Appendix H: Complex Numbers

1. complex numbers $z = a + bi$, where $a, b \in \mathbb{R}$, with $a$ being the real part, and $b$ being the imaginary part of the complex numbers

2. recall that $i^2 = -1$ or $\sqrt{-1} = i$

3. add, subtract, divide and multiply complex numbers

4. for a complex number $a + bi$, we define its conjugate to be $a - bi$

5. the polar form of a complex number $z$ is $r(\cos \theta + i \sin \theta)$, where $r = |z| = \sqrt{a^2 + b^2}$ and $\tan \theta = \frac{b}{a}$

b being the imaginary part of the complex numbers

6. recall that $i^2 = -1$ or $\sqrt{-1} = i$

7. add, subtract, divide and multiply complex numbers in polar form

8. also, the power of a complex number in polar form is given by DeMoivre’s Thm:

$$z^n = r^n(\cos(n\theta) + i \sin(n\theta))$$

9. the $n^{th}$ roots of a complex number

$$w_k = r^{1/n} \left[ \cos \left( \frac{\theta + 2k\pi}{n} \right) + i \sin \left( \frac{\theta + 2k\pi}{n} \right) \right],$$

where $k = 0, 1, 2, \ldots, n - 1$

10. using Taylor’s polynomials for $e^x$, $\cos x$ and $\sin x$, one can prove Euler’s Formula:

$$e^{iy} = \cos y + i \sin y$$