6.4 Work

1. total amount of effort required to perform a task.

2. notice the difference when given mass versus weight (multiply by $g = 9.8 \text{m/s}^2$)

3. Newton’s second law of motion: $F = m \cdot a$ (where $m$ is the mass of the object, and $a$ is the acceleration), or mass times the second derivative of the distance

4. if the force is constant, then $W = F \cdot d$ (where $F$ is the force that acts on the object, and $d$ is the displacement), or $W = \int F(x) \, dx$, where $F(x)$ is the constant force.

5. if force is not constant and it given as a function of $x$, say $f(x)$, then $W = \int f(x) \, dx$

6. Hooke’s law: $F = k \cdot d$, where $k$ is Hooke’s constant that depends on each spring, and $d$ is the displacement. Thus $W = \int k \cdot d \, dx$, where $d$ must be expressed in meters (since the unit for force is 1J = 1N \cdot 1m), or in feet (since the unit for work is ft-lb), but not in cm or inches.

7. gravitational force: $G = m \cdot g$, where $g = 9.8 \text{m/s}^2$ is the gravitational acceleration, and $m = \rho \cdot V$ is the mass of the object as the product of density and the unit volume.

6.5 Average Value of a Function

1. the average value function is the function that will give you the average value for each respective function:

$$f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx.$$ 

This makes sense since you can think of $\int_a^b f(x) \, dx = f_{\text{ave}} \cdot (b-a)$

2. MVT for Integrals gives a value $c$ at which $f(c)$ is the average value of $f$ over the interval: If $f$ is continuous on $[a,b]$, then there is

$$c \in [a,b] \text{ such that } f(c) = f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx$$

3. compare MVT for integrals with MVT for derivatives