MA 1114: Calculus II and Matrix Algebra, Fall 2006

Homework:

- Chapter 6:
  - 6.1. #1, 2, 5, 9, 11, 17, 19, 23, 27, 41.
  - 6.2: 1, 5, 9, 11, 15, 17, 49.
  - 6.3: 1, 5, 9, 15, 17, 21, 23.
  - 6.4: 1, 3, 7, 9.
  - 6.5: 5, 9, 13, 17.
- Chapter 7:
  - 7.2: 1, 5, 15, 17, 21, 31.
  - 7.3: 1, 5, 9, 11, 19.
  - 7.4: 9, 15, 19, 21, 23, 39, 53.
  - 7.8: 5, 9, 15, 19, 27, 31, 37, 49, 53.
- Chapter 11:
  - 11.1: 1, 5, 7, 13, 15, 19, 21, 31, 37, 39, 55, 59.
  - 11.2: 9, 17-31(odd).
  - 11.3: 9, 13-19(odd).
  - 11.4: 5-15(odd), 21, 23, 25.
  - 11.5: 3, 5, 7, 13, 19.
  - 11.6: 3-23(odd). 29.
  - 11.8: 3, 7, 9, 19-27(odd).
  - 11.9: 5-11(odd), 15, 17, 23.
  - 11.10: 5, 7, 15, 23, 27, 37, 39, 42.
  - 11.11: 1, 3, 7, 11.
  - 11.12: 5, 9, 19.
- Appendix G:
  - 1, 5, 7, 13, 15, 19, 23, 25, 29, 33, 37, 41. Matrix Algebra
- Chapter 1:
  - 1.1: 1(b), 2(b), 3(a, b), 4(a, b), 5(a, b), 6(a, b).
  - 1.2: 1, 2(a, b, c), 4(a, b), 5, 6(c), 13
  - 1.3: 1(c, f), 2(a, b, d), 4, 5(b, c), 6(c), 7, 8(d), 13(a).
  - 1.4: 1, 2, 5, 9, 10(e, f), 12(b, d).
- Chapter 2:
  - 2.1: 2, 3(a, b, d, g), 5, 6.
  - 2.2: 1, 3, 4, 7. 2.3: 1, 2. 5.
- Chapter 6:
  - 6.1: 1.

Exams:

- Exam 1 -- Mo. November 14th, 2005 (Ch 6 and 7) -- improper integrals of type 2
- Exam 2 -- Fri. November 18th, 2005 (Ch 11- sections 1-6)
- Exam 3 -- Wed. November 23rd, 2005 (Ch 11- sections 8-12)
- Exam 4 -- Fri. December 2nd, 2005 (App G and Ch 1 of Linear Algebra)
- Final Exam -- Fri. December 9th, 2005 (cumulative). The final exam is a 2-hour exam that can be taken from 11am-1pm or noon-2pm. If we need more time to review the chapters for the final exam, the Fri class will be used for review, and the exam will take place on Mo, Dec 12th from 1-3pm.

**Calendar:**

### November 2005

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**Objectives:**

Upon completion of this course, the student should be able to satisfy the following objectives.

**A. INTEGRAL CALCULUS**

1. Use integration to: find the area between curves; find volumes of solids of revolution; find total work done in appropriate problems; find the average value of a function.

2. Evaluate appropriate integrals by using trigonometric identities and/or trigonometric substitution.

3. Evaluate integrals of rational functions by the method of partial fractions.
4. Recognize improper integrals, determine whether they converge, and if possible, evaluate them.

B. SEQUENCES AND SERIES

5. Given a sequence, determine whether or not it converges, and if it does, find the limit.

6. Determine the convergence of a series by appropriate tests, including the ratio, comparison, p-series, and alternating series tests.

7. Find the interval of convergence for a power series.

8. Apply Taylor’s Theorem to find polynomial approximations to given functions and estimate their accuracy.

C. MATRIX ALGEBRA

9. Perform arithmetic operations on complex numbers: addition, subtraction, multiplication, division, raising to powers; determine the magnitude, argument, real part, imaginary part, and complex conjugate; convert between rectangular and polar forms.

10. State and apply De Moivre’s theorem; find n th roots of complex numbers. State Euler’s formula.

11. Find the general solution for m × n systems of linear equations using Gauss-Jordan elimination. Determine the type of solution set (inconsistent, unique solution, or infinitely many solutions) by Gauss elimination.

12. Perform algebraic operations on matrices and vectors: addition, subtraction, scalar multiplication, matrix multiplication, transposition.

13. Define and describe the basic properties of the inverse of a matrix. Find the inverse of a square matrix using Gauss-Jordan elimination.

14. Compute the determinant of a matrix either by elementary row operations or by cofactor expansion.

15. Use Cramer’s rule (where applicable) to solve 2 x 2 and 3 x 3 systems of linear equations. Explain why Cramer’s rule is inappropriate for large systems.

16. Find the eigenvalues and associated eigenvectors of 2 x 2 and 3 x 3 matrices, including cases of repeated or complex eigenvalues.

MA/Ca 8/05

Topics:

• Mon. Nov. 7th: 6.1 Areas between curves:
• formula 1 page 437 and formula 2 page 438= area between two curves is the difference of the integrals of the upper curve and the lower one. The limits on the integral are the x-values of the intersection of the curves.
• if one curve is not always above the other, the integral must be split as in formula 3 and Example 5 page 440. The intersection points of the curves are needed.
• if the curves are functions of y versus functions of x, then the limits on the integral are the y values of the intersection points, and the integrand is the difference of the curves as functions of y (i.e. the variable inside the integral should be y rather than x, and the integral end with "dy" instead of "dx"). See Example 6.
• 6.2 Volumes (Washer method):
  • Cross-section \( A(x) = \) are of the cross section perpendicular to the x-axis (it is a function of x).
  • Definition of the volume as the integral of the area of the cross section (notice that if we use \( A(x) \) then we need to have "dx" as part of the integral, and the limits on the integral are values of x. Similarly for \( A(y) \) we need to have "dy" as part of the integral, and the limits on the integral are values of y. ) See formula for \( A(y) \) on the top of page 450 (similar formula holds for \( A(x) \)).
  • Choose \( A(x) \) if the solid is obtained by rotating the area about the x-axis, and choose \( A(y) \) if the area is rotated about the y-axis.
  • Note that the area that is rotated about x-axis (or the y-axis) makes a difference in the solid obtained: in finding the area of the washer, you might need to subtract the area of the inner disk (see example 4).
• Tu. Nov 8th: 6.3 Volumes by Cylindrical Shells: mostly used when trying to find the \( A(x) \) or \( A(y) \) one ends up with \( R=r \) (i.e. the same function is inner and the outer radius -- see page 455)
  • volume is the integral of the product of the circumference and the height (formula 2)
  • need to "adjust the circumference" when the volume is obtained by rotating an area about any line different than x-axis/y-axis (see example 4).
• 6.4 Work: \( W \) is the integral of the force, where --click here for more details.
• 6.5 Average Value of a Function
  • average value of a function
  • MVT for Integrals gives a value \( c \) at which the value of \( f \) is the average value of \( f \) over the interval (compare to MVT for derivatives)
• Wed. Nov 9th: 7.2 Trigonometric Integrals:
  • strategy for evaluating sin-cos integrals -p. 484
  • strategy for evaluating sec-tan integrals -p. 486
  • other cases - there are no clear-cut guidelines so try substitution or parts
• 7.3 Trigonometric Substitutions (uses backwards substitution to solve integrals that do not contain trig functions. In solving the integral we use trig identities):
  • table page 490
• Thu. Nov 10th: 7.4 Partial Fractions (they help find the derivative of a ratio of polynomials by factoring the denominator):
o see cases 1-4 (If denominator is a polynomial that is raised to the power, then the numerator should have the degree one less than the degree of the denominator before raising it to the power.)
o rationalizing substitution: if the fraction is not rational (i.e. not a fraction of polynomials) uses substitution to get both the numerator and denominator polynomials.

- 7.8 Improper Integrals:
o integrals of type 1: defn. 1 page 531 (integrals over infinite intervals)
o integrals of type 2: defn. 3 page 534 (integrand is discontinuous or not defined on the whole interval)
o convergent integral: if the corresponding limit of the integral exists
o divergent integral: if the corresponding limit of the integral does not exist
o formula 2 page 533
o comparison theorem: compares two integrals to find out if one converges/diverges depending on the convergence of the other.

- Fri: No class. Exam over Ch 6 and Ch 7 on Mo.
- Mon. Nov. 14th: Exam over Ch6 and Ch7
- Tue. Nov 15th: 11.1 Sequences (a list of numbers, but not the sum)
o limit of a sequence p 703 (if the limit is finite/infinite, then the sequence converges/diverges)
o Thm 3 p 704: write the general term of the sequence as a function. Then, if the limit of the sequence is the limit of the function (recall the limit laws learned in MA1113).
o limit laws for sequences p 705
o squeeze theorem for sequences p 705
o Thm 6 used for alternating sequences
o limit of r^n
o increasing/decreasing sequences (monotone sequence = increasing or decreasing sequence)
  . To show increasing/decreasing try:
  ▪ (1) \( a_n > a_{(n+1)} \) or \( a_n < a_{(n+1)} \), or
  ▪ (2) take the derivative and see where it is positive or negative
o bounded sequences, bounded above sequences, and bounded below sequences (the bound has to be a fixed number, not a variable)
o if \( \lim a_n \) DNE, then the sequence \( \{a_n\} \) diverges
o if \( \lim a_n \) is infinity, then the sequence \( \{a_n\} \) diverges to infinity
o finding convergence of sequences:
  ▪ limit \( a_n \) is finite
  ▪ Squeeze Theorem
  ▪ formula 8 p 707 for \( r^n \)
  ▪ Monotonic Sequence Theorem (if the sequence is decreasing, one needs to show that it is bounded below in order for the sequence to be convergent. Similarly, if the sequence is increasing, one needs to show that it is bounded above in order for the sequence to be convergent.)

- 11.2 Series (is the sum of the terms of an infinite sequence) series can be finite (like the sum of \( 1/(2^n) \)), or infinite (like the sum of \( 2n \)). partial sum is part of the sum of the terms of an infinite sequence defn. of convergent and divergent series types of series and their convergence: geometric
(p715), telescoping (p 717), harmonic (p 717) difference between the sequence \{a_n\} and the sequence \{s_n\}. The \(n\)th term test: if the series converges, then \(\lim_{n \to \infty} a_n = 0\) (or the limit of the sequence \(a_n\) is 0). Converse is not true: If \(\lim_{n \to \infty} a_n = 0\) it doesn't mean that the series converges (for example the harmonic series). If the series converges, then \(\lim s_n = s\), where \(s\) is the sum of the series. Divergence test: if \(\lim a_n \text{ DNE or it is not 0, then the series diverges (again, if } \lim a_n = 0,\text{ then we don't know if the series converges or diverges) operations with series to show divergence of series use the divergence test (or see if it is a harmonic series). To show series are convergent: look for a geometric series with \(-1 < r < 1\) if \(\lim s_n\) exists and it is finite (not the \(\lim a_n\)). Then series converges to \(\lim s_n\) (for example telescoping series).

- **Wed. Nov 16th: 11.3 The Integral Test and Estimates of Sums (and p-series)**: the integral test p 724 (helps to decide if a series is convergent, but it doesn't give the sum of the series in the convergence case) p-series (p725) and its convergence (compare the form of the p-series to the geometric series) use the integral test to see if a series is convergent, and if it is then estimate the sum: for a continuous decreasing function we can estimate the sum of the series by the integral of the corresponding function, with the remainder given by the Remainder Estimate for the Integral Test p 727 11.4 Comparison Tests (use the convergence of known series) comparison test page 731: compare it to known series like the geometric, telescoping, harmonic, p-series limit comparison test page 732: consider the limit of the quotient of the series with a known series.

- **Thu. Nov 17th: 11.5 Alternating Series**: what is an alternating series alternating series test page 736 alternating harmonic series converges (even though the harmonic series does not converge) 11.6 Absolute Convergence, Ratio and Root Test: absolute and conditionally convergence absolute convergent implies convergence, i.e. if a series is abs. conv. then it is conv. examples of series that are absolute convergent, conditionally convergent, and just convergent a convergent series whose terms are positive is absolute convergent ratio test (mostly used for factorials, polynomials, power function \(a_n\)) root test (used for polynomials to power \(n\)).

- **Fri. Nov 18th**: Exam 2. Click here for review for convergence and divergence.

- **Mo. Nov 21st**: 11.8 Power Series: power series about a use ratio or root test to find the interval of convergence or to find the radius of convergence. Use a different test to find if the series converges at the endpoints of the interval of convergence.

- **11.9 Representation of Functions as Power Series**: use formula 1 page 754 to represent other functions as power series and to find the interval of convergence term-by-term differentiation and integration theorem p 756.

- **11.10 Taylor and Maclaurin Series**: (shows which functions have power series representation) Taylor Series of a function at "a" page 761 Maclaurin Series of a function page 762 (Taylor series at 0). When does a function equal its Taylor series? (see Theorem 8 page 763) formula 10 page 764 Taylor's theorem applications: (1) evaluating integrals as infinite series (p.768), and (2) finding limits (p. 768).

- **Tue. Nov 22nd**: 11.11 Binomial Series: Binomial theorem (recall that the coefficients come from Pascal's triangle --MA1113) Binomial Series use Binomial Series to find Maclaurin series for functions 11.12 Applications of Taylor Polynomials: approximating functions by polynomials read applications to physics.

- **Wed. Nov 23rd**: Review and Exam 3

- **Thu. and Fri**: No classes.

- **Appendix G**
o Formulas in the section, in particular De Moivre's Thm, Roots of complex numbers, Euler's formula.