1. Suppose $|A| = 6$ and $|B| = 5$. Find the number of functions $f : A \rightarrow B$.

Solution: Since each element of $A$ can be mapped to one of the 4 choices of elements of $B$, we have that the number of functions is $5^6 = 15,625$.

2. Suppose $|A| = 10$ and $|B| = 4$. Find the number of $1 \rightarrow 1$ functions $f : A \rightarrow B$.

Solution: To be a function, each element needs to get mapped to an element of $B$. Then some elements will get mapped to the same element, and so the function is not one-to-one. Thus the answer is 0 since there are more elements to map from $A$ than there are in $B$.

3. Using the ordinary alphabet and allowing repeated letters, find the number of words of length 8 that begin with L or end with R.

Solution: The number of words that start with L is $26^7$, and the number of elements that end with R is $26^7$, of which $26^6$ start with L and end with R. Thus there are $26^7 + 26^7 - 26^6 = 15,754,704,576$ words.

4. Find the number of subsets of $S = \{1, 2, 3, \ldots, 10\}$ that contain at least two elements.

Solution: From the total number of subsets $2^{10}$ we subtract the number of subsets that have one or fewer elements: 1 set with zero elements and 10 more sets of one element each. We thus have $2^{10} - 11 = 1,013$ such subsets.

5. Show that in a group of 14 people, there must be two whose birthdays fall in the same month.

Proof: There are 12 months (pigeonholes) and 14 people (the pigeons). Since there are more pigeons than pigeonholes, it follows by the Pigeonhole Principle that at least two must share a common month for their birthdays.
6. Suppose a restaurant serves a "special dinner" consisting of soup, salad, entree, dessert, and beverage. The restaurant has five kinds of soup, three kinds of salad, ten entrees, five desserts, and four beverages. How many different special dinners are possible? (Two special dinners are different if they differ in at least one selection.)

Solution: By the multiplication principle we have that there are $5 \cdot 3 \cdot 10 \cdot 5 \cdot 4 = 3,000$ special dinners.

7. How many 4-character long passwords of lower case letters and decimal digits are there, that begin with a vowel or end with a vowel? (The middle two characters can be any lower case letter or digit)

Solution: The number of passwords that start with a vowel is $5 \cdot 36^3$. The number of passwords that end with a vowel is $5 \cdot 36^3$. The number of passwords that start and end with a vowel is $5^2 \cdot 36^2$. By the exclusion-inclusion principle, the total number is $5 \cdot 36^3 + 5 \cdot 36^3 - 5 \cdot 36^2 \cdot 5 = 434,160$.

8. Consider all bit strings of length 12. How many begin with 11 and end with 10?

Solution: The number of bit string that begin with 11 and end with 10 is $1 \cdot 1 \cdot 2^8 \cdot 1 \cdot 1 = 256$.

9. Consider all bit strings of length 9. How many begin with 11 or end with 10?

Solution: The number of bit strings that begin with 11 is $2^7$. The number of bit strings that end with 10 is $2^7$. The number of bit strings that begin with 11 and end with 10 is $2^5$. By the inclusion-exclusion principle we have a total of $1 \cdot 1 \cdot 2^7 + 2^7 \cdot 1 \cdot 1 - 2^5 = 224$ bit strings.

10. A computer is programmed to print subsets of $\{1, 2, 3, 4, 5\}$ at random. If the computer prints 40 subsets, prove that some subset must have been printed at least twice.

Proof: There are $2^5 = 32$ subsets. Since 40 subsets are printed it follows that at least one will have been printed twice by the pigeonhole principle.