11.5 Alternating Series

1. an alternating series: the terms of the sequence alternate between positive and negative numbers

2. alternating series test: Let \( \sum_{n=1}^{\infty} (-1)^{n-1}b_n \) be an alternating series. If \( b_{n+1} \leq b_n \) for all \( n \) and \( \lim_{n \to \infty} b_n = 0 \), then the series converges.

3. alternating harmonic series \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \) converges (even though the harmonic series does not converge)

11.6 Absolute Convergence, Ratio and Root Test

1. absolute versus conditionally convergence:
   - a series \( \sum a_n \) converges absolutely if \( \sum |a_n| \) converges
   - a series \( \sum a_n \) converges conditionally if it converges but it doesn’t converge absolutely

2. absolute convergent implies convergence, i.e. if a series converges absolutely then it converges

3. a convergent series whose terms are positive will be absolute convergent

4. ratio test (mostly used for factorials, polynomials, power function \( a_n \)):
   - if \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1 \), then the series \( \sum_{n=1}^{\infty} a_n \) is absolutely convergent (and so also convergent)
   - if \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1 \), then the series \( \sum_{n=1}^{\infty} a_n \) is divergent
   - if \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L = 1 \), then the test is inconclusive, so try something else

5. root test (used for polynomials to power \( n \))
   - if \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = L < 1 \), then the series \( \sum_{n=1}^{\infty} a_n \) is absolutely convergent (and so convergent)
   - if \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = L > 1 \), then the series \( \sum_{n=1}^{\infty} a_n \) is divergent
   - if \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = L = 1 \), then the test is inconclusive, so try something else