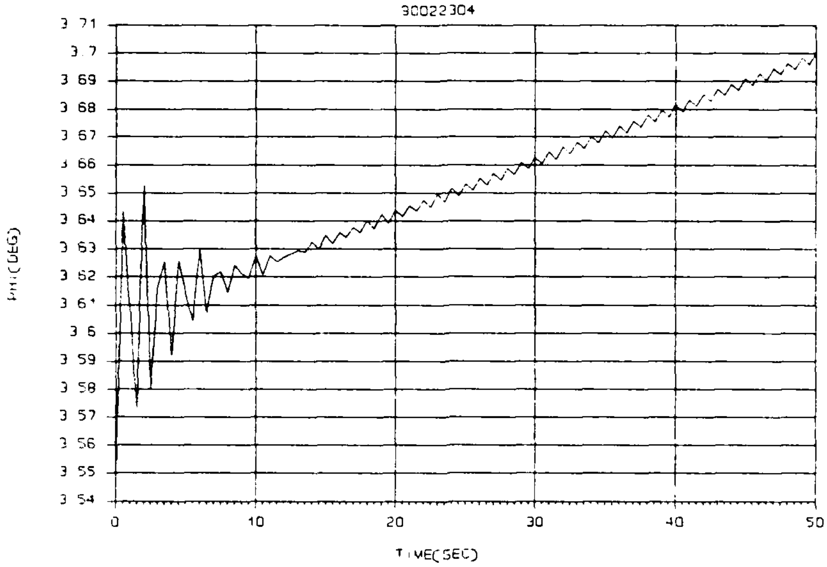
SYM, Is/It = 1.05, 15% FUEL



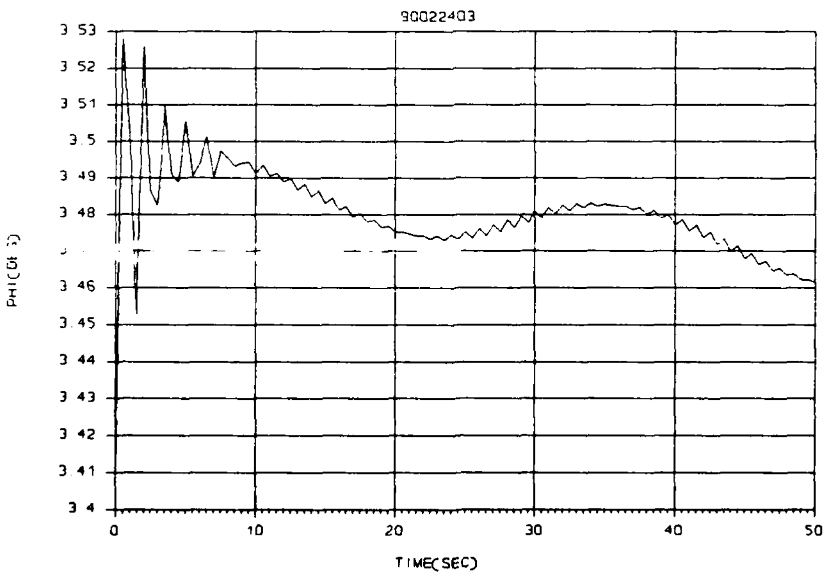

SYM, Is/It = 1.03, 26.2% FUEL
90022402
PHI(DEG)
3.68
3.67
3.66
3.65
3.64
3.63
3.62
3.61
3.6
3.59
3.58
3.57
3.56
3.55
3.54
0
10
20
30
40
50
TIME(SEC)


SYM, Is/It = 1.03, 20% FUEL
90022401
PHI(DEG)
3.69
3.68
3.67
3.66
3.65
3.64
3.63
3.62
3.61
3.6
3.59
3.58
3.57
3.56
3.55
3.54
0
10
20
30
40
50
TIME(SEC)

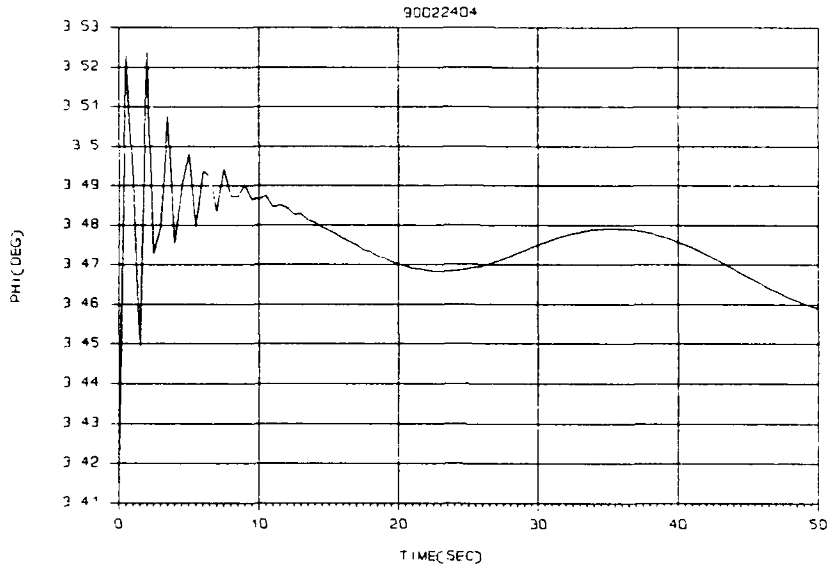
SYM, Is/It = 1 03, 15% FUEL



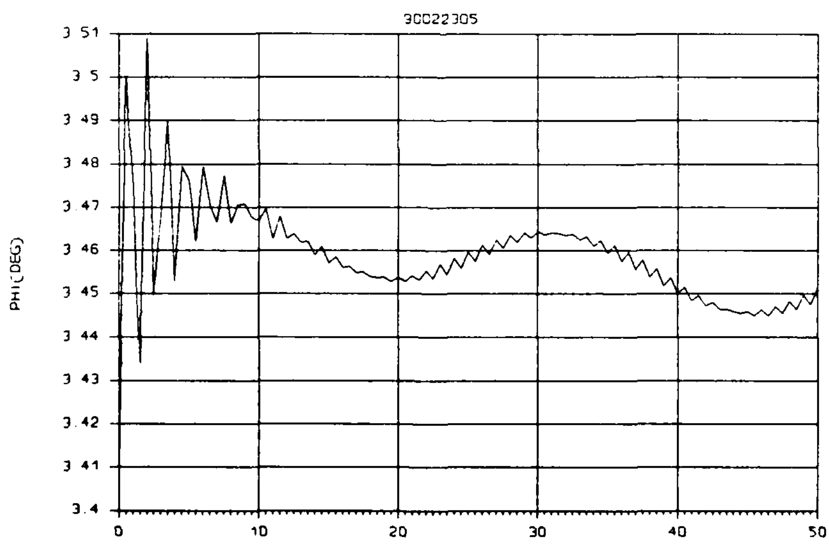
SYM, Is/It = 1.07, 26.2% FUEL

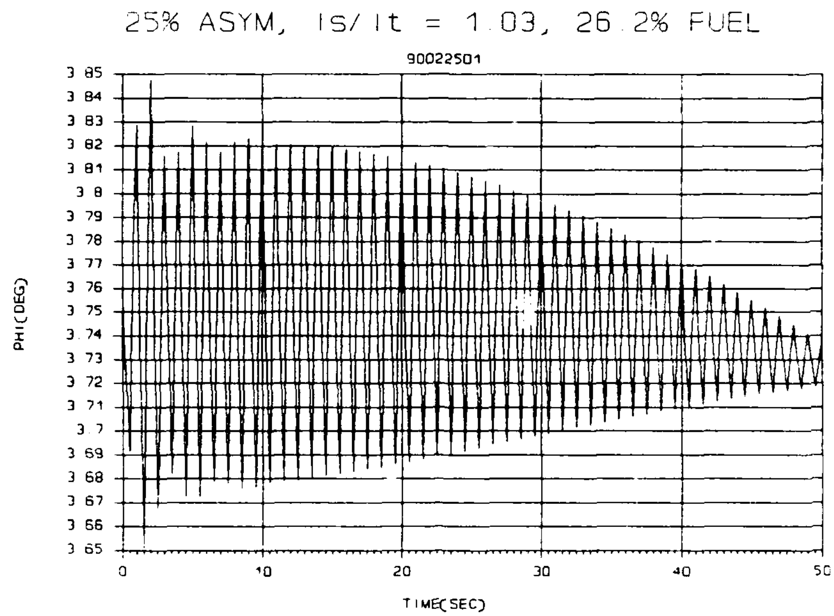


SYM, Is/It = 1.07, 20% FUEL



SYM, Is/It = 1.07, 15% FUEL




25% ASYM, Is/It = 1.03, 26.2% FUEL
90022501
PHI(DEG)
TIME(SEC)
3.85
3.84
3.83
3.82
3.81
3.8
3.79
3.78
3.77
3.76
3.75
3.74
3.73
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3.7
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3.66
3.65
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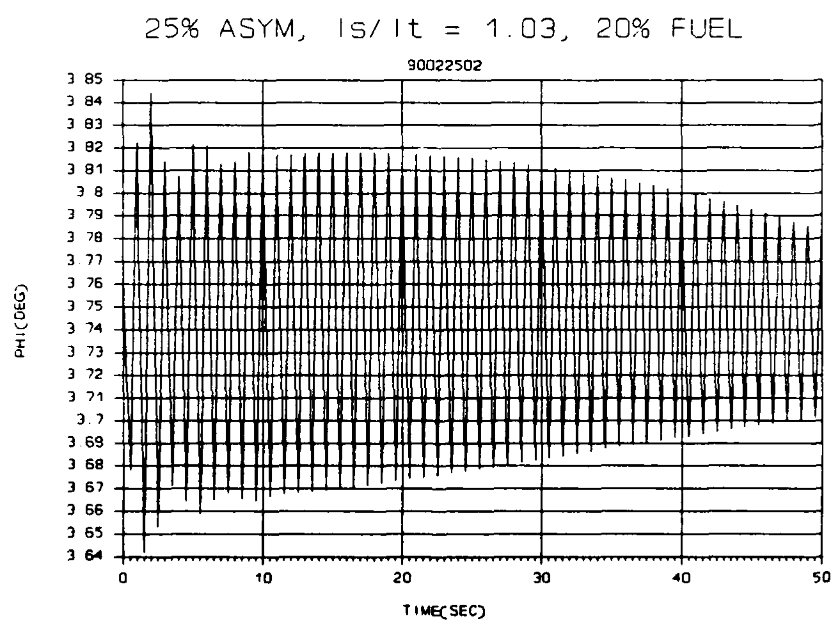
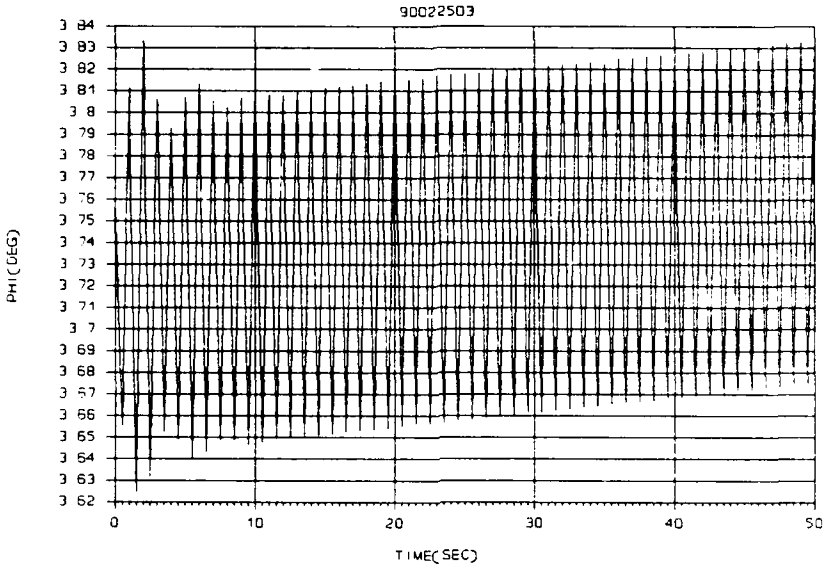
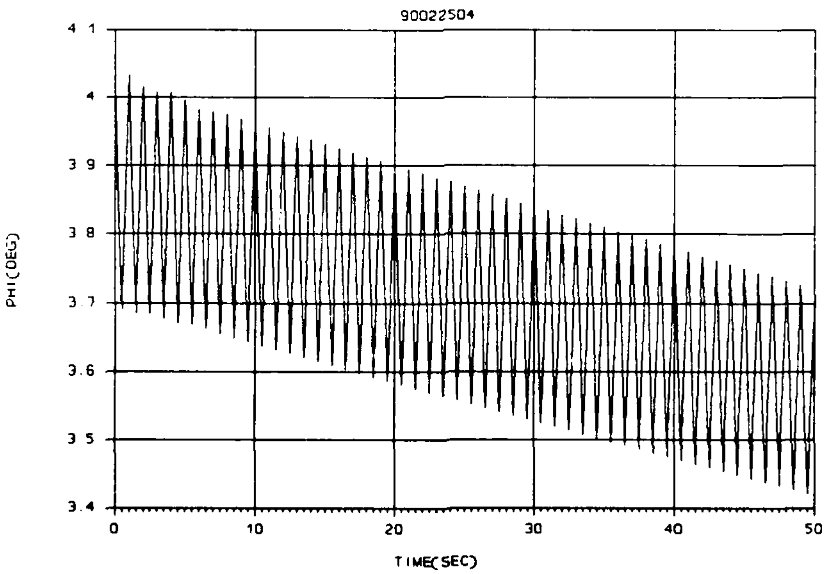
10
20
30
40
50


25% ASYM, Is/It = 1.03, 20% FUEL
90022502
PHI(DEG)
TIME(SEC)
3.85
3.84
3.83
3.82
3.81
3.8
3.79
3.78
3.77
3.76
3.75
3.74
3.73
3.72
3.71
3.7
3.69
3.68
3.67
3.66
3.65
3.64
0
10
20
30
40
50

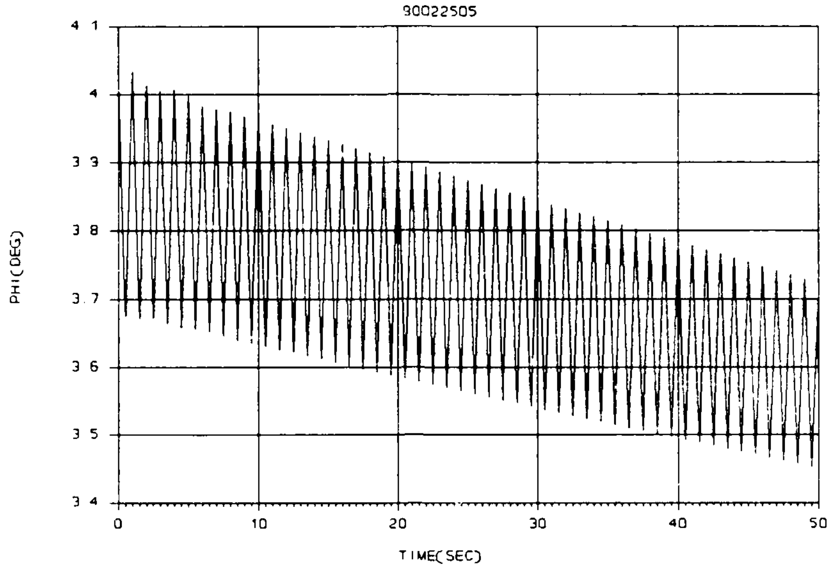
25% ASYM, Is/It = 1.03, 15% FUEL



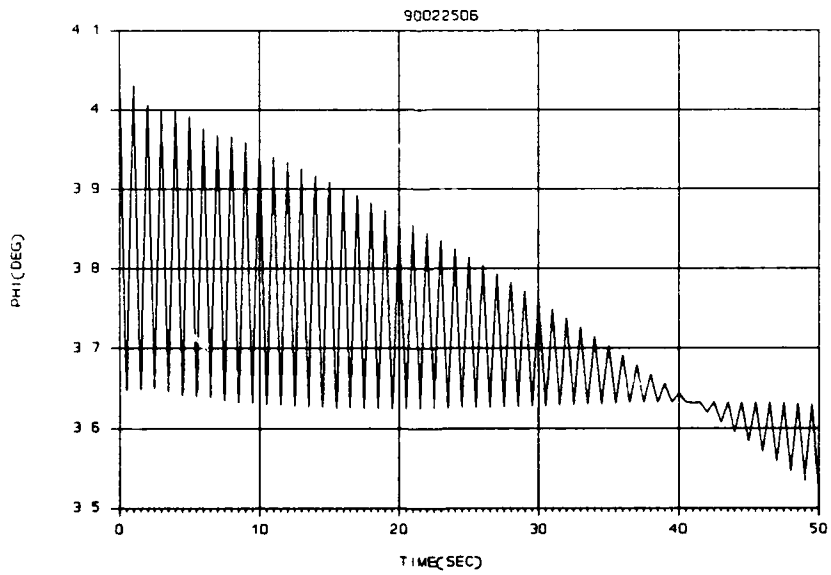
55% ASYM, Is/It = 1.03, 26.2% FUEL



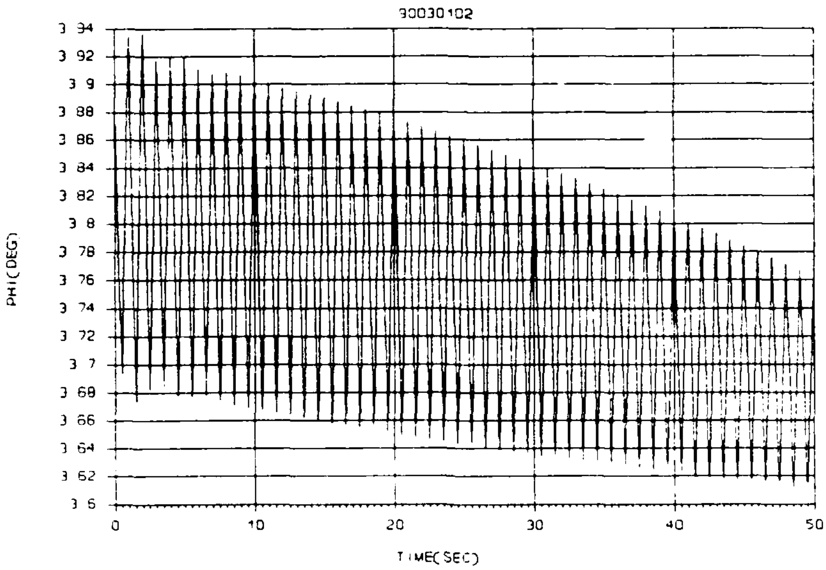
55% ASYM, Is/It = 1.03, 20% FUEL



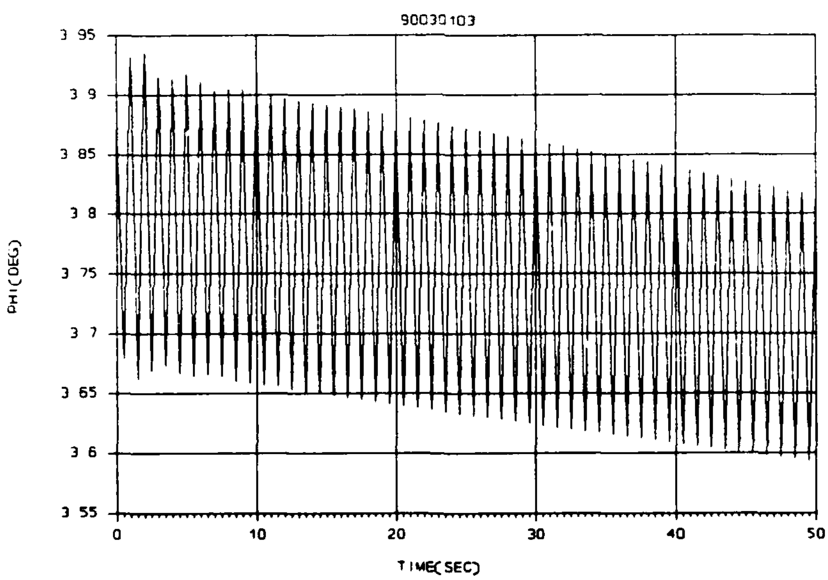
55% ASYM, Is/It = 1.03, 15% FUEL



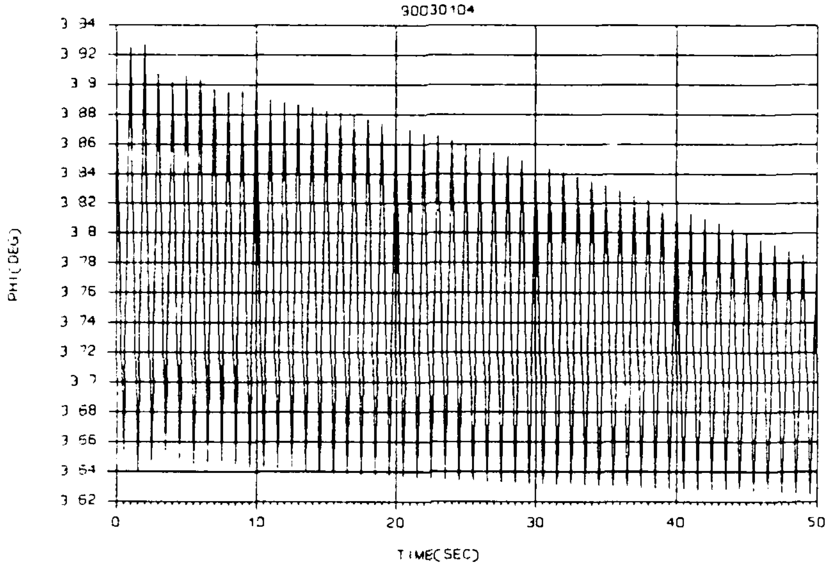
40% ASYM, Is/It = 1.03, 26.2% FUEL



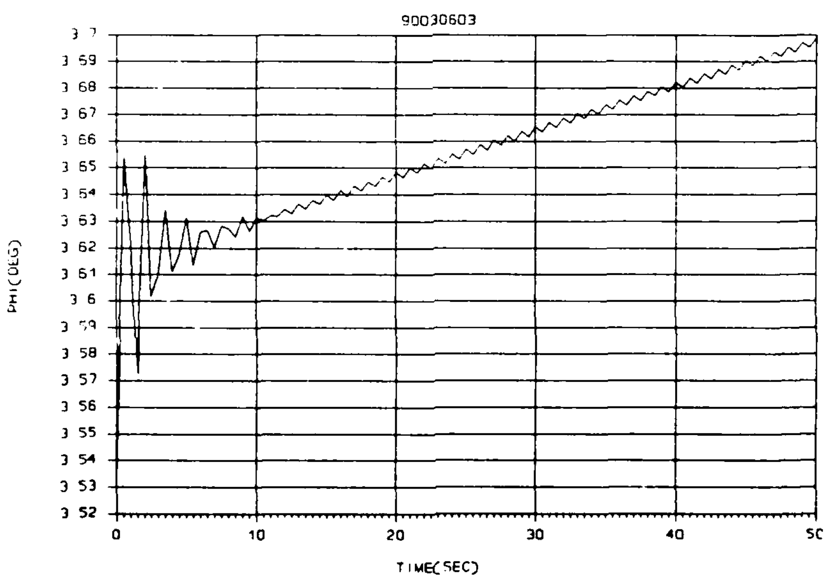
40% ASYM, Is/It = 1.03, 20% FUEL



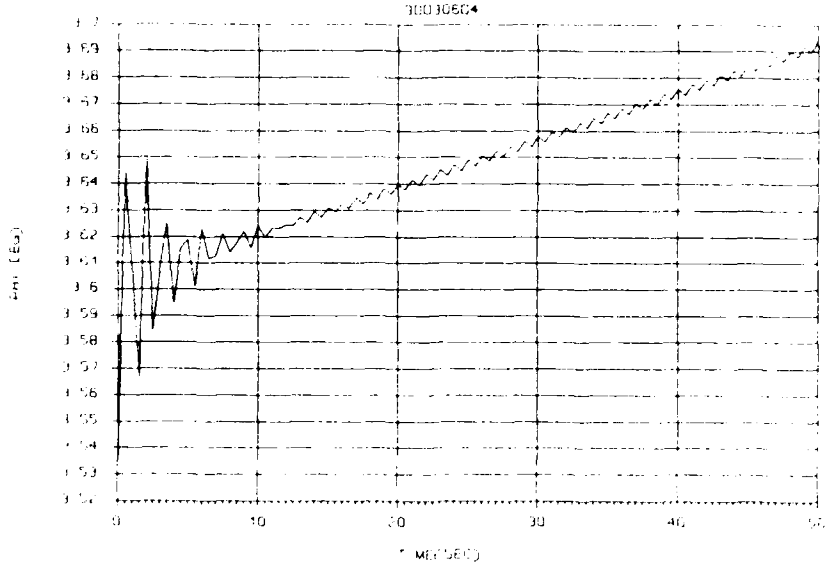
40% ASYM, Is/It = 1.03, 15% FUEL



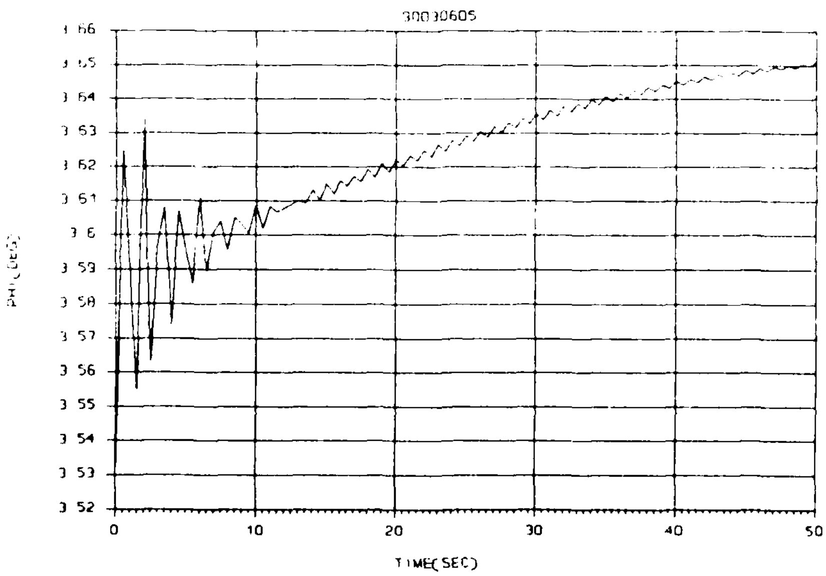
SYM, Is/It = 1.035, 26.2% FUEL

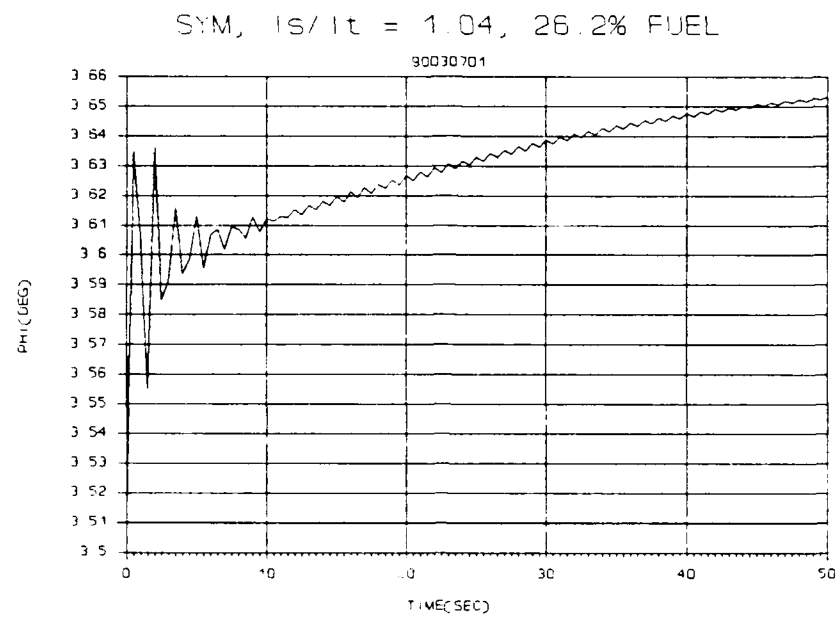


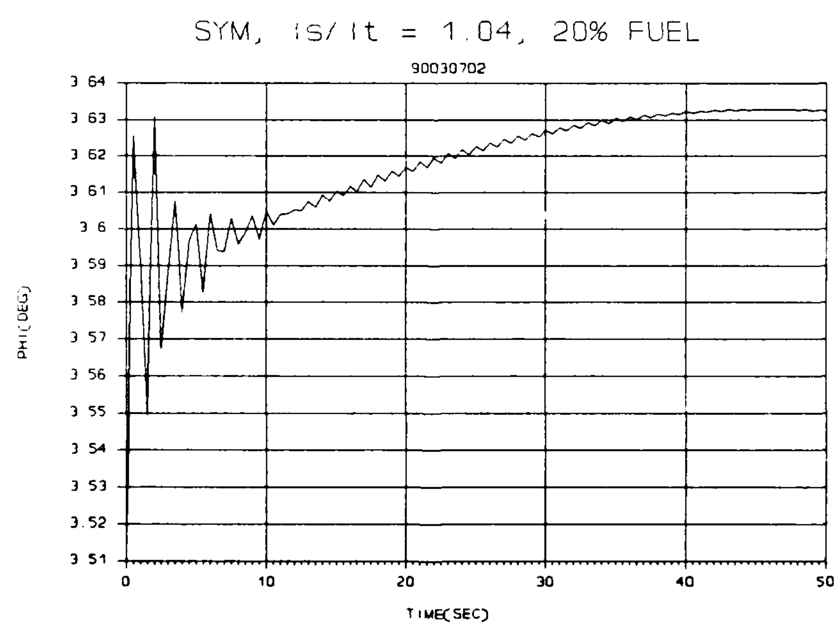
SYM, IS/IT = 1.635, 20% FUEL



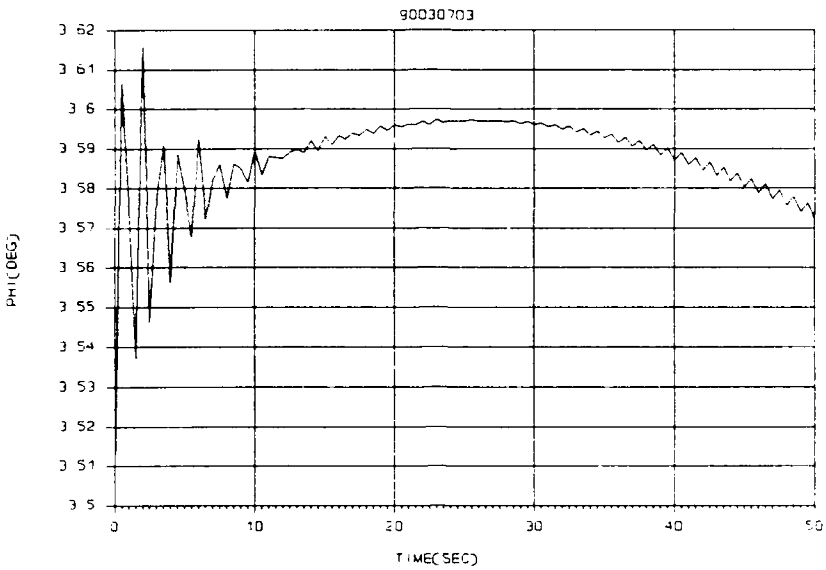
SYM, IS/IT = 1.635, 15% FUEL



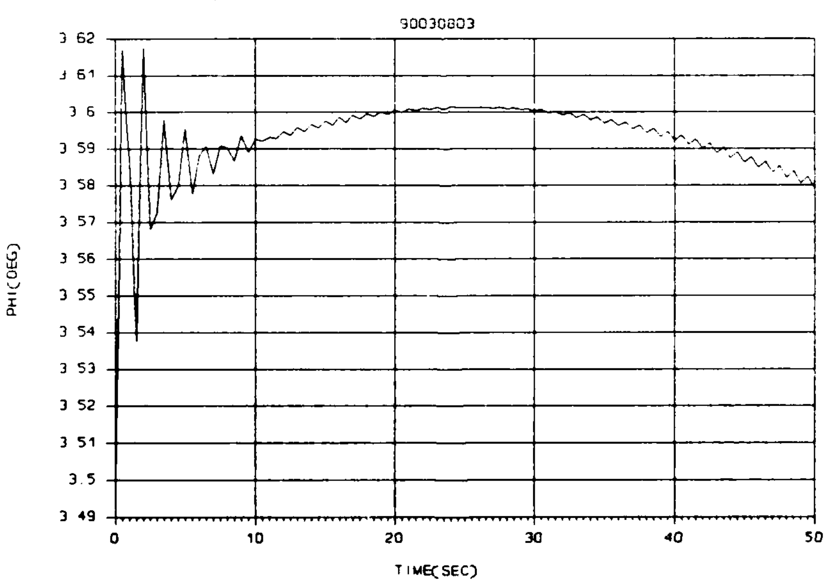

SYM, Is/It = 1.04, 26.2% FUEL
90030701
PHI(DEG)
TIME(SEC)
0
10
30
40
50


SYM, Is/It = 1.04, 20% FUEL
90030702
PHI(DEG)
TIME(SEC)
0
10
20
30
40
50

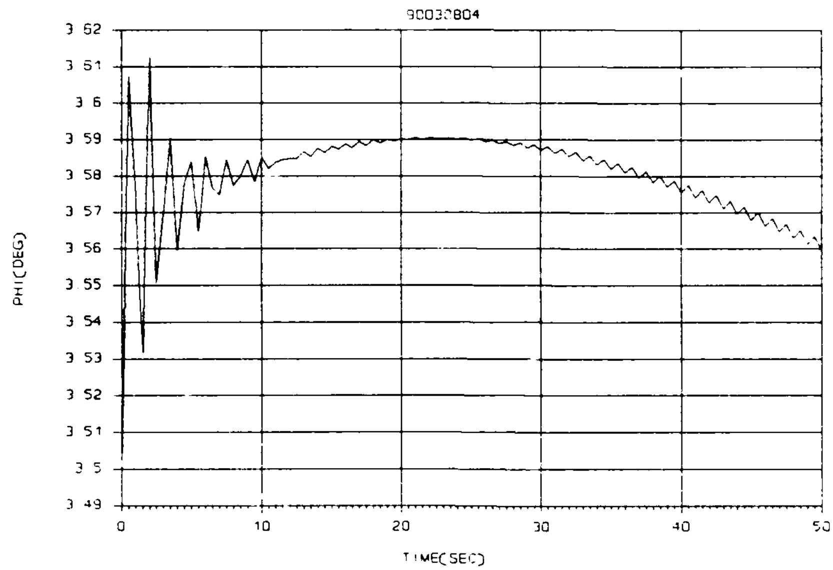
SYM, Is/It = 1.04, 15% FUEL



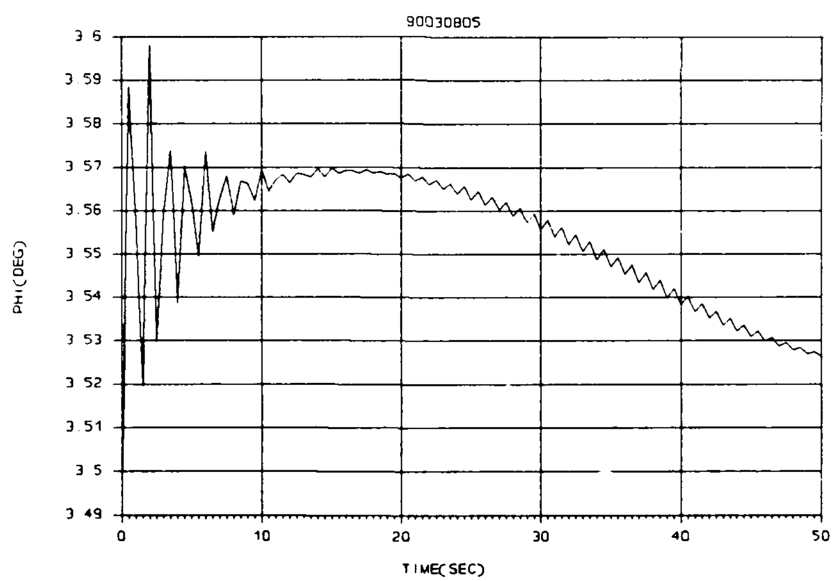
SYM, Is/It = 1.045, 26.2% FUEL



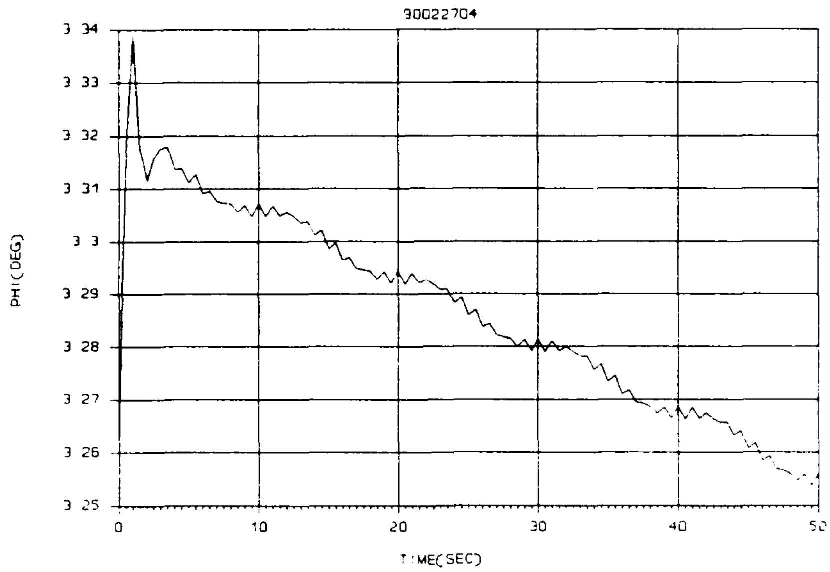
SYM, IS/It = 1.045, 20% FUEL



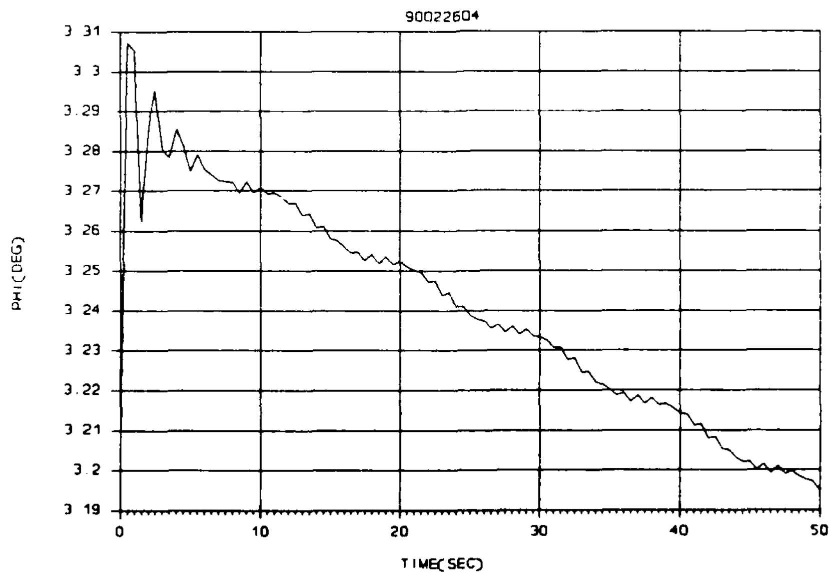
SYM, IS/It = 1.045, 15% FUEL



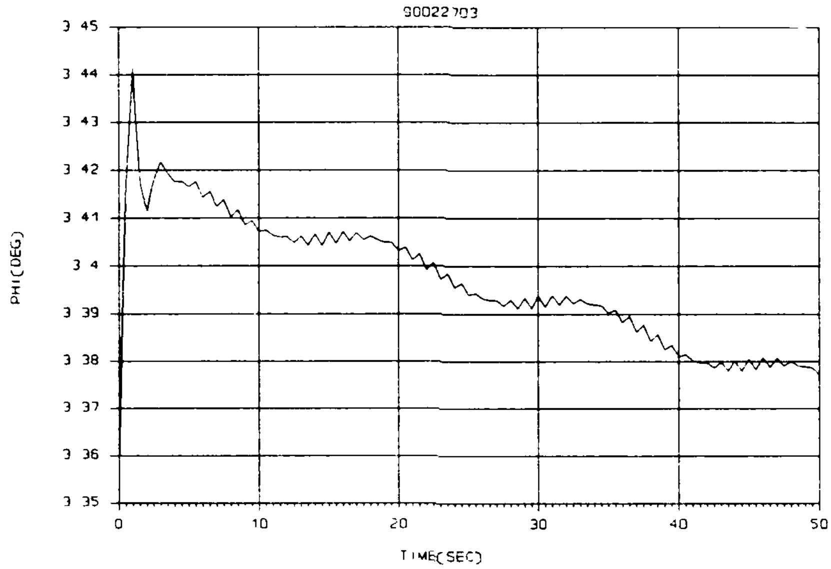
ON ORBIT, SYM, Is/It = 1 1, 75% FUEL



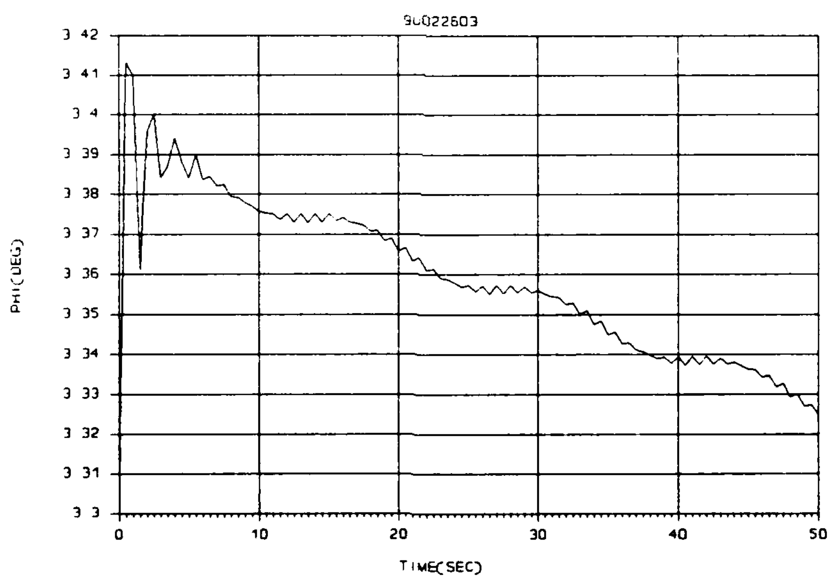
ON ORBIT, SYM, Is/It = 1.1, 50% FUEL

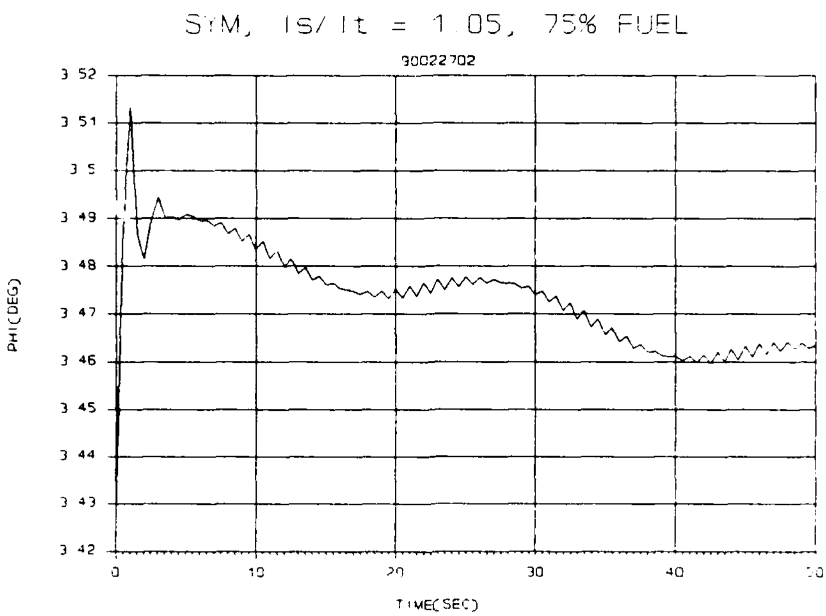


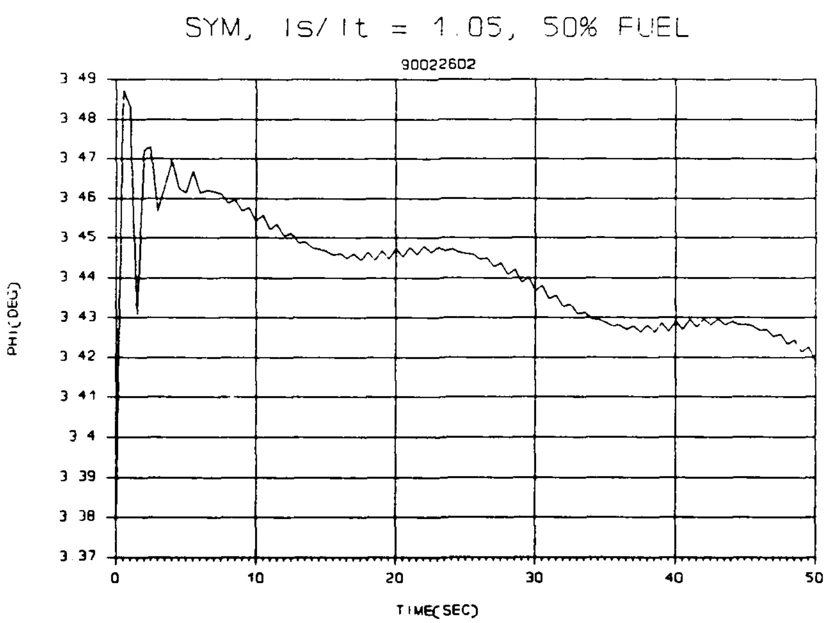
SYM, Is/It = 1.07, 75% FUEL

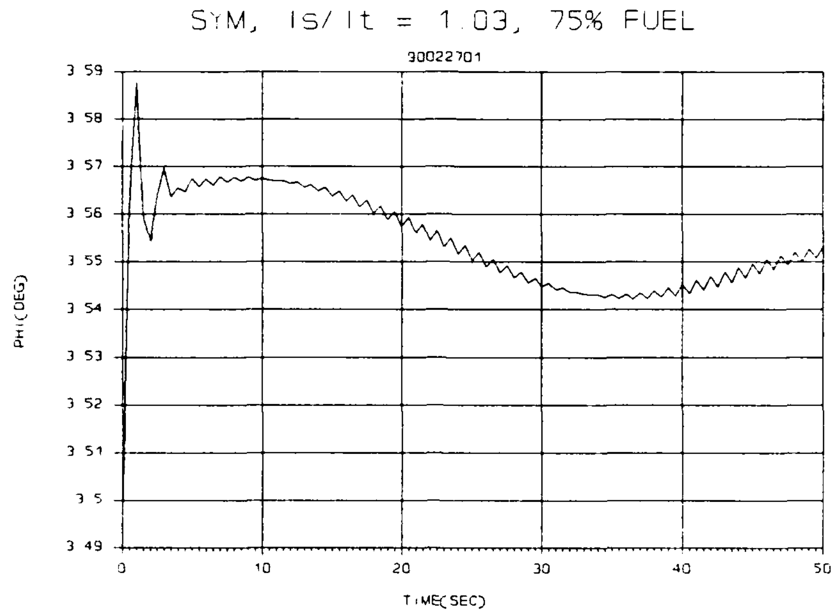


SYM, Is/It = 1.07, 50% FUEL




SYM, Is/It = 1.05, 75% FUEL
90022702
PHI(DEG)
TIME(SEC)


SYM, Is/It = 1.05, 50% FUEL
90022602
PHI(DEG)
TIME(SEC)


SYM, Is/It = 1.03, 75% FUEL
90022701
PHI(DEG)
TIME(SEC)
3.59
3.58
3.57
3.56
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3.54
3.53
3.52
3.51
3.5
3.49
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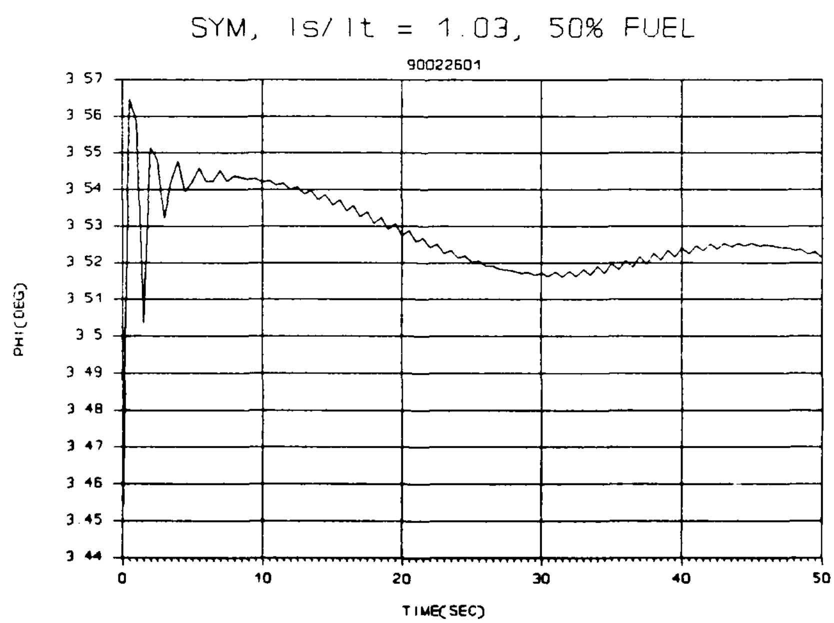
10
20
30
40
50


SYM, Is/It = 1.03, 50% FUEL
90022601
PHI(DEG)
TIME(SEC)
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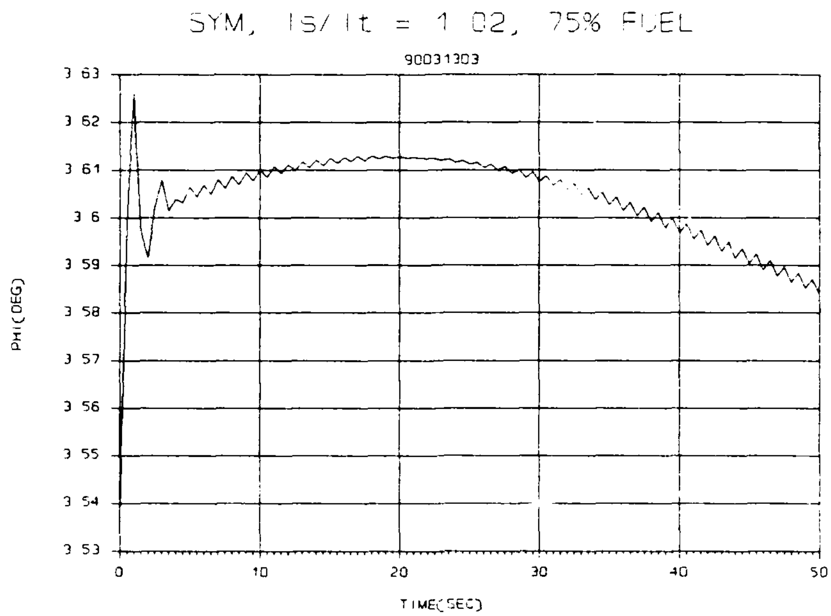
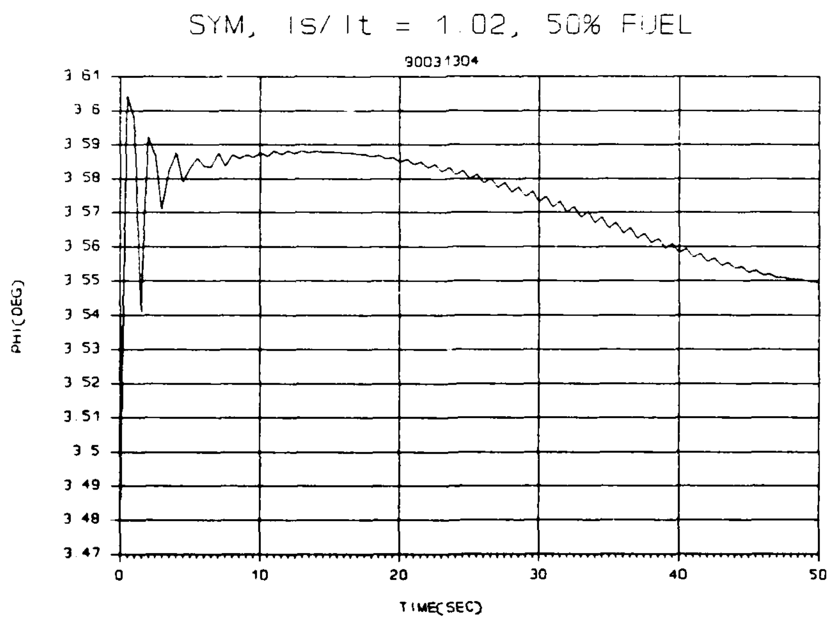
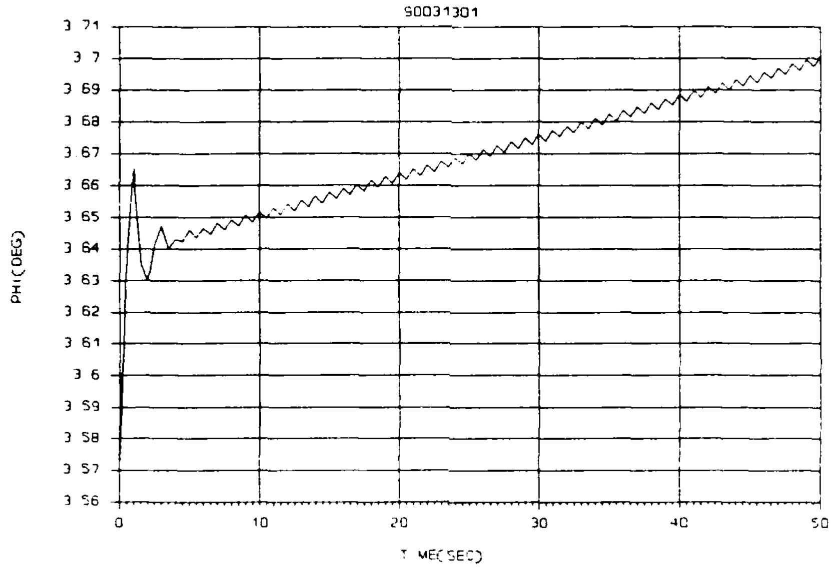
3.56
3.55
3.54
3.53
3.52
3.51
3.5
3.49
3.48
3.47
3.46
3.45
3.44
0
10
20
30
40
50

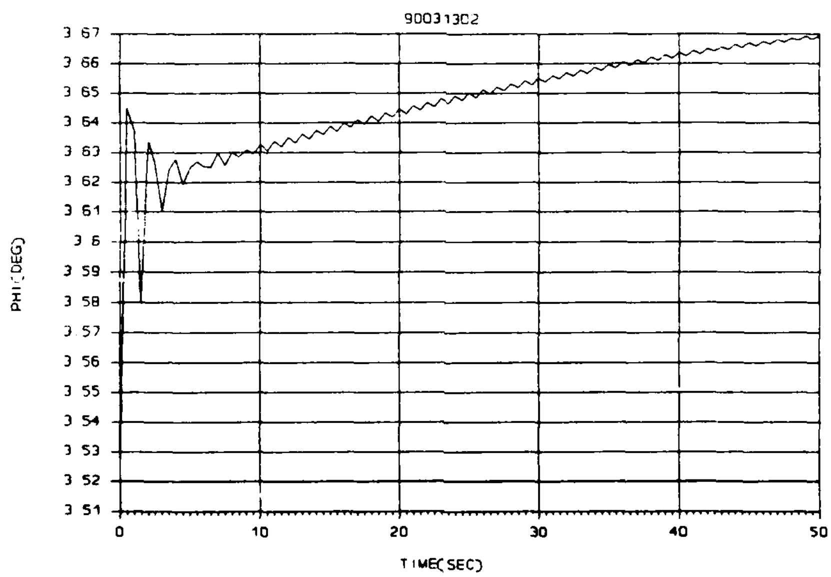

SYM, Is/It = 1.02, 75% FUEL
90031303
PHI(DEG)
TIME(SEC)


SYM, Is/It = 1.02, 50% FUEL
90031304
PHI(DEG)
TIME(SEC)

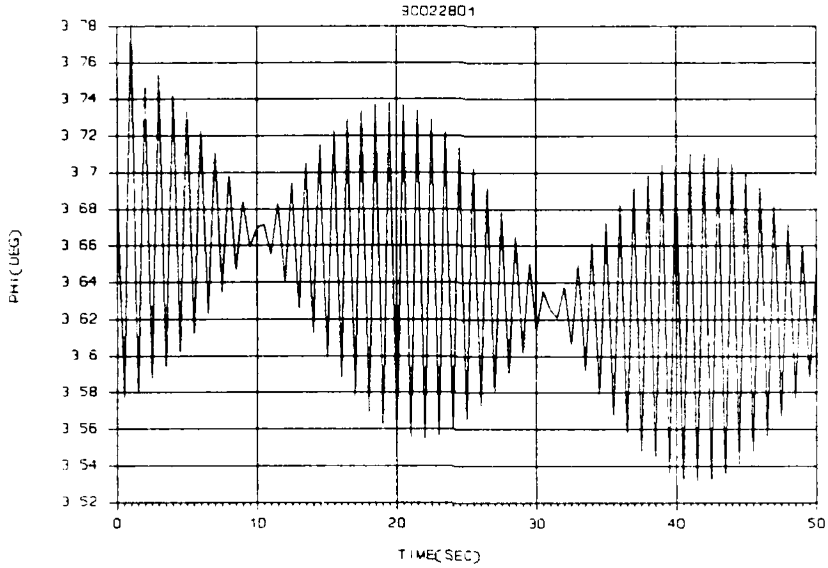
SYM, Is/It = 1.01, 75% FUEL



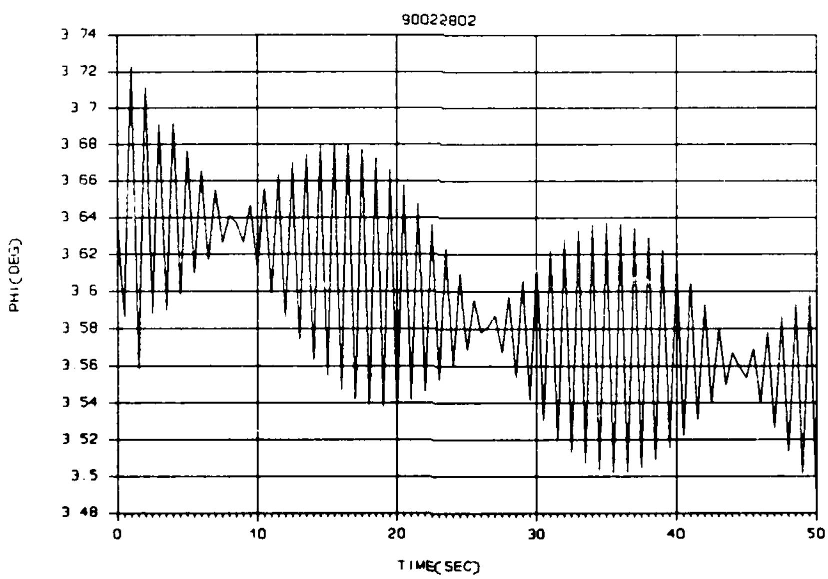
SYM, Is/It = 1.01, 50% FUEL



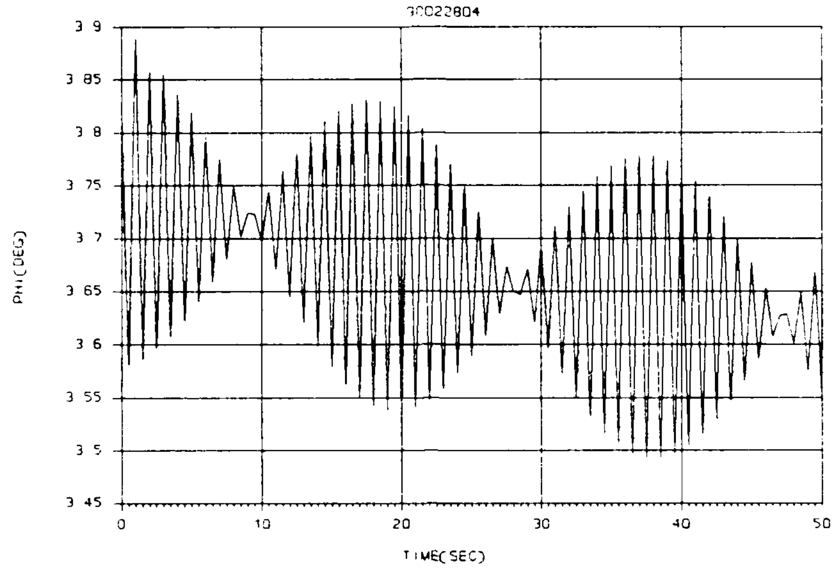
25% ASYM, Is/It = 1 03, 75% FUEL



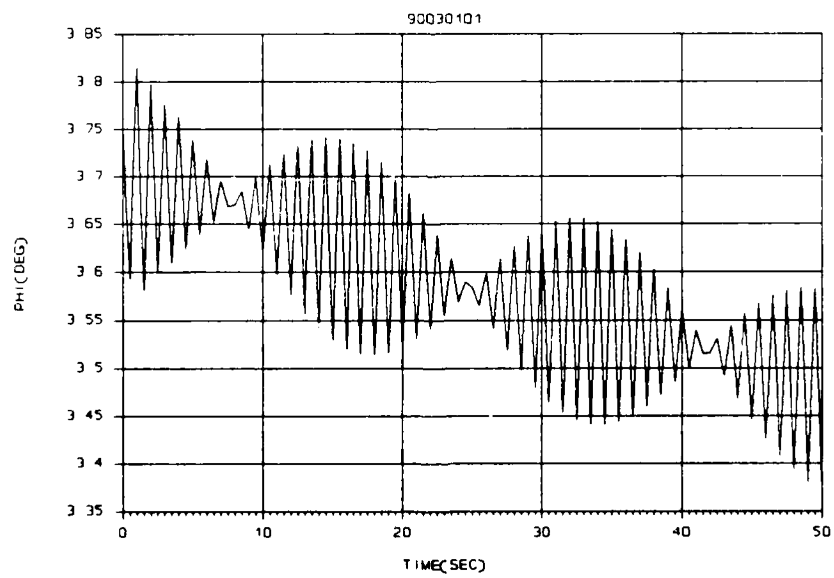
25% ASYM, Is/It = 1.03, 50% FUEL



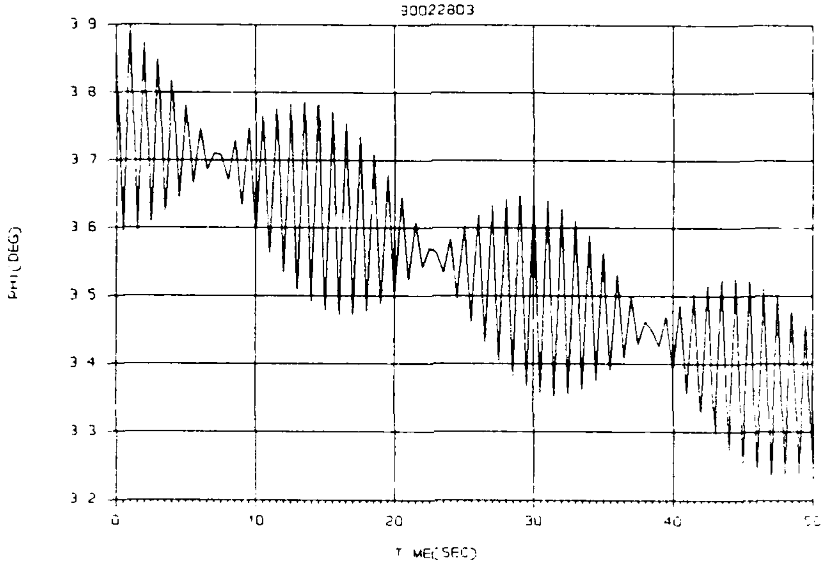
40% ASYM, Is/It = 1 03, 75% FUEL



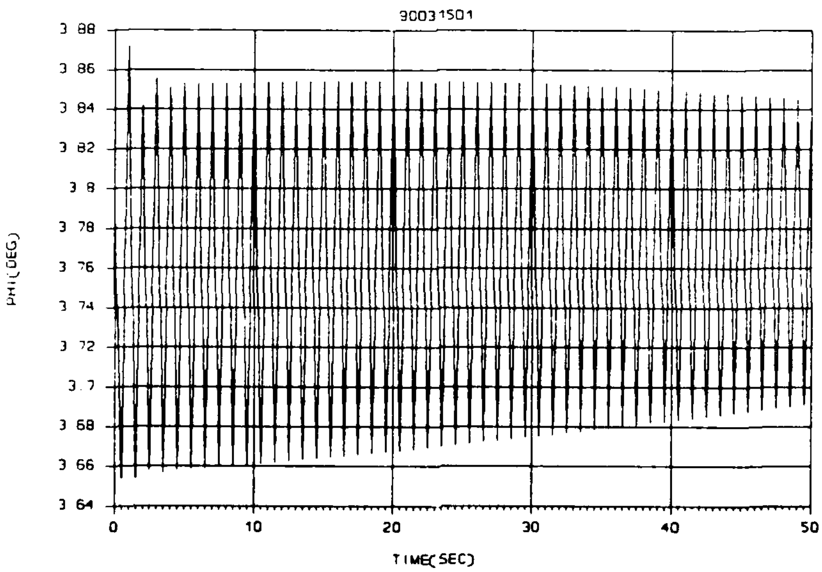
40% ASYM, Is/It = 1 03, 50% FUEL

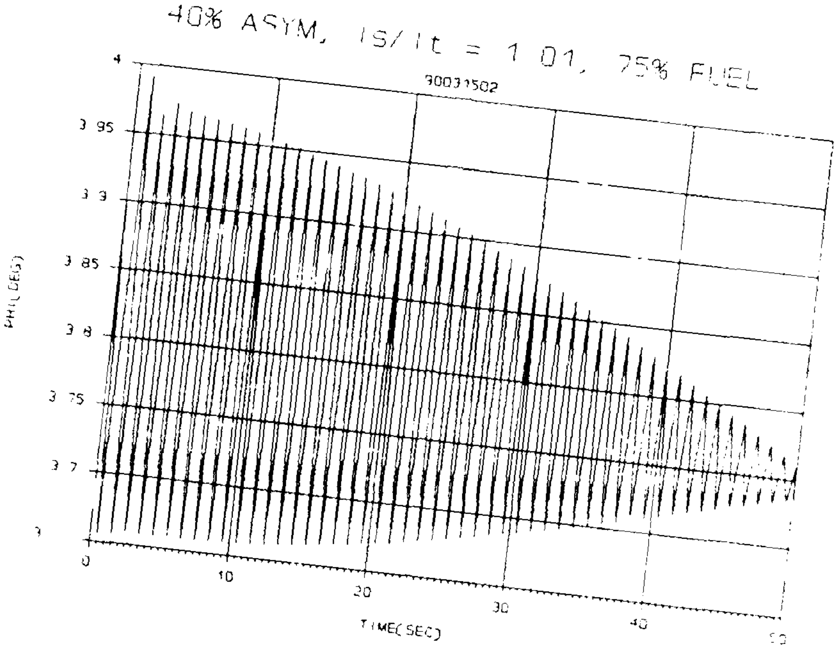


55% ASYM, Is/It = 1.03, 50% FUEL



25% ASYM, Is/It = 1.01, 75% FUEL




40% ASYM, Is/It = 1.01, 75% FUEL
90031502
PHI(DEG)
TIME(SEC)

APPENDIX B

ENERGY SINK PREDICTIONS

Included in this appendix are the spread sheet data for the energy sink predictions. The spread sheet takes as its input, w_p, w_r, I_1, I_2, I_p, and I_r. The spread sheet uses the following equations to calculate σ_r,

$$h_0 = I_p w_p + I_r w_r \tag{15}$$

$$\sigma_1 = [(I_p - I_2)w_p + I_r w_r]/I_1 \tag{8}$$

$$\sigma_2 = [(I_p - I_1)w_p + I_r w_r]/I_2 \tag{9}$$

$$\sigma_0 = h_0(I_1\sigma_1 + I_2\sigma_2)/(I_1^2\sigma_1 + I_2^2\sigma_2) \tag{20}$$

$$\sigma_p = \sigma_0 - w_p \tag{22}$$

$$\sigma_r = \sigma_0 - (w_p + w_r) \tag{23}$$

For the configurations used in this study, as long as σ_r is positive, the energy sink criteria predicts stability.

RUN NO.	Wp	Wr	I1	I2	Ip	Ir	ho	Sigma1	Sigma2	Sigma0	SigmaP	SigmaR	Predict	Result
EQUATION NUMBER							(15)	(8)	(9)	(20)	(22)	(23)		
SYMMETRIC, Is/It = 1.0, 75% FUEL														
90030601	0.000072	3.141519	6499.4	6488.4	8012.2	6493.9	20401.29	3.138878	3.144199	3.141609	3.141536	0.000017	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.0, 50% FUEL														
90030602	0.000072	3.141519	5995.1	5984.1	7507.9	5989.6	18816.99	3.138656	3.144425	3.141611	3.141538	0.000018	STABLE	UNSTABLE

RUN NO.	Wp	Wr	I1	I2	Ip	Ir	ho	Sigma1	Sigma2	Sigma0	SigmaP	SigmaR	Predict	Result
EQUATION NUMBER							(15)	(8)	(9)	(20)	(22)	(23)		
SYMMETRIC, Is/It = 1.001, 75% FUEL														
90030704	0.000072	3.141519	6499.4	6488.4	8018.7	6500.4	20421.71	3.142020	3.147347	3.144754	3.144681	0.003161	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.001, 50% FUEL														
90030705	0.000072	3.141519	5995.1	5984.1	7513.9	5995.6	18835.94	3.141800	3.147575	3.144758	3.144685	0.003165	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.005, 75% FUEL														
90020902	0.000072	3.141519	6499.4	6488.4	8044.7	6526.4	20593.40	3.154567	3.159935	3.157332	3.157259	0.015739	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.005, 50% FUEL														
90020801	0.000072	3.141519	5995.1	5984.1	7537.8	6019.5	18910.92	3.154324	3.160122	3.157293	3.157221	0.015701	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.01, 75% FUEL														
90031301	0.000072	3.141519	6499.4	6488.4	8077.1	6558.8	20605.18	3.170249	3.175623	3.173006	3.172933	0.031414	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.01, 50% FUEL														
90031302	0.000072	3.141519	5995.1	5984.1	7567.8	6049.5	19005.17	3.170045	3.175872	3.173029	3.172956	0.031436	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.01, 26.2% FUEL														
90021501	0.000072	3.141519	4469.5	4458.5	6026.9	4508.6	14164.29	3.169028	3.176846	3.173005	3.172932	0.031412	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.01, 20% FUEL														
	0.000072	3.141519	4182.5	4171.5	5737.1	4218.8	13253.86	3.168812	3.177168	3.173057	3.172984	0.031464	STABLE	
SYMMETRIC, Is/It = 1.01, 15% FUEL														
	0.000072	3.141519	3950.6	3939.6	5502.8	3984.5	12517.78	3.168506	3.177352	3.172995	3.172923	0.031403	STABLE	
25% ASYMMETRIC, Is/It = 1.01, 75% FUEL														
90031301	0.000072	3.141519	6872.3	6115.5	8077.1	6558.8	20605.18	2.998231	3.369256	3.173006	3.172933	0.031413	STABLE	UNSTABLE
40% ASYMMETRIC, Is/It = 1.01, 75% FUEL														
90031302	0.000072	3.141519	7096.1	5891.7	8077.1	6558.8	20605.18	2.903673	3.497237	3.173006	3.172933	0.031413	STABLE	STABLE

RUN NO.	Wp	Wr	I1	I2	Ip	Ir	ho	Sigma1	Sigma2	Sigma0	SigmaP	SigmaR	Predict	Result
EQUATION NUMBER							(15)	(8)	(9)	(20)	(22)	(23)		
SYMMETRIC, Is/It - 1.1, 75% FUEL														
	0.000072	3.141519	6499.4	6488.4	8661.6	7143.3	22441.44	3.452776	3.458630	3.455773	3.455701	0.314181	STABLE	STABLE
SYMMETRIC, Is/It = 1.1, 50% FUEL														
	0.000072	3.141519	5995.1	5984.1	8106.9	6588.6	20698.80	3.452548	3.458894	3.455791	3.455718	0.314198	STABLE	STABLE
SYMMETRIC, Is/It = 1.1, 26.2% FUEL														
90022001	0.000072	3.141519	4469.5	4458.5	6428.7	4910.4	15426.58	3.451451	3.459966	3.455776	3.455703	0.314183	STABLE	STABLE
SYMMETRIC, Is/It = 1.1, 20% FUEL														
90022003	0.000072	3.141519	4182.5	4171.5	6113	4594.7	14434.78	3.451161	3.460261	3.455778	3.455705	0.314185	STABLE	STABLE
SYMMETRIC, Is/It = 1.1, 15% FUEL														
90022002	0.000072	3.141519	3950.6	3939.6	5857.9	4339.6	13633.36	3.450888	3.460523	3.455771	3.455699	0.314179	STABLE	STABLE
5% ASYMMETRIC, Is/It = 1.1, 26.2% FUEL														
90022004	0.000072	3.141519	4516.2	4411.8	6428.7	4910.4	15426.58	3.415762	3.496590	3.455776	3.455703	0.314183	STABLE	STABLE
5% ASYMMETRIC, Is/It = 1.1, 20% FUEL														
90022005	0.000072	3.141519	4227.3	4126.7	6113	4594.7	14434.78	3.414587	3.497826	3.455778	3.455705	0.314185	STABLE	STABLE
5% ASYMMETRIC, Is/It = 1.1, 15% FUEL														
90022101	0.000072	3.141519	3993.7	3896.5	5857.9	4339.6	13633.36	3.413647	3.498800	3.455771	3.455699	0.314179	STABLE	STABLE
10% ASYMMETRIC, Is/It = 1.1, 26.2% FUEL														
90022102	0.000072	3.141519	4562.9	4365.1	6428.7	4910.4	15426.58	3.380803	3.533998	3.455776	3.455703	0.314183	STABLE	STABLE
10% ASYMMETRIC, Is/It = 1.1, 20% FUEL														
90022103	0.000072	3.141519	4272	4082	6113	4594.7	14431.78	3.378859	3.536128	3.455778	3.455705	0.314185	STABLE	STABLE
10% ASYMMETRIC, Is/It = 1.1, 15% FUEL														
90022104	0.000072	3.141519	4036.8	3853.4	5857.9	4339.6	13633.36	3.377201	3.537933	3.455771	3 455699	0.314179	STABLE	STABLE
15% ASYMMETRIC, Is/It = 1.1, 26.2% FUEL														
90022105	0.000072	3.141519	4609.5	4318.5	6428.7	4910.4	15426.58	3.346626	3.572131	3.455776	3.455703	0.314183	STABLE	STABLE

RUN NO.	Wp	Wr	I1	I2	Ip	Ir	ho	Sigma1	Sigma2	Sigma0	SigmaP	SigmaR	Predict	Result
EQUATION NUMBER							(15)	(8)	(9)	(20)	(22)	(23)		
15% ASYMMETRIC, Is/It = 1.1, 20% FUEL														
90022106	0.000072	3.141519	4316.8	4037.2	6113	4594.7	14434.78	3.343794	3.575367	3.455778	3.455705	0.314185	STABLE	STABLE
15% ASYMMETRIC, Is/It = 1.1, 15% FUEL														
90022107	0.000072	3.141519	4080	3810.2	5857.9	4339.6	13633.36	3.341443	3.578045	3.455771	3.455699	0.314179	STABLE	STABLE
20% ASYMMETRIC, Is/It = 1.1, 26.2% FUEL														
90022108	0.000072	3.141519	4656.2	4271.8	6428.7	4910.4	15426.58	3.313061	3.611182	3.455776	3.455703	0.314183	STABLE	STABLE
20% ASYMMETRIC, Is/It = 1.1, 20% FUEL														
90022109	0.000072	3.141519	4361.5	3992.5	6113	4594.7	14434.78	3.309525	3.615396	3.455778	3.455705	0.314185	STABLE	STABLE
20% ASYMMETRIC, Is/It = 1.1, 15% FUEL														
90022110	0.000072	3.141519	4123.1	3767.1	5857.9	4339.6	13633.36	3.306514	3.618981	3.455771	3.455699	0.314179	STABLE	STABLE
25% ASYMMETRIC, Is/It = 1.1, 26.2% FUEL														
90022111	0.000072	3.141519	4702.9	4225.1	6428.7	4910.4	15426.58	3.280163	3.651095	3.455776	3.455703	0.314183	STABLE	STABLE
25% ASYMMETRIC, Is/It = 1.1, 20% FUEL														
90022112	0.000072	3.141519	4406.3	3947.7	6113	4594.7	14434.78	3.275877	3.656424	3.455778	3.455705	0.314185	STABLE	STABLE
25% ASYMMETRIC, Is/It = 1.1, 15% FUEL														
90022113	0.000072	3.141519	4166.2	3724	5857.9	4339.6	13633.36	3.272309	3.660865	3.455771	3.455698	0.314179	STABLE	STABLE
35% ASYMMETRIC, Is/It = 1.1, 26.2% FUEL														
90022201	0.000072	3.141519	4850.9	4077.1	6428.7	4910.4	15426.58	3.180088	3.783629	3.455776	3.455703	0.314183	STABLE	STABLE
35% ASYMMETRIC, Is/It = 1.1, 20% FUEL														
90022202	0.000072	3.141519	4547	3807	6113	4594.7	14434.78	3.174512	3.791556	3.455777	3.455705	0.314185	STABLE	STABLE
35% ASYMMETRIC, Is/It = 1.1, 15% FUEL														
90022203	0.000072	3.141519	4300.8	3589.4	5857.9	4339.6	13633.36	3.169899	3.798142	3.455771	3.455698	0.314178	STABLE	STABLE
55% ASYMMETRIC, Is/It = 1.1, 26.2% FUEL														
90022204	0.000072	3.141519	4796.2	4131.8	6428.7	4910.4	15426.58	3.216355	3.733539	3.455776	3.455703	0.314183	STABLE	STABLE

RUN NO.	Wp	Wr	I1	I2	Ip	Ir	ho	Sigma1	Sigma2	Sigma0	SigmaP	SigmaR	Predict	Result
EQUATION NUMBER							(15)	(8)	(9)	(20)	(22)	(23)		
55% ASYMMETRIC, Is/It = 1.1, 20% FUEL														
90022301	0.000072	3.141519	4495.8	3858.2	6113	4594.7	14434.78	3.210664	3.741241	3.455777	3.455705	0.314185	STABLE	STABLE
55% ASYMMETRIC, Is/It = 1.1, 15% FUEL														
90022302	0.000072	3.141519	4252.5	3637.7	5857.9	4339.6	13633.36	3.205902	3.747713	3.455771	3.455698	0.314178	STABLE	STABLE
SYMMETRIC, Is/It = 1.03, 75% FUEL														
90022701	0.000072	3.141519	6499.4	6488.4	8207	6688.7	21013.28	3.233038	3.238519	3.235849	3.235776	0.094256	STABLE	STABLE
SYMMETRIC, Is/It = 1.03, 50% FUEL														
90022601	0.000072	3.141519	5995.1	5984.1	7687.6	6169.3	19381.53	3.232823	3.238766	3.235865	3.235792	0.094272	STABLE	STABLE
SYMMETRIC, Is/It = 1.03, 26.2% FUEL														
90022402	0.000072	3.141519	4469.5	4458.5	6116.2	4597.9	14444.83	3.231796	3.239769	3.235851	3.235778	0.094258	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.03, 20% FUEL														
90022401	0.000072	3.141519	4182.5	4171.5	5820.6	4302.3	13516.18	3.231531	3.240052	3.235859	3.235786	0.094266	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.03, 15% FUEL														
90022304	0.000072	3.141519	3950.6	3939.6	5581.7	4063.4	12765.65	3.231248	3.240270	3.235826	3.235753	0.094233	STABLE	UNSTABLE
25% ASYMMETRIC, Is/It = 1.03, 75% FUEL														
90022801	0.000072	3.141519	6856.1	6131.7	8207	6688.7	21013.28	3.064837	3.426909	3.235848	3.235776	0.094256	STABLE	STABLE
25% ASYMMETRIC, Is/It = 1.03, 50% FUEL														
90022802	0.000072	3.141519	6296.2	5683	7687.6	6169.3	19381.53	3.078225	3.410360	3.235864	3.235792	0.094272	STABLE	STABLE
25% ASYMMETRIC, Is/It = 1.03, 26.2% FUEL														
90022501	0.000072	3.141519	4741.9	4186.1	6116.2	4597.9	14444.83	3.046149	3.450585	3.235850	3.235778	0.094258	STABLE	UNSTABLE
25% ASYMMETRIC, Is/It = 1.03, 20% FUEL														
90022502	0.000072	3.141519	4442.8	3911.2	5820.6	4302.3	13516.18	3.042203	3.455681	3.235859	3.235786	0.094266	STABLE	UNSTABLE
25% ASYMMETRIC, Is/It = 1.03, 15% FUEL														
90022503	0.000072	3.141519	4200.7	3689.5	5581.7	4063.4	12765.65	3.038872	3.459913	3.235825	3.235753	0.094233	STABLE	UNSTABLE

RUN NO.	Wp	Wr	I1	I2	Ip	Ir	ho	Sigma1	Sigma2	Sigma0	SigmaP	SigmaR	Predict	Result
EQUATION NUMBER							(15	(8)	(9)	(20)	(22)	(23)		
40% ASYMMETRIC, Is/It = 1.03, 75% FUEL														
90022804	0.000072	3.141519	7070.1	5917.7	8207	6688.7	21013.28	2.972072	3.550833	3.235848	3.235775	0.094255	STABLE	STABLE
40% ASYMMETRIC, Is/It = 1.03, 50% FUEL														
90030101	0.000072	3.141519	6476.9	5502.3	7687.6	6169.3	19381.53	2.992347	3.522357	3.235864	3.235791	0.094272	STABLE	STABLE
40% ASYMMETRIC, Is/It = 1.03, 26.2% FUEL														
90030102	0.000072	3.141519	4905.4	4022.6	6116.2	4597.9	14444.83	2.944621	3.590832	3.235850	3.235777	0.094257	STABLE	STABLE
40% ASYMMETRIC, Is/It = 1.03, 20% FUEL														
90030103	0.000072	3.141519	4599	3755	5820.6	4302.3	13516.18	2.938880	3.599427	3.235858	3.235785	0.094265	STABLE	STABLE
40% ASYMMETRIC, Is/It = 1.03, 15% FUEL														
90030104	0.000072	3.141519	4350.8	3539.4	5581.7	4063.4	12765.65	2.934035	3.606639	3.235825	3.235752	0.094232	STABLE	STABLE
55% ASYMMETRIC, Is/It = 1.03, 75% FUEL														
	0.000072	3.141519	7284.1	5703.7	8207	6688.7	21013.28	2.884758	3.684056	3.235848	3.235775	0.094255	STABLE	
55% ASYMMETRIC, Is/It = 1.03, 50% FUEL														
90022803	0.000072	3.141519	6657.6	5321.6	7687.6	6169.3	19381.53	2.911131	3.641959	3.235864	3.235791	0.094271	STABLE	STABLE
55% ASYMMETRIC, Is/It = 1.03, 26.2% FUEL														
90022504	0.000072	3.141519	5068.9	3859.1	6116.2	4597.9	14444.83	2.849643	3.742963	3.235849	3.235777	0.094257	STABLE	STABLE
55% ASYMMETRIC, Is/It = 1.03, 20% FUEL														
90022505	0.000072	3.141519	4755.2	3598.8	5820.6	4302.3	13516.18	2.842345	3.755651	3.235857	3.235785	0.094265	STABLE	STABLE
55% ASYMMETRIC, Is/It = 1.03, 15% FUEL														
90022506	0.000072	3.141519	4500.9	3389.3	5581.7	4063.4	12765.65	2.836190	3.766361	3.235824	3.235752	0.094232	STABLE	STABLE
SYMMETRIC, Is/It = 1.035, 26.2% FUEL														
90030603	0.000072	3.141519	4469.5	4458.5	6138.5	4620.2	14514.89	3.247471	3.255483	3.251544	3.251472	0.109952	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.035, 20% FUEL														
90030604	0.000072	3.141519	4182.5	4171.5	5841.5	4323.2	13581.84	3.247230	3.255792	3.251578	3.251505	0.109985	STABLE	UNSTABLE

RUN NO.	Wp	Wr	I1	I2	Ip	Ir	ho	Sigma1	Sigma2	Sigma0	SigmaP	SigmaR	Predict	Result
EQUATION NUMBER							(15)	(8)	(9)	(20)	(22)	(23)		
SYMMETRIC, Is/It = 1.035, 15% FUEL														
90030605	0.000072	3.141519	3950.6	3939.6	5601.5	4083.2	12827.86	3.246994	3.256060	3.251593	3.251520	0.110000	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.04, 26.2% FUEL														
90030701	0.000072	3.141519	4469.5	4458.5	6160.9	4642.6	14585.26	3.263216	3.271266	3.267309	3.267236	0.125716	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.04, 20% FUEL														
90030702	0.000072	3.141519	4182.5	4171.5	5862.4	4344.1	13647.50	3.262928	3.271532	3.267297	3.267225	0.125705	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.04, 15% FUEL														
90030703	0.000072	3.141519	3950.6	3939.6	5621.2	4102.9	12889.75	3.262659	3.271769	3.267281	3.267208	0.125688	STABLE	UNSTABLE
SYMMETRIC, Is/It = 1.045, 26.2% FUEL														
90030803	0.000072	3.141519	4469.5	4458.5	6183.2	4664.9	14655.32	3.278890	3.286980	3.283003	3.282930	0.141410	STABLE	MARGINAL
SYMMETRIC, Is/It = 1.045, 20% FUEL														
90030804	0.000072	3.141519	4182.5	4171.5	5883.3	4365	13713.16	3.278627	3.287272	3.283017	3.282944	0.141424	STABLE	MARGINAL
SYMMETRIC, Is/It = 1.045, 15% FUEL														
90030805	0.000072	3.141519	3950.6	3939.6	5640.9	4122.6	12951.64	3.278325	3.287479	3.282968	3.282896	0.141376	STABLE	MARGINAL
SYMMETRIC, Is/It = 1.05, 75% FUEL														
90022702	0.000072	3.141519	6499.4	6488.4	8336.9	6818.6	21421.37	3.295827	3.301415	3.298691	3.298618	0.157099	STABLE	STABLE
SYMMETRIC, Is/It = 1.05, 50% FUEL														
90022602	0.000072	3.141519	5995.1	5984.1	7807.4	6289.1	19757.90	3.295602	3.301660	3.298701	3.298628	0.157108	STABLE	STABLE
SYMMETRIC, Is/It = 1.05, 26.2% FUEL														
90022308	0.000072	3.141519	4469.5	4458.5	6205.5	4687.2	14725.38	3.294565	3.302693	3.298697	3.298624	0.157104	STABLE	STABLE
SYMMETRIC, Is/It = 1.05, 20% FUEL														
90022309	0.000072	3.141519	4182.5	4171.5	5904.1	4385.8	13778.50	3.294250	3.302937	3.298661	3.298588	0.157068	STABLE	STABLE

RUN NO.	Wp	Wr	I1	I2	Ip	Ir	ho	Sigma1	Sigma2	SigmaO	SigmaP	SigmaR	Predict	Result
EQUATION NUMBER							(15)	(8)	(9)	(20)	(22)	(23)		
SYMMETRIC, Is/It = 1.05, 15% FUEL														
90022203	0.000072	3.141519	3950.6	3939.6	5660.7	4142.4	13013.84	3.294071	3.303268	3.298736	3.298663	0.157143	STABLE	STABLE
25% ASYMMETRIC, Is/It = 1.05, 26.2% FUEL														
90022405	0.000072	3.141519	4730.8	4197.2	6205.5	4687.2	14725.38	3.112597	3.508300	3.298696	3.298624	0.157104	STABLE	STABLE
25% ASYMMETRIC, Is/It = 1.05, 20% FUEL														
90022406	0.000072	3.141519	4432.4	3921.6	5904.1	4385.8	13778.50	3.108524	3.513409	3.298660	3.298598	0.157068	STABLE	STABLE
25% ASYMMETRIC, Is/It = 1.05, 15% FUEL														
90022407	0.000072	3.141519	4190.9	3699.3	5660.7	4142.4	13013.84	3.105198	3.517838	3.298725	3.298663	0.157143	STABLE	STABLE
SYMMETRIC, Is/It = 1.07, 75% FUEL														
90022703	0.000072	3.141519	6499.4	6488.4	8466.8	6948.5	21829.46	3.358616	3.364310	3.361534	3.361461	0.219941	STABLE	STABLE
SYMMETRIC, Is/It = 1.07, 50% FUEL														
90022603	0.000072	3.141519	5995.1	5984.1	7927.2	6408.9	20134.26	3.358380	3.364553	3.361537	3.361464	0.219944	STABLE	STABLE
SYMMETRIC, Is/It = 1.07, 26.2% FUEL														
90022403	0.000072	3.141519	4469.5	4458.5	6294.8	4776.5	15005.92	3.357333	3.365616	3.361542	3.361470	0.219950	STABLE	STABLE
SYMMETRIC, Is/It = 1.07, 20% FUEL														
90022404	0.000072	3.141519	4182.5	4171.5	5987.7	4469.4	14041.14	3.357045	3.365897	3.361538	3.361465	0.219945	STABLE	STABLE
SYMMETRIC, Is/It = 1.07, 15% FUEL														
90022305	0.000072	3.141519	3950.6	3939.6	5739.6	4221.3	13261.71	3.356813	3.366186	3.361566	3.361493	0.219973	STABLE	STABLE
SYMMETRIC, Is/It = 1.02, 75% FUEL														
90022703	0.000072	3.141519	6499.4	6488.4	8142.1	6623.8	20809.39	3.201667	3.207095	3.204452	3.204379	0.062859	STABLE	STABLE
SYMMETRIC, Is/It = 1.02, 50% FUEL														
90022603	0.000072	3.141519	5995.1	5984.1	7627.7	6109.4	19193.35	3.201434	3.207319	3.204447	3.204374	0.062854	STABLE	STABLE

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