4. Solutions of Problems on Fourier Analysis of Continuous Time Signals: Unit 1

4.1 Expansion of Periodic Signals by Complex Exponentials: the Fourier Series

Problem 4.1.1

Determine the Fourier Series coefficients for the following periodic signals:

a) \( x(t) = 10 \cos (100 \pi t + 0.1 \pi); \)

In this case we can determine the expansion simply by using Euler’s formulas:

\[ x(t) = 5 e^{j0.1 \pi} e^{j100\pi t} + 5 e^{-j0.1 \pi} e^{-j100\pi t} \]
which in itself is a Fourier Series. In fact we can just rearrange it as

\[ x(t) = (5e^{j0.1\pi})e^{j2\pi F_0 t} + (5e^{-j0.1\pi})e^{-j2\pi F_0 t} \]

with \( F_0 = 50 \text{ Hz} \). Therefore the period is \( T_0 = 1/50 \text{ sec} = 20 \text{ msec} \) and the Fourier Series coefficients are \( a_{\pm 1} = 5e^{j0.1\pi} \), and \( a_k = 0 \) for all \( k \neq \pm 1 \);

b) \( x(t) = 10 | \cos (100 \pi t) | \);

As we can see from the plot, the period is \( T_0 = 0.01 \text{ sec} = 100 \text{ msec} \), and the Fourier Series coefficients have to be obtained by integration, as

\[ a[k] = \frac{e^{-jk\pi}(-1.51365-(0.983632j)k)}{-0.25+k^2} \]

c) \( x(t) \) as shown:
As we can see from the plot, the period is \( T_0 = 4 \text{ sec} \), and the Fourier Series coefficients have to be obtained by integration, as

\[
a_k = \frac{196.726 j}{-40000 + k^2}
\]

d) \( x(t) \) as shown:

As we can see from the plot, the period is \( T_0 = 4 \text{ sec} \), and the Fourier Series coefficients have to be obtained by integration, as

\[
a_k = \frac{-0.101321 e^{-ikn} (0.101321 + 0.159155 i k) + 0.159155 i k}{k^2}
\]

e) \( x(t) \) as shown:
As we can see from the plot, the period is $T_0 = 4 \text{ sec}$, and the Fourier Series coefficients have to be obtained by integration, as

$$a_k = \frac{e^{-i k \pi} \left( (0. - 0.1591 i) + e^{i k \pi} \left( (0. + 0.3183 i) - (0. + 0.159155 i) e^{i k \pi} \right) \right)}{k}$$