Experimental generation of non-Kolmogorov Turbulence using a Liquid Crystal Spatial Light Modulator

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ABSTRACT

Several experiments showed that the classical Kolmogorov power spectral density of the refractive-index sometimes does not properly describe the statistics of the atmosphere. In this paper we show an experimental testbed able to generate non-classical Kolmogorov turbulence by using a liquid crystal spatial light modulator. The testbed is used at Naval Postgraduate School for laboratory investigation of laser beam propagation in maritime environment where a power law different from classical Kolmogorov, $11/3$, could be present. Applications of this testbed are ship to-ship free space optical communication, imaging and high energy laser weapons

Keywords: Atmospheric turbulence generation, non-Kolmogorov power spectrum, optical turbulence in maritime environment, laser beam propagation, spatial light modulator

1. INTRODUCTION

Atmospheric turbulence is one of the most serious drawbacks to laser beam propagation in free space [1]. As a laser beam begins to propagate through the Earth's atmosphere, the varying index of refraction caused by the turbulent air will alter the optical path of the light. The Earth's atmosphere can be described as a locally homogeneous medium in which its properties vary with respect to temperature, pressure, wind velocities, humidity and other factors. Also, the Earth's atmosphere temporally changes in a quasi-random fashion and therefore it acts like many random lenses present in the air. All of these processes are usually simply referred to as "atmospheric turbulence." The Kolmogorov model of energy distribution in a turbulent medium is a useful statistical model to describe the fluctuation of the index of refraction due to temperature changes [1]. This model is supported by a variety of experimental measurements and is quite widely used in simulations for the propagation of electromagnetic waves through a random medium. Although for many years, scientific communities have investigated optical turbulence present in the atmosphere and new important results have been carried out, the atmosphere's statistical behavior is still not well understood. For example, the power spectrum of the index of refraction sometimes does not follow the classical Kolmogorov power law, $11/3$, and it rather follows different power laws [2-4]. NRL researchers recently measured scintillation in maritime environment [9], some conclusion are that classical Rytov approximation [1] might not be representative of propagation in a maritime environment, scintillation shows a much faster transition into strong saturation regime, refractive index structure parameter, which is a measure of the strength of the fluctuations in the refractive index in the atmosphere, is more stable during the day, and, more importantly, refractive index power spectra is sometimes different from Kolmogorov [11-12].The Navy has an interest in a laser-based directed energy weapon (DEW) system because of the need for anti-ship cruise missile and tactical air defense [5-6]. US Naval Research Laboratory has undertaken a multi-year research effort in free space optical communications in maritime environment [7-12]. For these and other reasons is important and strategic to do more investigation on laser propagation through atmospheric turbulence in a maritime environment [17],

where Kolmogorov power spectrum does not describe well the statistical behavior of the index of refraction fluctuations in the atmosphere.

The refractive index structure parameter is known to behave differently in marine and terrestrial environments, since a large body of water acts quite differently from soils [13]. Friehe [14] investigated the influence of humidity on the optical refractive index in the maritime environment, and evidence of non-Kolmogorov power spectrum is even shown. Finally, significant evidence of power law deviation from classical Kolmogorov turbulence is shown in [15], where is investigated the effect of intermittency in fully developed turbulence on the spectra of fluctuations of "passive scalars".

In this paper we describe a testbed used at Naval Postgraduate School for laboratory investigation of laser beam propagation in maritime environment where a power law different from classical Kolmogorov, 11/3, could be present. Applications of this testbed are, for instance, ship to-ship free space optical communication, imaging and high energy laser weapons.

2. NON-CLASSICAL KOLOMOGOROV POWER SPECTRUM

The traditional Kolmogorov spectrum is primarily used in theoretical studies due to its simplicity and it is only valid in the inertial subrange, where no inner scale and outer scale effects are included.

In this paper we use the power spectrum introduced in [4] with a power law α that can deviate from the classical Kolmogorov. The non-Kolmogorov power spectrum is given by

$$\Phi_n(\kappa,\alpha) = A(\alpha)\cdot\tilde{C}_n^2\cdot\frac{1}{\left(\kappa^2+\kappa_0^2\right)^{\frac{\alpha}{2}}}\cdot\exp\left(-\frac{\kappa^2}{\kappa_m^2}\right) \quad 0\leq\kappa<\infty,\ 2<\alpha<4 \tag{1}$$

where α is the spectral index or power law, $\tilde{C}_n^2 = \beta\cdot C_n^2$ is a generalized structure parameter with units $\mathrm{m}^{3-\alpha}$,

$A(\alpha)$ is defined by

$$A(\alpha) = \frac{1}{4\pi^2}\cdot\Gamma(\alpha-1)\cdot\cos\left(\frac{\alpha\cdot\pi}{2}\right); \tag{2}$$

$\kappa_0 = \frac{2\pi}{L_0}$, L_0 is the outer scale parameter;

$\kappa_m = \frac{C(\alpha)}{l_0}$, l_0 is the inner scale parameter ;

and the symbol $\Gamma(x)$ is the gamma function.

The constant $C(\alpha)$ is defined by

$$C(\alpha) = \left\{\frac{2}{3}\pi\cdot A(\alpha)\cdot\Gamma\left(\frac{5-\alpha}{2}\right)\right\}^{\frac{1}{\alpha-5}} \tag{3}$$

The procedures to carry out $A(\alpha)$ and $C(\alpha)$ are shown in Andrews and Phillips [1], pag.74-75, or in [4] where essentially the asymptotic behavior of the structure function has been considered.

To show the non-Kolmogorov power law, we plot in Fig.1 the power spectrum as a function of wavenumber κ for three different values of α, we set $l_0 = 0.01m$; $L_0 = 10m$, $\tilde{C}_n^2 = 1.7 \cdot 10^{-14} m^{3-\alpha}$.
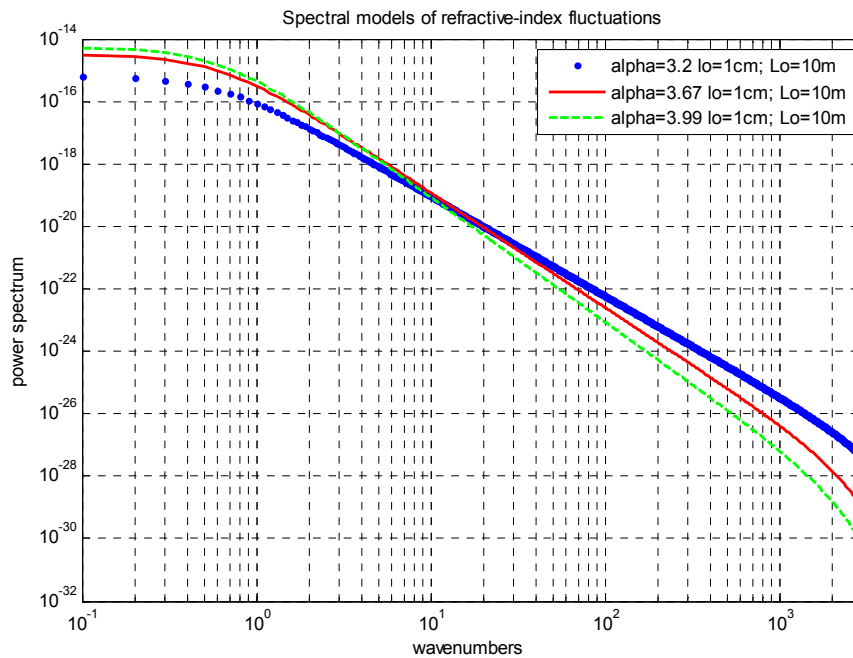

Fig. 1 –Power spectrum as a function of wavenumber for three different power law values

We deduce from Fig. 1 that a different power law from the classic value of Kolmogorov, $\alpha = 11/3$, changes the slope of the spectrum and obviously impacts on atmospheric turbulence parameters such as phase structure function, scintillation and long term beam spread [17]. Therefore applications of laser beam propagation in maritime environment, such as ship-to-ship free space optical communication and imaging should consider this non-Kolmogorov effect into the budget link analysis.

3. TURBULENCE GENERATION METHOD

The Naval Research Laboratory has developed a new method for generating atmospheric turbulence and a testbed that simulates its aberrations far more inexpensively and with greater fidelity than many other methods [16]. This system uses a Liquid Crystal (LC) Spatial Light Modulator (SLM) which allows the simulation of atmospheric seeing conditions ranging from very poor to very good and different algorithms may be easily employed on the device for comparison. These simulations can be dynamically generated and modified very quickly, easily and multiple devices can be used to simulate multiple layers of turbulence in a laboratory environment. Current testing with using multiple LC SLMs is under investigation at the Naval Research Laboratory and the Naval Postgraduate School.

The method described in [16] permits to generate the classical Kolmogorov atmospheric turbulence. In this paper we extend the method even for non-Kolmogorov turbulence. However here we use Zernike polynomials instead than Karhunen-Loeve polynomials, therefore the coefficients will not be statistical independent. A future step will be to carry out and implement the coefficients for phase expansion by using Karhunen-Loeve polynomials.

It is well known that Zernike polynomials represent aberrations from low to high order with increasing order index. A wavefront can generally be represented by

$$\Phi = \sum_{j=1}^{M} a_j Z_j \tag{4}$$

where the a_j's are the amplitudes of the aberrations for the jth Zernike order, Z_j, and M is the total number of orders in which the wavefront is represented.

Noll [18] analyzed the phase distortions due to the atmosphere in terms of Zernike polynomials for Kolmogorov turbulence; Boreman and Dainty [19] described how to calculate in closed form the a_j's coefficients of the Zernike expansions for non-Kolmogorov turbulence using the power spectrum (1). As main result, for zero inner scale and infinite outer scale, they carry out that

$$\left\langle \left| a_j \right|^2 \right\rangle = \frac{(n+1)}{\pi} \left(\frac{D}{r_0} \right)^{\alpha-2} \frac{\Gamma\left(\frac{2n+2-\alpha}{2} \right) \cdot \Gamma\left(\frac{\alpha+4}{2} \right) \cdot \Gamma\left(\frac{\alpha}{2} \right) \cdot \sin\left(\pi \frac{\alpha-2}{2} \right)}{\Gamma\left(\frac{2n+4+\alpha}{2} \right)} \tag{5}$$

valid for $n \geq 1$ and $3 < \alpha < 4$. In expression (5) n is the radial order as shown in (16), D is the aperture diameter, and r_0 is the Fried parameter as defined in (17).

Also, Boreman and Dainty show in [19] that the a_j's coefficients of the Zernike expansions for non-Kolmogorov turbulence using power spectrum (1) and including the effect of inner and outer scale can be numerically obtained by numerically solving

$$\left\langle \left| a_j \right|^2 \right\rangle = 4(n+1) \left(\frac{D}{r_0} \right)^{\alpha-2} \cdot \frac{\int_0^\infty \left[\kappa^2 + \left(\frac{\pi D}{L_0} \right)^2 \right]^{-\frac{\alpha}{2}} \kappa^{-1} J_{n+1}^2 (\kappa) d\kappa}{\int_0^\infty \left[\kappa^2 + \left(\frac{\pi D}{L_0} \right)^2 \right]^{-\frac{\alpha}{2}} \kappa \left(1 - \frac{4 J_1^2 (\kappa)}{\kappa^2} \right) d\kappa} \tag{6}$$

However, in this paper we will use only equation (5), therefore we will not consider inner and outer scale effects.

It is well known that the temporal phase variances of atmospheric turbulence have a Gaussian random distribution, therefore, by taking Equation (4) and modifying it such that there is Gaussian random noise factored in gives,

$$\Phi = \sum_{j=1}^{M} \left(1 + X_j \right) a_j Z_j \tag{7}$$

where, X_j is the amount of noise for the jth mode based on a zero-mean unitary Gaussian random distribution and it can be generated by using a software random number generator that uses the Gaussian distribution. However generating a continuous transition for the atmospheric turbulence realization temporally will require another method. The X_j's can

be modified from being just random numbers to a continuous function of time for each mode. Thus, Equation (7) can be rewritten as

$$\Phi = \sum_{j=1}^{M} \left[1 + X_j(t) \right] a_j Z_j \qquad (8)$$

where the $X_j(t)$ function here is generated by, first, creating a vector with some random numbers with the zero mean unitary Gaussian distribution. Next, a spline curve is fit to this vector, as shown in Fig. 2, and this spline curve is now the $X_j(t)$ temporal function for generating the wavefronts in the atmospheric turbulence simulation. This process is repeated for each of the M modes in the model. Without this splining technique, the change between phase screens would be discontinuous and would not provide an accurate representation of the atmosphere for testing an adaptive optics system. In reality, the Earth's atmosphere is a continuous medium. With this Spline technique, the temporal transition of the wavefronts in the atmospheric turbulence simulation is continuous and smooth.
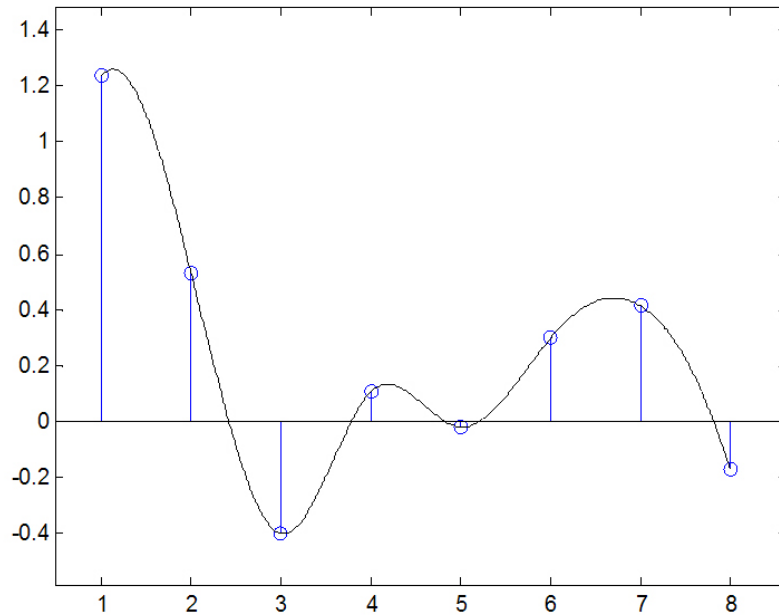

Fig.2-Sample Xi(t) temporal function generated from a vector of a few Gaussian random number

Finally, Wilcox [16] showed that the method of generating phase distortions for atmospheric turbulence simulation via a Spline technique is virtually the same as the Frozen Seeing method with the advantage of being far less computationally intensive on a computer system. A comparison between the new Spline method and the Frozen Seeing method for temporal simulation of atmospheric turbulence was performed [16]. After generating and analyzing 10 different realizations of atmosphere from the Frozen Seeing method and performing the cross-correlation between the two methods' temporal transitions it was found that the two methods are similar and consistent with each other.

4. SLM CONTROL SOFTWARE

While the diffractive nature of the SLM decreases the available light and provides additional alignment challenges, the device provides a great deal of flexibility in generating and applying aberrations in the optical path. Atmospheric scenarios can be changed quickly in software without having to change hardware. This provides a significant advantage

over other popular hardware such as rotating plates imprinted with specific atmospheric statistics, which cannot be changed without constructing new plates.

The Naval Research Laboratory (NRL) has developed software to apply atmospheric aberrations on the SLMs using a Matlab graphical user interface (GUI). The SLM control GUI is shown in Fig.3. The atmosphere generated in software is based on non-Kolmogorov statistics as previously described. As analytical models are developed to describe horizontal turbulence and include effects particular to the maritime environment, the software can be modified to accommodate these model changes.

The NRL software GUI allows the user to specify telescope and site parameters such as the telescope diameter, D, and the atmospheric coherence length or Fried parameter, r_0. Non-Kolmogorov turbulence generation can be performed with the Zernike polynomial expansion (7), the Karhunen-Loève expansion can be used only for Kolmogorov turbulence. The Karhunen-Loève modes contain a statistically independent set of coefficients based on Zernike modes, and are often used in turbulence simulation. Next step will be to extend the non-Kolmogorov turbulence generation even for the Karhunen-Loève expansion.

Once site parameters and any initial alignment biases or static aberrations are selected, the controller generates a user-specified number of phase screens to represent the atmospheric simulation. At the present, the value of non-Kolmogorov power law, α, is not shown on the GUI, but it is set inside a Matlab function which is part of the code. In Fig. 4 an example of phase screen is shown. A secondary annular obscuration can be included in the simulation to simulate a telescope's secondary mirror. By pushing the button "Generate Turbulence" on the GUI several frames of turbulence are generated as shown in the **Video reachable at http://dx.doi.org/**10.1117/12.903681.1.

The simulation is then applied on the ferroelectric Spatial Light Modulator, from Boulder Nonlinear Systems, which is able to operate up to 1 kHz, it has 512 x 512 pixels and an active area of 7.7mm x 7.7mm.

The simulation can be changed easily and quickly as new atmospheric and site parameters are desired for testing, and the underlying code can be changed as new models of turbulence become available.
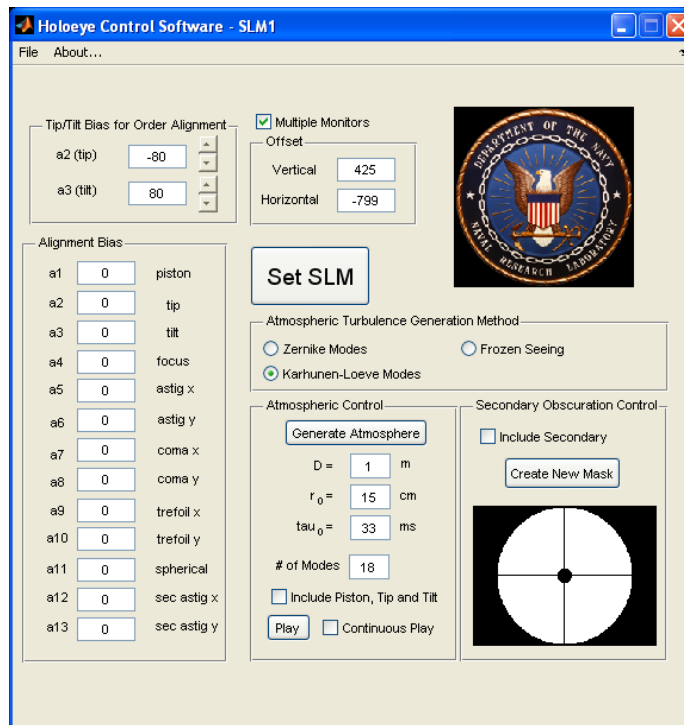

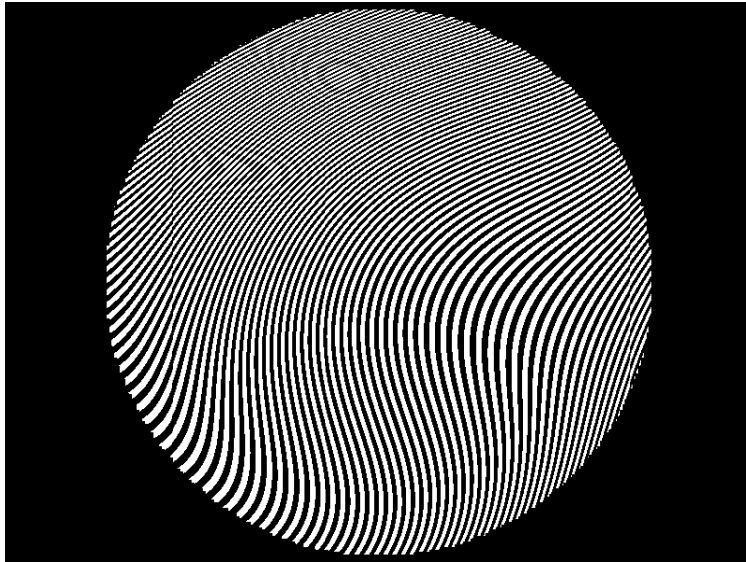Fig. 3 – Matlab graphical user interface developed by NRL

Fig. 4 - An example of phase screen applied on BNS SLM

5. EXPERIMENTAL TESTBED

An experimental setup, as shown in Fig. 5, has been built at Naval Postgraduate School to conduct the practical tests.
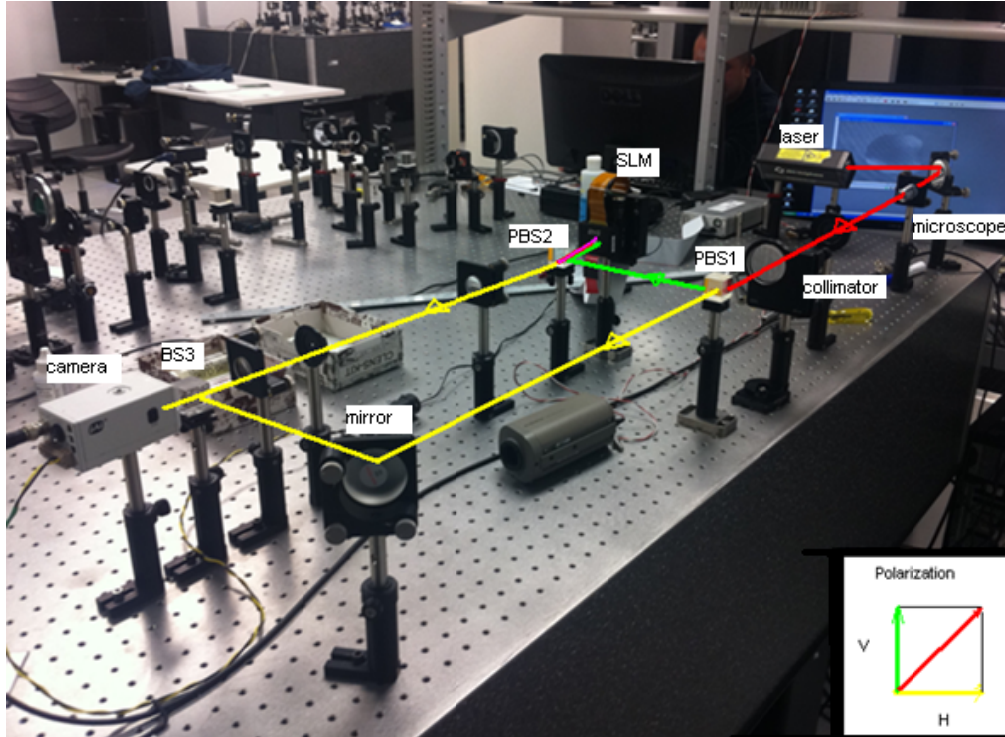

Fig. 5 – Experimental setup built at Naval Postgraduate School

The system is configured with two parallel beams that, after coming through a microscope from the laser, are split by a polarized beam splitter, PBS1. One of these beams is the reference beam that is propagated until the camera by using a mirror and a beam splitter, BS3. The Spatial Light Modulator is a ferroelectric from Boulder Nonlinear Systems with 512 x 512 pixels and an active area of 7.7mm x 7.7mm. The second beam going from PBS1 goes through another beam splitter PBS2 and is directed onto the SLM. After being reflected from the SLM the beam comes back through PBS2 and into a couple of lenses and an iris placed at the focus to select only the first diffraction order of the SLM. It then travels to BS4, where it interferes with the reference beam. A tip-tilt bias on the phase was introduced to "send" light to the first diffraction order. The amplitude and phase information is carried by the first diffraction order selected by the iris.

6. CONCLUSION

In this paper we described a method of using a Spatial Light Modulator to simulate non-Kolmogorov turbulence at laboratory. The method of generating phase distortions for atmospheric turbulence simulation via a Spline technique is virtually the same as the Frozen Seeing method with the advantage of being far less computationally intensive on a computer system. This advantage can be exploited in the development of a software package that can drive any LC SLM to simulate atmospheric turbulence at almost any wavelength for any telescope diameter at any location. AO and Free Space Laser Communication systems can also be tested for performance evaluations.

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