B2. Spectral Estimation by the DFT

Problems

Problem 1. A continuous time signal $x(t) = A \cos(2\pi F_0 t)$ is sampled at a frequency $F_S$ and we take the FFT of $N$ samples. Call $k_i$ the indices where the FFT has the largest magnitude. For each one of the cases below:
Q1: determine the indices $k_i$ where you expect to have the maximum values;
Q2: the approximate value of the peaks.

a) $A = 1.7$, $F_0 = 1.0kHz$, $F_S = 3.0kHz$, $N = 256$;
b) $A = 3.2$, $F_0 = 375.0Hz$, $F_S = 3.0kHz$, $N = 256$;
c) $A = 2.5$, $F_0 = 1.0kHz$, $F_S = 1.6kHz$, $N = 256$;
d) $A = 1.2$, $F_0 = 1.0kHz$, $F_S = 3.0kHz$, $N = 128$;

Problem 2. You have two sinusoids with frequencies $F_1$, $F_2$ Hz, data length $T_0$ sec and sampling frequency $F_S$ Hz. Assume the two sinusoids to have similar power. For each one of the cases below:
Q1: determine whether you can resolve the two frequencies;
Q2: if you can, at which indices the DFT has peaks;
Q3: if you cannot, how would you change the data length and/or the sampling frequency so that you can see both frequencies.

a) $F_1 = 800Hz$, $F_2 = 1000Hz$, $F_S = 4,000Hz$, $T_0 = 10m sec$
b) $F_1 = 800Hz$, $F_2 = 1000Hz$, $F_S = 1,500Hz$, $T_0 = 10m sec$
c) $F_1 = 800Hz$, $F_2 = 1000Hz$, $F_S = 4,000Hz$, $T_0 = 2m sec$
d) $F_1 = 800Hz$, $F_2 = 1000Hz$, $F_S = 40,000Hz$, $T_0 = 2m sec$
e) $F_1 = 800Hz$, $F_2 = 1000Hz$, $F_S = 4.0MHz$, $T_0 = 2m sec$
Solutions.

Problem 1: [video](http://faculty.nps.edu/rcristi/eo3404/b-discrete-fourier-transform/problems/problem2-1.html)

Problem 2: [video](http://faculty.nps.edu/rcristi/eo3404/b-discrete-fourier-transform/problems/problem2-2.html)