Gaussian beam propagation in maritime atmospheric turbulence: long term beam spread and beam wander analysis

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ABSTRACT

Laser beam propagation in maritime environment is particularly challenging, not only for scattering and absorption due to high humidity, but also for a different behavior of atmospheric turbulence with respect to terrestrial propagation. Recently, a new power spectrum for the fluctuations of the refractive index in the Earth's atmosphere has been introduced to describe maritime atmospheric turbulence. This maritime power spectral model shows a characteristic bump, similar to Hill's bump, that appears when the product between the wavenumber and the inner scale is around unity, $\kappa \cdot l_0 \sim 1$. In this paper, under weak turbulence conditions, we use the mentioned maritime power spectrum to analyze *long term beam spread*, *beam wander* and *Strehl ratio* of a Gaussian beam wave propagating through maritime atmospheric turbulence.

Keywords: Atmospheric turbulence, laser propagation, maritime power spectrum, beam spreading, long term beam spread, angle of arrival, image jitter, Strehl ratio.

1. INTRODUCTION

Atmospheric turbulence in maritime environment is particularly challenging. A paper [1] reported recently a new power spectral model for the fluctuactions of refractive index in the Earth's atmosphere in a marine environment. Theoretical irradiance fluctuation expressions are derived from the new spectrum, both for plane and spherical waves in weak optical turbulence. The results suggested that the new spectral model is adequate for describing the marine atmosphere. Also, the results showed significantly different refractivity measurements in the marine atmosphere, especially at wave numbers near the characteristic bump [2][3][4]. The bump in the refractive index power spectrum, which appears when the product of the wave number with inner scale is around unity, $\kappa \cdot l_0 \sim 1$, produces a corresponding bump in the variance of the log intensity, and phase structure function. Scintillation expressions valid in all regimes of optical turbulence for propagation in the maritime environment, based on the aforementioned power spectrum, have been developed for spherical waves [4]. Results showed that the spherical wave scintillation values for the marine spectrum are smaller than their terrestrial counterparts (modified spectrum) in medium-to-strong turbulence condition; however, in weak turbulence, the scintillation for marine spectrum is larger than with a terrestrial spectrum [1]. NRL researchers recently have measured scintillation in maritime environment [5]. Some of preliminary conclusions are that classical Rytov approximation [6] may not be representative of propagation in marine environment. In this case scintillation shows a much faster transition into strong saturation regime. The refractive index structure parameter, which is a measure of the strength of the fluctuations in the refractive index in the atmosphere, is more stable during the day (it could be an advantage for laser communications). Finally, the refractive index power spectrum is different from a Kolmogorov spectrum[7]. Others measurements and modeling of optical turbulence in maritime environment are shown in [8][9]. The refractive index structure parameter is known to behave differently in marine and terrestrial environments, since a large body of water acts quite differently from soils [10]. Friehe [11] investigated the influence of humidity on the optical refractive index in the maritime environment, and evidence of non-Kolmogorov power spectrum is even shown. Hill's experiments [12] showed the impact of temperature-humidity cospectrum on wave propagation, therefore impact of humidity for laser beam propagation in maritime environment should be better investigated. In addition, evidence of stable refractive layers above the water, under conditions that the air temperature is warmer than the water, is shown in [13].

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In this paper, the focus is on modeling of atmospheric turbulence in a maritime environment for future investigation of adaptive optics technique for compensation of wavefront aberration. As first step, in this paper, we use a maritime power spectrum to analyze *long term beam spread*, *beam wander* and *Strehl ratio* of a Gaussian beam wave propagating through maritime atmospheric turbulence, under weak turbulence conditions.

2. MARITIME POWER SPECTRUM

Maritime environment is particularly challenging and atmospheric turbulence follows a different behavior from Kolmogorov statistics[14][7]. The traditional Kolmogorov spectrum is primarily used in theoretical studies due to its simplicity, but is only valid in the inertial sub range, and does not account for the characteristic bump at high wave numbers which appears just prior to the dissipation range, when the product of the wavenumber with inner scale is around unity, $\kappa \cdot l_0 \sim 1$. The bump in the spectrum was first seen in the Hill numerical spectrum and later adapted to an analytical form in the modified atmospheric spectrum introduced by Andrews.

A paper [16] recently showed a refractive index power spectrum which presents a multiplicative factor able to describe the aforementioned characteristic bump. The maritime power spectrum is given by

$$\Phi_{n,Ma}(\kappa) = 0.033 \cdot C_n^2 \cdot \frac{1}{\left(\kappa^2 + \kappa_0^2\right)^{\frac{11}{6}}} \cdot \exp\left(-\frac{\kappa^2}{\kappa_H^2}\right) \cdot f(\kappa, \kappa_H), \quad \kappa > 0 \qquad (1)$$

where

$$f(\kappa, \kappa_H) = 1 - 0.061 \cdot \frac{\kappa}{\kappa_H} + 2.836 \left(\frac{\kappa}{\kappa_H}\right)^{\frac{7}{6}} \qquad (2)$$

$\kappa_H = \frac{3.41}{l_0}$; $\kappa_0 = \frac{2\pi}{L_0}$; l_0 is the inner scale and L_0 is the outer scale.

Even though more experiments are necessary for its validation, the spectrum (1) is a good candidate to describe atmospheric turbulence in a maritime environment. Fig.1 shows the scaled spectral models of refractive-index fluctuations plotted as a function of the product of the wavenumber and inner scale. The characteristic bump at high wave numbers appears just prior to the dissipation range, when the product of the wavenumber with inner scale is around unity, *klo*~1.

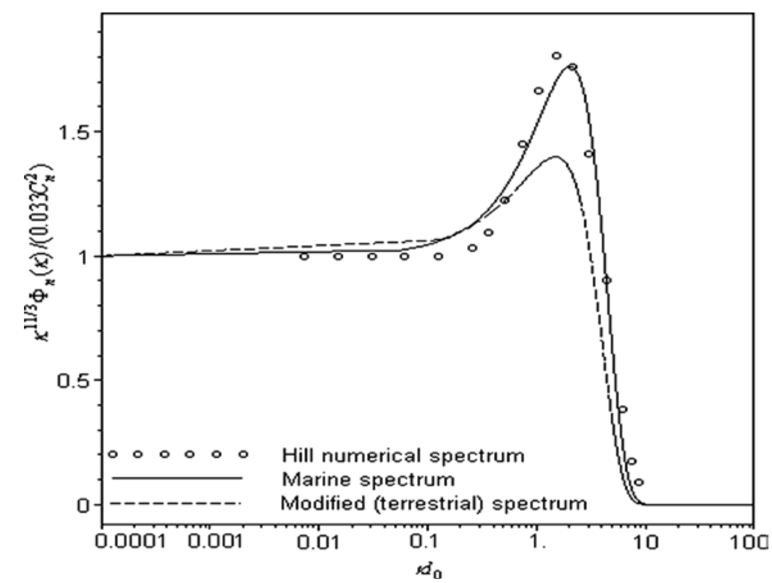

Fig 1- Scaled spectral models of refractive-index fluctuations plotted as a function of the product between wavenumber and inner scale lo

3. LONG TERM BEAM SPREAD IN MARITIME ENVIRONMENT

It is well known that the beam spot size of a laser beam propagating in turbulence is affected from two main effects: beam spreading and beam wander. Random temperature fluctuations of the atmosphere yield random refractive index fluctuations, therefore a laser beam propagating through the atmosphere is randomly deviated from the direction of propagation (beam wander) and it is more affected from beam spreading than a diffraction limited beam in absence of turbulence. Physically the turbulence acts like many lenses of different size that randomly change the effective optical path of the beam. The combined effects of beam wander and beam spreading is called *long term beam spread* which represents the effective beam spot size and it is used as one of the main parameters to evaluate the intensity profile along the path.

We assume that the transmitted Gaussian beam at the input plane has finite waist W_0 and phase front radius of curvature given by F_0.

The analytical form of long term beam spread for a Gaussian beam wave is [6]

$$W_{LT} = W\sqrt{1+T_{Ma}} \tag{3}$$

where W is the diffraction limited spot size radius and T_{Ma} is the spread term due to maritime turbulence, which includes small scale beam spreading and beam wander atmospheric effects in maritime environment.

From equation (3) it is evident that to analyze long term beam spread W_{LT}, we need to calculate the T_{Ma} term. We follow the same procedure as discussed in [6], but here we use the maritime power spectrum (1). We carry out that the spread term T_{Ma} of a Gaussian beam propagating through a maritime atmospheric turbulence is given by:

$$\begin{aligned} T_{Ma} &= 4\pi^2 k^2 L \cdot \left(\int_0^1\int_0^\infty \kappa \cdot \phi_{n,Ma}(\kappa) d\kappa d\xi - \int_0^1\int_0^\infty \kappa \cdot \phi_{n,Ma}(\kappa) \exp\left(-\frac{\Lambda L \kappa^2 \xi^2}{k}\right) d\kappa d\xi \right) \\ &= 2\pi^2 \cdot k^2 \cdot L \cdot 0.033 \cdot C_n^2 \cdot \kappa_H^{-\frac{5}{3}} \\ &\times \left\{ \Gamma\left(-\frac{5}{6}\right) \cdot \left[1 - {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{3}{2}; -\Lambda Q_H\right)\right] \right. \\ &-0.061 \cdot \Gamma\left(-\frac{1}{3}\right) \cdot \left[1 - {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\Lambda Q_H\right)\right] \\ &\left. +2.836 \cdot \Gamma\left(-\frac{1}{4}\right) \cdot \left[1 - {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\Lambda Q_H\right)\right] \right\} \end{aligned} \tag{4}$$

Where C_n^2 is the refractive index structure parameter, which is constant for a horizontal path of length L , $\xi = 1 - \frac{z}{L}$, $\Lambda = \frac{2L}{kW^2}$ (z is the propagation distance), Γ is the gamma function, $Q_H = \frac{L}{k} \cdot \kappa_H^2$, $k = \frac{2\pi}{\lambda}$ and ${}_2F_1(a,b;c;-x)$ is the hypergeometric function.

Equation (4) does not include outer scale effects because the objective of this paper is to analyze the impact of the 'bump', which is an inner scale effect, on the effective beam spread. However, it is well known that a finite value of the outer scale always reduce the long term beam spread [6]

We plotted in Fig.2 and Fig.3 the T_{Ma} term and the long term beam spread as a function of the inner scale for three different spectrums: Von Karman, Modified terrestrial and Maritime power spectrums.

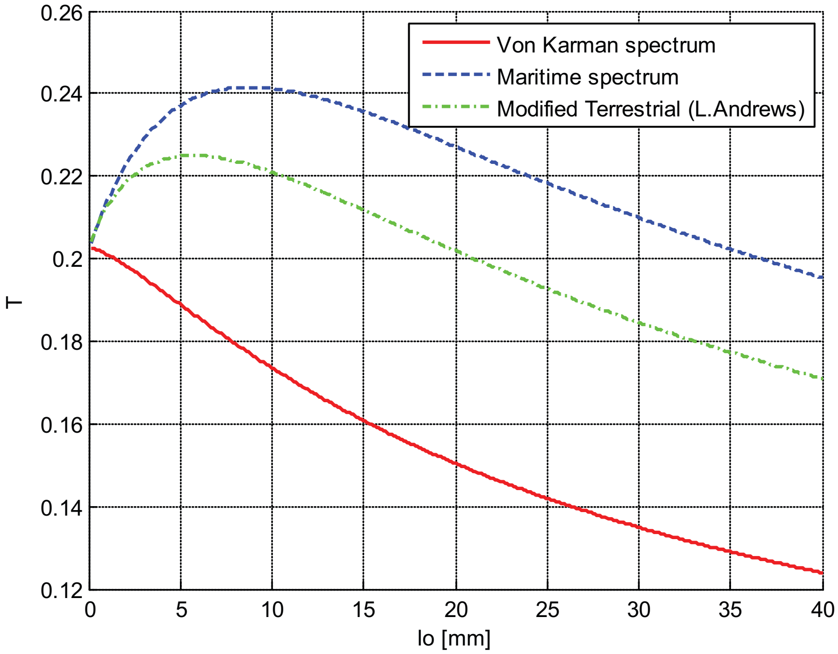

Fig 2- T as a function of inner scale for Cn^2=3e-14,L=1km,Wo=0.01m, collimated beam

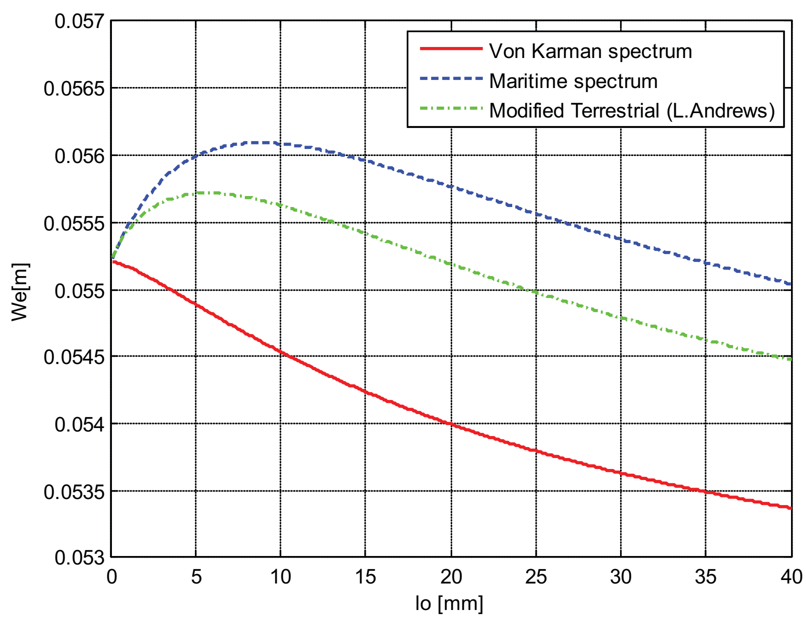

Fig 3- Long Term Beam Spread as a function of the inner scale for Cn^2=3e-14,L=1km,Wo=0.01m, collimated beam

We deduce from Fig.2 and Fig.3 that the Maritime power spectrum predicts higher values of long term beam spread than both Modified and Von Karman power spectrum. The higher 'bump' presents into the maritime power spectrum (1) leads to higher values of the long term beam spread, particularly around the inner scale value of 8 mm (for this specific set of parameters). Also we deduce that if the inner scale value increases, the long term beam spread increases until to reach a maximum value and then it starts to decrease for higher inner scale values.

4. ANGLE OF ARRIVAL, IMAGE JITTER AND BEAM WANDER ANALYSIS

Angle-of-arrival fluctuations of an optical wave in the plane of the receiver aperture are associated with image dancing in the focal plane of an imaging system. Fluctuations in the angle of arrival β_a can be described in terms of the phase structure function [6]. To understand this, let ΔS denote the total phase shift across a collecting lens of diameter $2W_G$ and Δl the corresponding optical path difference. These quantities are related by

$$k\Delta l = \Delta S \tag{5}$$

If we assume that β_a is small so that $\sin\beta_a \cong \beta_a$, then, under the geometrical optics method (GOM), the angle of arrival is defined by

$$\beta_a = \frac{\Delta l}{2W_G} = \frac{\Delta S}{2kW_G}.\left[radian\right] \tag{6}$$

Further assuming that the mean $\langle \beta_a \rangle = 0$, we deduce the variance of the angle of arrival

$$\langle \beta_a{}^2 \rangle = \frac{\langle (\Delta S)^2 \rangle}{(2kW_G)^2} = \frac{D_S(2W_G, L)}{(2kW_G)^2}, \tag{7}$$

where $D_S(2W_G, L)$ is the phase structure function with the radial distance $\rho = 2W_G$.

Also, angle of arrival at the receiver is related to root mean square image jitter at focal plane by[1]

$$rms\_image\_jitter = f \cdot \sqrt{\langle \beta_a{}^2 \rangle} \tag{7}$$

where f is the focal length.

In addition, the beam wander is related to angle of arrival fluctuations by

$$\langle r_c{}^2 \rangle \cong L^2 \cdot \langle \beta_a{}^2 \rangle, \text{ for } W_G = W_0 \tag{8}$$

where L is the path length and W_0 is the spot radius at the transmitter.

This relationship, as reported in [6], derives from the observation that beam wander at the receiver plane can be modeled as if it arises from a random tilt angle at the transmitter plane, similar to angle of arrival fluctuations of a reciprocal propagating wave with the receiver diameter replaced by the transmitter beam diameter.

Therefore we focus our analysis on the beam wander because this parameter is related to both angle of arrival and image jitter.

The beam wander is a large scale effect due mainly to large turbulence cells size which act like lens along the beam path. These turbulence cells contribute to generate a wandering effect on the beam if they are larger than the beam size, therefore we need to introduce a filter function able to filter out all the turbulence cells that do not contribute to beam wander.

We first introduce two parameters to describe the Gaussian beam at the transmitter:

$$\Theta_0 = 1 - \frac{L}{F_0}, \quad \Lambda_0 = \frac{2L}{kW_0^{\,2}}, \tag{9}$$

where F_0 is the phase front radius of curvature (it is infinite for a collimated beam). After that, we introduce the filter function as follow

$$H_{LS}(\kappa, z) = \exp\left[-\kappa \cdot W^2(z)\right] = \exp\left[-\kappa^2 W_o^2 \left(\Theta_0 + \bar{\Theta}_0 \xi\right)^2 + \Lambda_0\left(1-\xi^2\right)\right] \tag{10}$$

where $\xi = 1 - \frac{z}{L}$ and we follow the same procedure as showed in [6], but we use the maritime power spectrum. Our analysis leads to:

$$\left\langle r_c^{\,2} \right\rangle_{Ma} = 4\pi^2 k^2 W^2 \int_0^L \int_0^\infty \kappa \cdot \phi_{n,Ma}(\kappa_H, \kappa) \cdot H_{LS}(k,z) \cdot \left[1 - \exp\left(-\frac{\Lambda L \kappa^2 \xi^2}{k}\right)\right] d\kappa dz$$

$$= 4\pi^2 \cdot 0.033 \cdot C_n^2 \cdot L^3 \cdot \left\{ \left[\frac{\Gamma\left(-\frac{1}{6}\right)}{\Gamma\left(\frac{11}{6}\right)} \cdot \frac{\kappa_0^{\frac{1}{3}}}{3} + \Gamma\left(\frac{1}{6}\right) \cdot \int_0^1 \xi^2 \left[\frac{1}{\kappa_H^2} + W_0^2 \cdot \left(\Theta_0 + \bar{\Theta}\xi\right)^2\right]^{-\frac{1}{6}} \right] \right.$$

$$- \frac{0.061}{\kappa_H} \left[\frac{\Gamma\left(\frac{5}{2}\right) \cdot \Gamma\left(-\frac{2}{3}\right)}{\Gamma\left(\frac{11}{6}\right)} \cdot \frac{\kappa_0^{\frac{4}{3}}}{3} + \Gamma\left(\frac{2}{3}\right) \cdot \int_0^1 \xi^2 \left[\frac{1}{\kappa_H^2} + W_0^2 \cdot \left(\Theta_0 + \bar{\Theta}\xi\right)^2\right]^{-\frac{2}{3}} \right]$$

$$\left. + \frac{2.836}{\kappa_H^{\frac{7}{6}}} \left[\frac{\Gamma\left(\frac{31}{12}\right) \cdot \Gamma\left(-\frac{3}{4}\right)}{\Gamma\left(\frac{11}{6}\right)} \cdot \frac{\kappa_0^{\frac{3}{2}}}{3} + \Gamma\left(\frac{3}{4}\right) \cdot \int_0^1 \xi^2 \left[\frac{1}{\kappa_H^2} + W_0^2 \cdot \left(\Theta_0 + \bar{\Theta}\xi\right)^2\right]^{-\frac{3}{4}} \right] \right\} \tag{11}$$

To emphasize the refractive nature of beam wander, we dropped the last term in (10) and we used the geometrical optics approximation

$$1 - \exp\left(-\frac{\Lambda L \kappa^2 \xi^2}{k}\right) \cong \frac{\Lambda L \kappa^2 \xi^2}{k}$$

Also, if we suppose a low inner scale value, the ratio $\frac{1}{\kappa_H^2}$ inside the integral can be omitted and expression (11) can be expressed in closed form as follow

$$
\left\langle r_c^{\,2} \right\rangle_{Ma} \cong \frac{4}{3}\pi^2 \cdot 0.033 \cdot C_n^2 \cdot L^3 \cdot \left\{ \left[\frac{\Gamma\left(-\frac{1}{6}\right)}{\Gamma\left(\frac{11}{6}\right)} \cdot \kappa_0^{\frac{1}{3}} + \Gamma\left(\frac{1}{6}\right) \cdot W_0^{-\frac{1}{3}} \cdot {}_2F_1\left(\frac{1}{3},1;4;1-\left|\Theta_0\right|\right) \right] \right.
$$

$$
-\frac{0.061}{\kappa_H}\left[\frac{\Gamma\left(\frac{5}{2}\right)\cdot\Gamma\left(-\frac{2}{3}\right)}{\Gamma\left(\frac{11}{6}\right)} \cdot \kappa_0^{\frac{4}{3}} + \Gamma\left(\frac{2}{3}\right) \cdot W_0^{-\frac{4}{3}} \cdot {}_2F_1\left(\frac{4}{3},1;4;1-\left|\Theta_0\right|\right) \right]
$$

$$
\left. +\frac{2.836}{\kappa_H^{\frac{7}{6}}}\left[\frac{\Gamma\left(\frac{31}{12}\right)\cdot\Gamma\left(-\frac{3}{4}\right)}{\Gamma\left(\frac{11}{6}\right)} \cdot \kappa_0^{\frac{3}{2}} + \Gamma\left(\frac{3}{4}\right) \cdot W_0^{-\frac{3}{2}} \cdot {}_2F_1\left(\frac{3}{2},1;4;1-\left|\Theta_0\right|\right) \right] \right\} \qquad (12)
$$

We plot in Fig.4 the beam wander (11) as a function of the inner scale for several outer scale values. We deduce from Fig.4 that the inner scale slightly affects the beam wander which is instead remarkably affected by the outer scale. This is a consequence that the beam wander is affected mainly from large turbulence cells size.

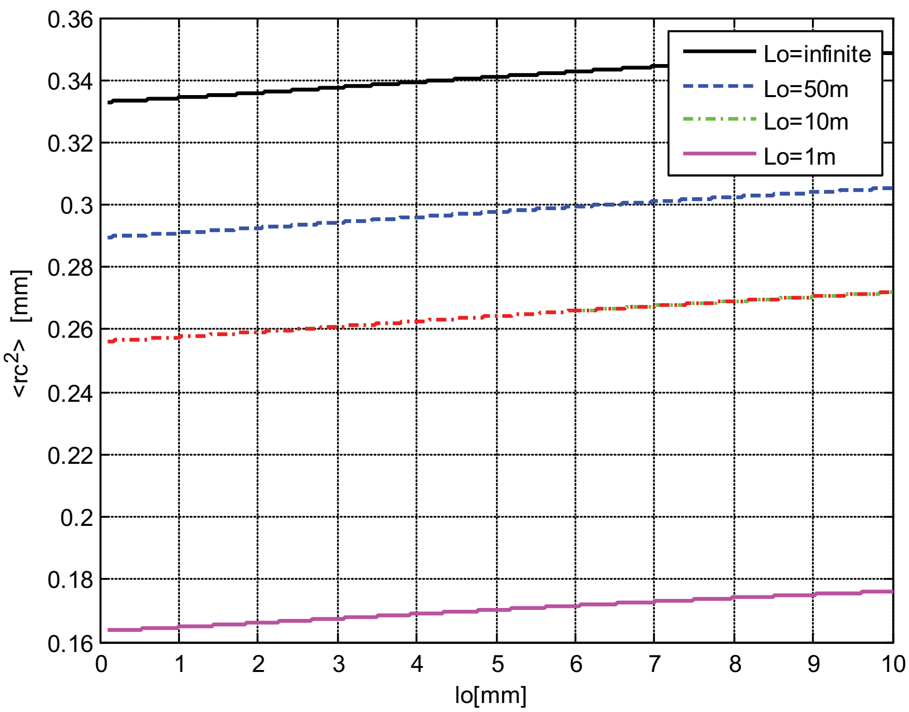

Fig 4- Beam Wander as a function of the outer scale for Cn^2=3e-14,L=1km,Wo=0.01m, collimated beam

We plot in Fig.5 and Fig.6 the beam wander (11) as a function of the beam spot radius for several inner scale and outer scale values.

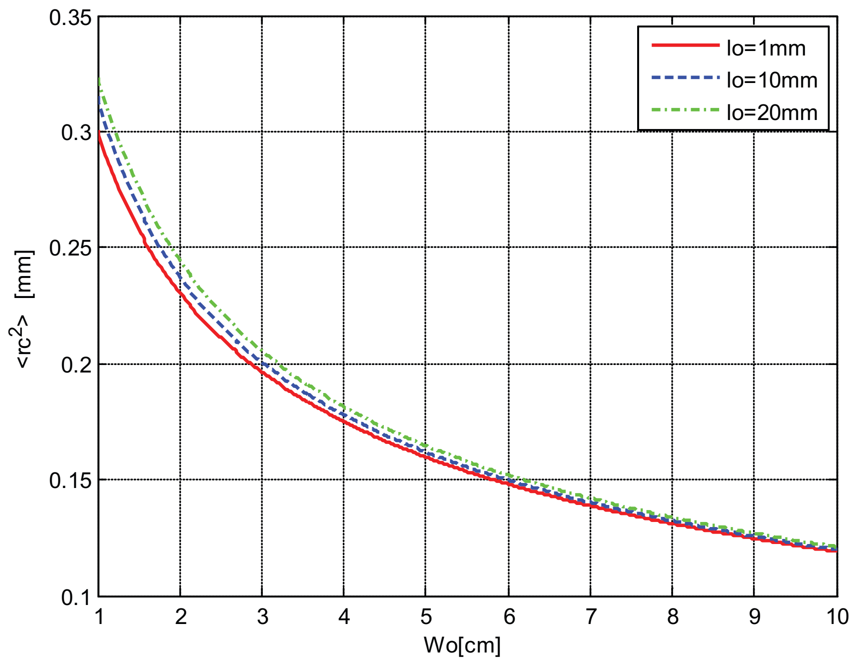

Fig 5- Beam Wander as a function of the spot radius for several inner scale values, setting Cn^2=3e-14,L=1km,Wo=0.01m, collimated beam

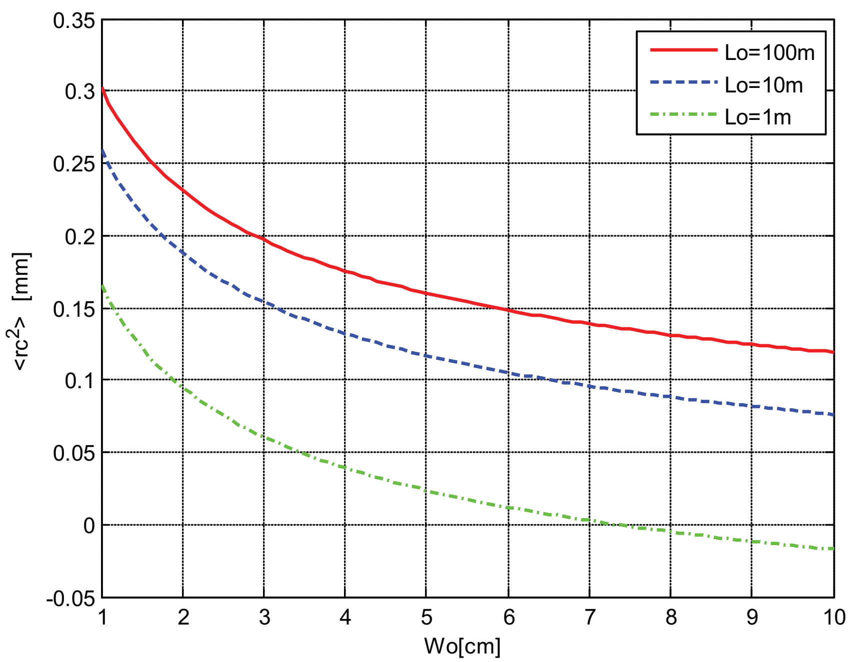

Fig 6- Beam Wander as a function of the spot radius for several outer scale values, setting Cn^2=3e-14,L=1km,Wo=0.01m, collimated beam

We deduce from Fig.5 that the inner scale slightly affects the beam wander which is instead remarkably affected by the beam spot radius. Again, this is a consequence that the beam wander is affected mainly from large turbulence cells size because an increase of the beam spot size is physically equivalent to filter out turbulence cells size which contribute to the beam wander. Finally, we deduce from Fig.6 that both the outer scale and beam spot radius remarkably affects the beam wander.

5. STREHL RATIO ANALYSIS

A useful parameter to evaluate optical system performance is the Strehl ratio which is defined as the ratio between the peak irradiance received by an optical system propagating in turbulence, $\langle I(0,L)\rangle$, and the peak irradiance without propagation in atmospheric turbulence, $I^0(0,L)$. The irradiance is related to the spot size of the beam, therefore the Strehl ratio can be written as [6]

$$SR_{Ma} = \frac{\langle I(0,L)\rangle}{I^0(0,L)} \cong \frac{W^2}{W^2_{LT_{Ma}}} = \frac{1}{1+T_{Ma}} \tag{13}$$

We plot in Fig.7 the Strehl ratio as function of inner scale for different power spectrums. It is shown in Fig.7 that a Gaussian beam propagating in maritime environment is more affected by beam spreading than in terrestrial environment, particularly for inner scale values near the bump of the maritime power spectrum. This increased beam spreading leads to a reduction of Strehl ratio which should be considered for budget link analysis in Lasercom or other free space optics applications.

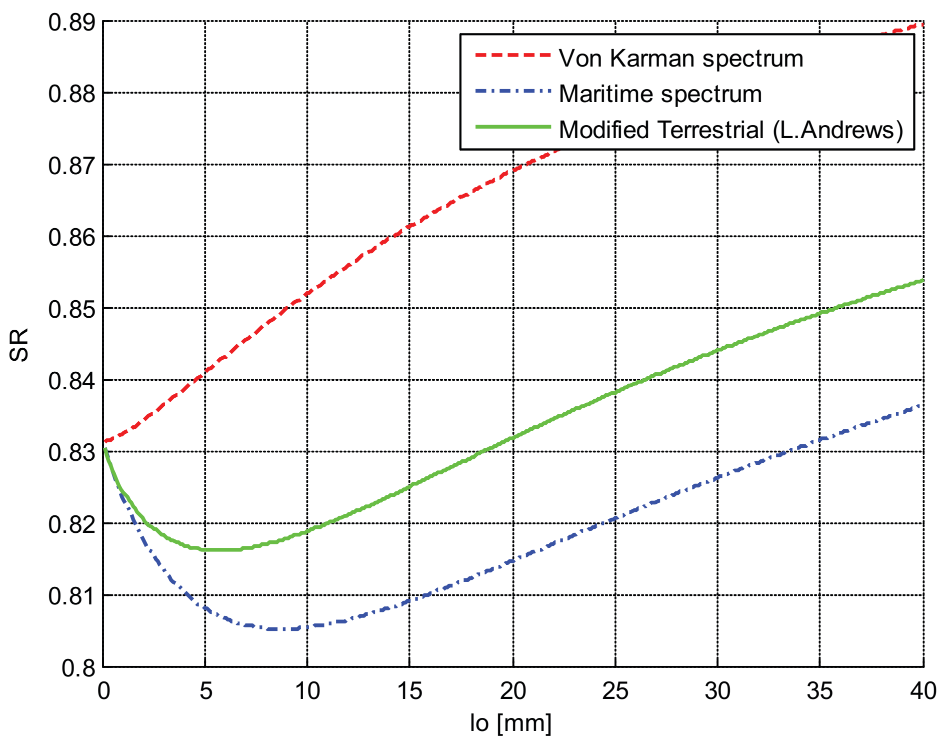

Fig 6- Strehl ratio as a function of the inner scale for several power spectrum, setting Cn^2=3e-14,L=1km,Wo=0.01m, collimated beam

6. DISCUSSION

In this paper we have used a maritime power spectrum, in a weak turbulence regime, to analyze long term beam spread, beam wander, and Strehl ratio of a Gaussian beam propagating through the atmosphere.
The results showed that the higher 'bump' presents into the maritime power spectrum leads to higher values of long term beam spread and beam wander. Therefore, the spot size of a Gaussian beam propagating in maritime environment is more effected by beam spreading than in terrestrial propagation, particularly for inner scale values near the bump of the maritime power spectrum. This increased beam spreading leads to a reduction of Strehl ratio that should be considered into the budget link analysis of applications of laser beam propagation in maritime environment, such as ship-to-ship free space optics communication and imaging.

7. ACKNOWLEDGMENTS

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REFERENCES

[1] K. J. Grayshan; F. StrÖMqvist Vetelino; C. Y. Young. *A marine atmospheric spectrum for laser propagation.* Waves in Random and Complex Media. Media,18, pag.173-184, February 2008
[2] R.J. Hill and S.F.Clifford. *Modified spectrum of atmospheric temperature fluctuations and its application to optical propagation.* JOSA.vol.68, .7, July 1978.
[3] L.C. Andrews. *An Analytical Model for the Refractive Index Power Spectrum and Its Application to Optical Scintillations in the Atmosphere.* Journal of Modern Optics, vol.39, n.9, 1992
[4] F. Stromqvist Vetelino, Katelyn Grayshan, and C. Y. Young. *Inferring path average Cn^2 values in the marine environment*. J. Opt. Soc. Am. A, vol.24, No.10,October 2007.
[5] Linda M. Wasiczko, Christopher I. Moore, Harris R. Burris, Michele Suite, Mena Stell, William Rabinovich. *Studies of atmospheric propagation in the maritime environment at NRL.* Proc. of SPIE vol.6951, 2007.
[6] Larry C. Andrews, Ronald L. Phillips. *Laser Beam Propagation through Random Media*. Spie Press, second edition - 2005.
[7] Mark Chang, Carlos O. Font, Freddie Santiago, Yaitza Luna, Erik Roura and Sergio Restaino. *Marine environment optical propagation measurements*. Proceedings of the SPIE, Volume 5550, pp. 40-46 (2004).
[8] Paul A. Frederickson, Stephen Hammel, and Dimitris Tsintikidis. Measurements and modeling of optical turbulence in a maritime environment. Proceeding of SPIE, Volume 6303, 2006
[9] Mark P.J.L. Chang, and el al. Comparing horizontal path Cn^2 measurements over 0.6 Km in the tropical littoral environment and in the desert. Proc. of SPIE, vol. 6551, 2007
[10] K.R. Weiss-Wrana. *Turbulence statistics in littoral area.* Proceding of SPIE 6364, 2006
[11] Friehe, C. A., La Rue, J. C., Champagne, F. H., Gibson, C. H., and Dreyer, G. F. *Effects of temperature and humidity fluctuations on the optical refractive index in the marine boundary layer.* Journal of the Optical Society of America, 65, 1502- (1975).
[12] R.J. Hill. *Spectra of fluctuations in refractivity, temperature, humidity, and the temperature-humidity cospectrum in the inertial and dissipation ranges (atmospheric effects on radio propagation)*. Radio Science, vol.13, pag. 953-961, 1978
[13] Linda M. Wasiczko, Rita Mahon, Christopher I. Moore, Harris R. Burris, Michele Suite, William Rabinovich. *Power Spectra of a Free Space Optical Link in a Maritime Environment.* Proc. of SPIE vol.7464, 2009.
[14] Italo Toselli, Larry C. Andrews, Ronald L. Phillips, Valter Ferrero. *Free space optical system performance for laser beam propagation through non-Kolmogorov turbulence.* Optical Engineering vol. 47, February 2008.