B2. Spectral Estimation by the DFT

Problems

Problem 1. A continuous time signal \( x(t) = A \cos(2 \pi F_0 t) \) is sampled at a frequency \( F_s \) and we take the FFT of \( N \) samples. Call \( k_i \) the indeces where the FFT has the largest magnitude. For each one of the cases below:

Q1: determine the indeces \( k_i \) where you expect to have the maximum values;
Q2: the approximate value of the peaks.

a) \( A = 1.7, F_0 = 1.0k\text{Hz}, F_s = 3.0k\text{Hz}, N = 256; \)
b) \( A = 3.2, F_0 = 375.0\text{Hz}, F_s = 3.0k\text{Hz}, N = 256; \)
c) \( A = 2.5, F_0 = 1.0k\text{Hz}, F_s = 1.6k\text{Hz}, N = 256; \)
d) \( A = 1.2, F_0 = 1.0k\text{Hz}, F_s = 3.0k\text{Hz}, N = 128; \)

Problem 2. You have two sinusoids with frequencies \( F_1, F_2 \text{ Hz} \), data length \( T_0 \text{ sec} \) and sampling frequency \( F_s \text{ Hz} \). Assume the two sinuoids to have similar power. For each one of the cases below:

Q1: determine whether you can resolve the two frequencies;
Q2: if you can, at which indeces the DFT has peaks;
Q3: if you cannot, how would you change the data length and/or the sampling frequency so that you can see both frequencies.

a) \( F_1 = 800\text{Hz}, F_2 = 1000\text{Hz}, F_s = 4,000\text{Hz}, T_0 = 10\text{m sec} \)
b) \( F_1 = 800\text{Hz}, F_2 = 1000\text{Hz}, F_s = 1,500\text{Hz}, T_0 = 10\text{m sec} \)
c) \( F_1 = 800\text{Hz}, F_2 = 1000\text{Hz}, F_s = 4,000\text{Hz}, T_0 = 2\text{m sec} \)
d) \( F_1 = 800\text{Hz}, F_2 = 1000\text{Hz}, F_s = 40,000\text{Hz}, T_0 = 2\text{m sec} \)
e) \( F_1 = 800\text{Hz}, F_2 = 1000\text{Hz}, F_s = 4.0\text{MHz}, T_0 = 2\text{m sec} \)
Solutions.

Problem 1: video

http://faculty.nps.edu/rcristi/eo3404/b-discrete-fourier-transform/problems/problem2-1.html
http://faculty.nps.edu/rcristi/eo3404/b-discrete-fourier-transform/problems/b-problem2-1.mp4

Problem 2: video

http://faculty.nps.edu/rcristi/eo3404/b-discrete-fourier-transform/problems/problem2-2.html
http://faculty.nps.edu/rcristi/eo3404/b-discrete-fourier-transform/problems/b-problem2-2.mp4