7.4 Partial Fractions

1. partial fractions help find the derivative of a ratio of polynomials by factoring the denominator

2. try to break down the main fraction $\frac{R(x)}{Q(x)}$ as sums of other fractions for which we know the antiderivative: generally a constant over a polynomial of degree 1 is the best choice, and the answer is the natural log of the denominator (with the appropriate constants).

3. let the integrand be the function $\frac{R(x)}{Q(x)}$. If $Q(x)$ has factors that are raised to the power, then the numerator of the new fractions should have the degree one less than the degree of the denominator before being raised to the power.

4. rationalizing substitution: if the fraction is not rational (i.e. not a fraction of polynomials, but rather there is some square root in the function) then use substitution to get both the numerator and denominator polynomials. Letting $u$ be the square root will usually do the trick.
7.8 Improper Integrals

1. they are the integrals of functions that go to $\pm\infty$

2. integrals of type 1: integrals over infinite intervals: use limits to $\pm\infty$ to evaluate it.

3. integrals of type 2: integrand is discontinuous or not defined at some value in the interval: break down the integral as the sum of two or more integrals, and use limits to the point of discontinuity to evaluate it.

4. convergent integral: if the corresponding limit of the integral exists

5. divergent integral: if the corresponding limit of the integral does not exist

6. $\int_{1}^{\infty} \frac{1}{x^p}$
   - converges if $p > 1$
   - diverges if $p \leq 1$

7. comparison theorem helps to estimate whether an integral (that cannot be evaluated) converges or not: knowing that the integral $I_1$ converges or diverges, we find out if integral $I_2$ converges or diverges.