Review for series convergence and divergence.

Types of series: geometric $\sum_{n=1}^{\infty} ar^{n-1}$, telescoping like $\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$, harmonic $\sum_{n=1}^{\infty} \frac{1}{n}$.

$p$-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, alternating harmonic $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

Types of convergence: convergence (when $\sum_{n=0}^{\infty} a_n$ converges to a finite number $L$),

absolute convergence (when $\sum_{n=0}^{\infty} |a_n|$ converges),

conditional convergence (when $\sum_{n=0}^{\infty} a_n$ converges but $\sum_{n=0}^{\infty} |a_n|$ does not converge),

divergence (when the sum does not converge to a finite number)

To show a series is divergent:

(1) test for divergence: $\lim_{n \to \infty} a_n$ DNE or it is not 0, then the series diverges

(2) geometric series with $r \leq -1$ or $r \geq 1$

(3) harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$

(4) $p$-series with $p \leq 1$

(5) integral test: if $\int_{1}^{\infty} f(x) \, dx$ diverges, where $f(x)$ is the corresponding function

(6) comparison test: show that the series is greater than a series that diverges

(7) limit comparison test: show that the limit of the quotient of the series and a series that diverges is a finite nonzero number

(8) ratio test: $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| > 1$

(9) root test: $\lim_{n \to \infty} \sqrt[n]{|a_n|} > 1$
To show a series is convergent (or absolute convergent):

1. If \( \lim_{n \to \infty} s_n \) exists and it is finite (not \( \lim_{n \to \infty} a_n \)), then series converges to \( \lim_{n \to \infty} s_n \).

2. Geometric series with \(-1 < r < 1\).

3. Telescopic series help find \( \lim_{n \to \infty} s_n \).

4. \( p \)-series with \( p > 1 \).

5. Integral test: if \( \int_1^\infty f(x) \, dx \) converges, where \( f(x) \) is the corresponding function.

6. Comparison test: show that the series is less than a series that converges.

7. Limit comparison test: show that the limit of the quotient of the series and a series that converges is a finite nonzero number.

8. Alternating series test: show that the series is decreasing and its limit is 0.

9. Any of the tests above could be used to find absolute convergence by using the test to show that \( \sum |a_n| \) converges. If the series has only positive terms, then it is abs. conv.

10. Alternating harmonic series \( \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \) converges conditionally.

11. Ratio test: \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1 \), gives abs. conv. and so conv.

12. Root test: \( \lim_{n \to \infty} \sqrt[n]{|a_n|} < 1 \), gives abs. conv. and so conv.

Inconclusive tests:

1. Divergence test: if \( \lim_{n \to \infty} a_n = 0 \).

2. Limit comparison test: if the limit of the quotient of the two series is zero or infinity.

3. Ratio test: \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1 \).

4. Root test: \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = 1 \).