2 Chapter 2: Determinants

2.1 The Determinant of a Matrix

1. each square matrix has a real number associated with it, namely its determinant, which tells if a matrix has an inverse (i.e. matrix is nonsingular) or not (i.e. matrix is singular)

2. A is nonsingular iff det $A \neq 0$

3. determinant of an $n \times n$ matrix (p.95):
   
   (a) COFACTOR METHOD: Finds the determinant of a matrix by expanding along a row or along a column (see Example 1 page 94). Expansion does not have to be done along the 1st row, but it is commonly done this way.
   
   (b) $A$ is a triangular matrix (either upper or lower) $\Rightarrow$ det($A$) = $a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}$ (i.e. the product of the entries of the diagonal)
   
   (c) if $A$ is a $2 \times 2$ matrix, then det($A$) = $a_{11}a_{22} - a_{12}a_{21}$.
   
   (d) a row/column of a matrix $A$ is all zero $\Rightarrow$ det($A$) = 0
   
   (e) two rows/columns of a matrix $A$ are identical $\Rightarrow$ det($A$) = 0

4. minor (as a matrix): $M_{ij}$

5. cofactor (as a determinant): $A_{ij} = (-1)^{i+j}$ det($M_{ij}$)

6. det($A^T$) = det($A$)