1 Chapter 1: Matrices and Systems of Equations

1.1 Systems of Linear Equations

1. systems of linear equations and their geometric interpretation as the intersection of lines

2. inconsistent system: system has no solutions

3. consistent system: at least one solution

4. equivalent systems: if they have the same set of solutions

5. diagonal entries: the numbers on the diagonal of a matrix $a_{11}, a_{22}, \ldots, a_{n,n}$

6. strict triangular form of a system: the diagonal entries are nonzero, and the entries below the diagonal are all zero

7. coefficient matrix (or simply the matrix) of a system = the coefficients of the variables in system of equations

8. augmented matrix = the coefficient matrix with the additional column of the entries on the right hand side of the system of equations.

9. elementary row operations = matrix operations that are used to solve the original system of equations. They are: Interchanging two rows, multiplying a row by a nonzero constant, and replacing a row by its sum with a multiple of another row.

10. pivotal row

11. Systems of linear equations can be solved by:

Method 1 back substitution p.6

Method 2 using the elementary row operations on the augmented matrix to reduce it to a strict triangular form p.8–9. The elementary row operations do not alter the solution of the system. This method fails if the pivot ends up being 0 at some point, producing an inconsistent system of equations. See new methods in next section.
1.2 Row Echelon Form

1. lead variables
2. free variables
3. row echelon form of a matrix p.15:
   - the first nonzero entry in each row is 1
   - if row $k$ does not consist entirely of zeros, the number of leading zero entries in row $k + 1$ is greater than the number of leading zero entries in row $k$
   - if there are rows that are entirely zeros, they are below the nonzero ones
4. What’s the difference between the strict triangular form and the row echelon form? The strict triangular form must have a nonezero entry in $a_{ii}$, for all $1 \leq i \leq n$
5. Gaussian elimination – Method 3 for solving a system of equations using row operations to put a matrix into row echelon form: Start with the augmented matrix, and reduce it to REF (Row Echelon Form)
6. reduced row echelon form of a matrix p.18: it is an REF matrix whose first nonzero entries in each row are the only nonzero entries in the particular column
7. Gauss-Jordan reduction – Method 4 for solving (mostly underdetermined) systems of equations using row operations to put a matrix into (RREF) Reduced Row Echelon Form.
8. underdetermined systems: fewer nonequivalent equations than unknowns
9. overdetermined systems: more nonequivalent equations than unknowns
10. homogeneous systems = systems of equations whose right hand side is zero. Homogeneous systems are always consistent (i.e. they will always have solutions):
   
   (a) If there are the same number of variables as equations, the system has a unique solution: $x_1 = 0, x_2 = 0, \ldots, x_n = 0$. 
   
   (b) If there are more variables than equations, then there are infinitely many solutions because of the free variables.