Performance Limits of Fair-Access in Underwater Sensor Networks

Yang Xiao and Miao Peng
Dept. of Computer Science
The University of Alabama
Tuscaloosa, AL 35487, USA
yangxiao@ieee.org,
mpeng@crimson.ua.edu

John Gibson, Geoffrey G. Xie
Dept. of Computer Science
Naval Postgraduate School
Monterey, CA 93943 USA
jhgibson@nps.edu, xie@nps.edu

Ding-Zhu Du
Dept. of Computer Science
University of Texas at Dallas
Richardson, TX 75083 USA
dzdu@utdallas.edu

***Abstract*—This paper investigates fundamental performance limits of medium access control (MAC) protocols for particular underwater multi-hop sensor networks under a fair-access criterion requiring that sensors have an equal rate of underwater frame delivery to a base station. Tight upper bounds on network utilization and tight lower bounds on minimum time between samples are derived for fixed linear topology. The paper also examines the implication of the end-to-end performance bounds regarding the traffic rate and sensing time interval of individual sensors.**

I. INTRODUCTION

Fundamental performance limitations must be well understood when establishing a network protocol to ensure the protocol is appropriate for a particular network design choice. The underwater acoustic sensor networks (UASNs) considered in this paper are multi-hop. Each network node performs sensing, transmission, and relay. All data frames are destined to a dedicated data-collection node, called the base station (BS), that is responsible for relaying the frames to a dislocated command center over a radio or wired link.

For this study we consider a regular topology, the linear or string network, designed by researchers from UC Santa Barbara for moored oceanographic applications [1], in which an array of equally spaced underwater marine sensors are suspended from a mooring buoy. All data in the network flows to a base station above water which is responsible for storing and relaying all collected data to a command center over an aerial radio link. During an event of interest, e.g., a storm, it is desirable that the command center acquires near real-time readings from all the sensors in order to calibrate them as the event progresses [1]. An equally appropriate employment would include a collection of seismic sensors, perhaps a long grid topology, along a potential tsunami path that would monitor the movement of the wave phenomena for a relatively short distance and relay the collected data samples through the base station to an observatory station as the radio signal would travel nearly 200,000 times faster than the acoustic signal. For such real-world applicable networks, we observe that it is critical for the MAC protocol to ensure each sensor has an equitable opportunity to forward its local observations to the command system in order to establish trends or detect anomalies.

In this paper, we adopted a notion of fairness from our previous work in [5] for sensor data delivery to this environment and to support application of a fair-access criterion to MAC protocols under consideration for use in UASNs. In our previous work, the studies focus on land (non-acoustic) sensor networks. This paper derives tight bounds on the network utilization and frame latency performance of fair-access MAC protocols for linear topologies in underwater sensor networks. This paper addresses the impact of non-trivial propagation delay, a definitive character of underwater acoustic networks. Tight upper bounds on network utilization and tight lower bounds on minimum time between samples are derived for a nominal fixed linear topology. The significance of these bounds is two-fold: First, they are universal, i.e., they hold for any MAC protocol conforming to the fair-access criterion, such as contention-based protocols (e.g., Aloha or CSMA based) or contention-free protocols (TDMA, etc.). Second, they are provably tight, i.e., they can be achieved by a version of time division multiple access (TDMA) protocol that is self-clocking, and therefore does not require system-wide clock synchronization. The paper also examines the implication of the end-to-end performance bounds regarding the traffic rate and sensing time interval of individual sensors. The challenge of this work also lies in the fact that the propagation delay impact in underwater sensor networks is difficult to model due to the time varying nature of the environment.

The existence of a computationally tractable optimal fair-access protocol is interesting since it has been shown that the general problem of optimal scheduling for a multi-hop network is NP-complete [2]. It may be because we consider only a particular topology where the routing structure is simple. The data forwarding paths of a linear or grid network can be modeled as a tree. While tree-based scheduling may be too restrictive for arbitrary ad hoc networks [3], such an approach seems appropriate for networks where all traffic must flow to a collective base station, essentially forming a root node. The flow of traffic along the branches of the tree must be de-conflicted with the flow of traffic along other branches so that collisions or interference between branches is eliminated or minimized. Individual node transmission windows may be adaptive [4] or static as described herein. While a multi-hop star topology may be of particular interest, a linear one is directly applicable to buoyed networks. Further, if the branches of the star are non-interfering, then it is the final hop of the star by which each branch connects to the base station that must be carefully controlled to limit collisions. In particular, if the one-hop neighbors of the base station form a natural ring structure a simple token passing scheme, perhaps out-of-band from the data streams, may provide sufficient control to mitigate access issues for multiple "strings" sharing a common base station.

We also examined the implication of the end-to-end performance bounds on the traffic generation rate and sensing interval of individual sensors. This paper presents an analysis that confirms the maximum feasible offered load by each sensor node is inversely proportional to the size of the network, which implies that multiple smaller networks may be inherently preferable to fewer larger networks.

In short, the specific contributions of this paper include a consideration of the fair-access concept as it applies to UASNs, a formal analysis of utilization and delay performance of specific linear UASNs that require fair-access, a scheduling algorithm to achieve the optimal utilization, and theoretical limits on the sustainable traffic load per sensor node for these particular sensor networks.

II. BACKGROUND

Similar to our previous work [5], a sensor network is defined as follows. Consider a wireless sensor network including a base station (BS) and n sensor nodes, denoted as $O_i, i = 1,2,...,n$. Sensor nodes generate sensor data frames and send them to the BS. Some sensor nodes perform an additional role of forwarding/routing frames to the BS, i.e., a frame may need to be relayed by several nodes to reach the BS.

From [5], let $U(n)$ denote the utilization of the above network, i.e., the fraction of time that the BS is busy with receiving correct data frames. Let G_i denote the contribution of (i.e., data generated by) sensor O_i to the total utilization. The following holds: $U(n) = \sum_{i=1}^{n} G_i$. However, in [5], prorogation delay is assumed to be negligible, but in this paper we cannot assume so as the propagation delay in acoustic networks is considerably more than in wired or wireless networks of the same physical expanse. Implicit in the utilization is the impact of propagation delays. As noted, these delays can be significant for UASNs, especially when compared to more traditional RF-based wireless networks.

Suppose that the network *is required* to use a MAC protocol that ensures all hosts are provided the capability to contribute equally to the composite throughput. From [5], the fair criterion is presented as follows.

Fair-access Criterion Definition: *A MAC protocol used by the sensor network satisfies the fair-access criterion if all sensor nodes contribute equally to the network utilization, i.e., the following condition holds:*

$$G_1 = G_2 = ... = G_n . \quad (1)$$

Optimization Objective and Assumptions: Consider a sensor network such as described above. The optimization problem is to maximize $U(n)$ under the fair-access criterion [5]. In the remainder of the paper, we investigate this problem under the following assumptions:

a. All data frames are of the same size.
b. All sensor nodes have the same transmission capacity.
c. Acknowledgments are either implicit via piggyback or, if explicit, are out-of-band.
d. In-network sensor data processing is not used.
e. If two sensor nodes are within one-hop, one sensor node's transmission will interfere with the other's reception
f. Internal node processing delays, associated with frame storage and queuing within a node, are negligible.

Linear Topology [5]: The topology is illustrated in Fig.1. There are n sensor nodes and a BS placed in a linear fashion. Assume that the transmission range of each node is just one hop and the interference range is less than two hops. In other words, only neighboring nodes have overlapping transmission ranges. As shown in Fig.1, O_i generates sensor data frames and sends the frames to O_{i+1}. O_i also relays data frames received from O_{i-1} to O_{i+1}. Finally, O_n forwards data to the BS, which collects all the data frames.

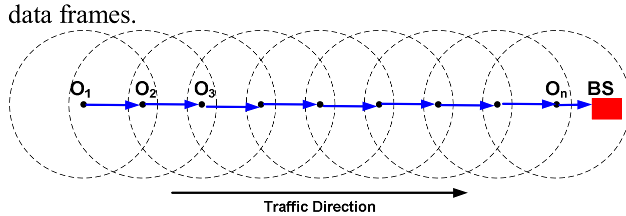

Figure 1. A linear topology

In our previous work we derived upper bounds on $U(n)$ and lower bounds on the effective inter-transmission delay of a node, that is, the time between samples for a given node, for a linear topology. This bound is reiterated here.

Theorem 1: For the linear topology under traditional RF-based wireless networks, under fair-access, $U(n)$ is upper bounded by the optimal utilization, $U_{opt}(n)$:

$$U(n) \le U_{opt}(n) = \begin{cases} n/[3(n-1)], & n > 1 \\ 1, & n = 1 \end{cases} \quad (2)$$

An asymptotic lower limit for the optimal utilization exists and is $\frac{1}{3}$.

Moreover, the inter-sample time for each node, denoted by $D(n)$ *, is lower bounded by the minimum effective transmission delay for a node, or minimum cycle time,* $D_{opt}(n)$:

$$D(n) \ge D_{opt}(n) = \begin{cases} 3(n-1)T, & n>1 \\ T, & n=1 \end{cases} \quad (3)$$

where T is the transmission time of one data frame.

In [5] we proved that the performance bounds introduced in Theorem 1 are indeed achievable under traditional RF-based wireless networks. Particularly, we presented a TDMA scheduling algorithm that conforms to the fair-access criterion and showed it achieves the performance bounds. Note that herein the optimal utilization is under the constraint of the fair-access criterion. Otherwise, by simply allowing *only* O_n to transmit, the optimal utilization is 1. The TDMA algorithm provided in [5], which we term *optimal fair scheduling,* is described below.

Optimal Fair Scheduling for Linear Topology: the cases of $n = 1, 2, or 3$, respectively, are simple and omitted for sake of brevity. For the general case of $n > 3$, let $d = D_{opt} = 3(n-1)$. A schedule with cycle d can be created as follows. O_1 transmits in timeslots $(d \cdot j)+1; j = 0,1,...$; O_i $(i = 2,....n)$ transmits relayed frames to O_{i+1} in timeslots from $(d \cdot j)+f(i)$ through $(d \cdot j)+f(i)+i-2$, and transmits one of its own frames to O_{i+1} in timeslot $(d \cdot j)+f(i)+i-1; j = 0,1,...$, where $f(i)$ is recursively defined as follows:

$$f(i) = \begin{cases} 1, & i=1 \\ f(i-1)+(i-1), & i>1 \end{cases} \quad (4)$$

Note that if we allow self-clocking among sensors by listening to the wireless media, the above TDMA scheme can be implemented easily without requiring system-wide clock synchronization.

In [5], we also addressed the impact of end-to-end performance bounds on the traffic load limitation of each sensor. Let ρ denote the traffic load generated by each sensor node. For the network depicted in Fig. 1, since each node can transmit at most one original frame, which requires a period of T in every $3(n-1)T$ time period, then, we must have that $\rho \le T/x = 1/[3(n-1)]$ if $n > 2$. Furthermore, a data frame contains protocol overhead (typically control fields in a header and/or trailer). Thus, ρ must be adjusted to account for this overhead. Denote m to be the fraction of actual data bits in a frame. From [5], we have the following theorem.

Theorem 2:*, For the linear topology illustrated in Fig. 1* under traditional RF-based wireless networks*, under the fair-access criterion, the maximum feasible per node traffic load is*

$$\frac{m}{3(n-1)}, \text{ if } n > 2 \quad (5)$$

III. Underwater Sensor Network

Consider an underwater sensor network, where the transmission medium is the water column itself and the carrier is an acoustic signal. We derive upper bounds on $U(n)$ and lower bounds on the minimum transmission delay, or time between samples, for general linear topologies, under the fair-access criteria. We consider the impact of non-negligible propagation delay. We denote transmission time and propagation delay by T and τ, respectively. Let us give an intuitive analysis before the formal proof. The fair-access criterion requires that $G_1 = G_2 = ... = G_i = ... = G_n$ for the network. Let x denote the time period during which the BS successfully receives at least one original data frame from each sensor node in the network. It is clear that x is a random variable. If we can derive the minimum value of x, and if the minimum value of x is achieved, the maximum utilization is also achieved. During the time period x, the BS has busy time (denoted as b) in which it is receiving frames and idle time (denoted as y) while it is either blocked or waiting for its upstream neighbor to send. Thus, $x = b + y$. Note that x is the cycle time for the network under the fair-access criteria and determines the effective inter-transmission delay for a node for an ordering of relayed frames. If no frame is transmitted by O_n during a period, there must exist an idle period with the same length in BS. Therefore, for deriving the idle period in the BS, we just need to derive the period during which O_n could not transmit frames.

Theorem 3*: For the linear topology, under fair-access, $U(n)$ is upper bounded by the optimal utilization $U_{opt}(n)$ for all τ $(\tau \le T/2)$:*

$$U(n) \le U_{opt}(n) = \begin{cases} nT/[3(n-1)T - 2(n-2)\tau], & n>1 \\ 1, & n=1 \end{cases} \quad (6)$$

and the maximum utilization $U_{opt}(n)$ can be achieved by a special case. An asymptotic lower limit for the optimal utilization exists and is $1/(3-2\tau/T)$. The inter-sample time for each node, denoted by $D(n)$, is lower bounded by the minimum effective inter-transmission delay for a node, or the minimum cycle time, $D_{opt}(n)$:

$$D(n) \ge D_{opt}(n) = \begin{cases} 3(n-1)T - 2(n-2)\tau, & n>1 \\ T, & n=1 \end{cases} \quad (7)$$

Proof of Theorem 3: 1) For $n > 2$: During the time period x, the BS needs to receive at least n frames from O_n. Thus, O_n transmits at least n frames (including $n-1$ relayed frames and one of its generated frames). We have $b \ge nT$. Likewise, in order for O_n to receive $(n-1)$ frames from O_{n-1}, O_n needs to listen to at least $(n-1)$ frames, during which (there is τ time delay) the BS must be idle. Furthermore, when O_{n-2} transmits, O_n must not transmit during a corresponding period during which the frames arrive at O_{n-1}. This is because O_n's transmissions will interfere with the frame reception by O_{n-1} from O_{n-2}, since O_n and O_{n-2} are only two-hops apart. O_{n-2} needs to transmit at least $(n-2)$ frames to O_{n-1} during which O_n could not transmit. But here we note that the period during which O_n is blocked by receiving frames from O_{n-1} may overlap with the period during which O_n is blocked for reception by O_{n-1} from O_{n-2} due to the propagation delays. This analysis is illustrated in Fig. 2, in which, O_n receives frame A in $(t, t+T)$ and O_{n-2} transmits frame B in (t_1, t_1+T) so that O_n is blocked in (t_1, t_1+T) since they are within two-hops, assuming the propagation delay is the same between both node pairs. Thus, it is apparent that some overlap of the induced idle periods may occur without frame loss.

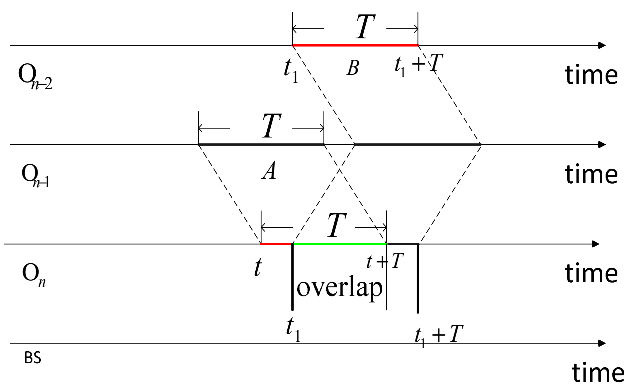

Figure 2. Overlapping period

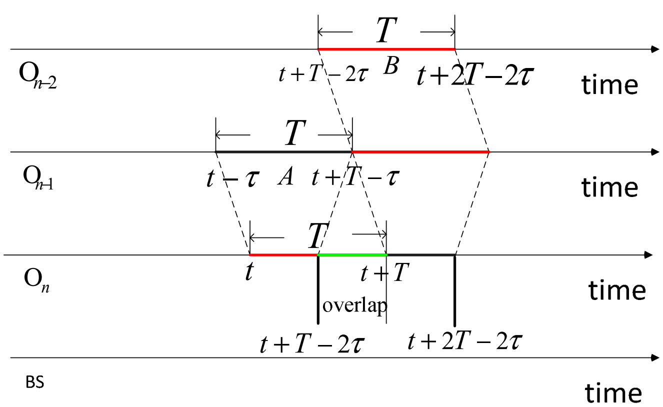

Figure 3. Maximal overlapping

Now, what we need to do is to maximize the overlapping period. When overlapping is maximum, the idle period generated independently by frame B is minimum. For maximizing the throughput of O_{n-1}, let O_{n-1} just complete transmission of frame A, then begin to receive frame B. This analysis is illustrated in Fig. 3, O_n receives frame A in $(t, t+T)$, that implies O_{n-1} transmitted frame A in $(t-\tau, t-\tau+T)$. Let O_{n-2} transmit frame B in $(t+T-2\tau, t+2T-2\tau)$ so that its first bit reaches O_{n-1} in $t+T-\tau$. From Fig. 3, it is easy to see that if $T-2\tau \ge 0$, for $\tau \le T/2$, the maximum overlapping period is $(t+T-2\tau, t+T)$.

Thus, the minimum time during which O_n may not transmit in order to prevent collision with frame B at O_{n-1} is

$$(t+2T-2\tau)-(t+T)=T-2\tau$$

So the total time when O_n must be idle, assuming that each frame is sent individually, is

$$y \ge (n-1)T+(n-2)(T-2\tau)$$

Therefore, we have

$$x=b+y \ge nT+(n-1)T+(n-2)(T-2\tau)$$

Since $D(n)=x$, we have derived equation (6) for the case of $n>2$. During the time period x, the BS may receive more than n frames, but only n frames can be counted into the utilization under the fair-access criterion. Since we must minimize x to achieve the optimal utilization, we have

$$U(n) \le \frac{nT}{nT+(n-1)T+(n-2)(T-2\tau)}=\frac{nT}{3(n-1)-2(n-2)\tau}$$

which proves equation (6) for the case of $n>2$.

2) For $n=2$: Since we want $G_1=G_2$, during the time period x, O_2 transmits at least two frames (one relayed frame and its own). We have $b \ge 2T$. O_2 needs to listen to at least one frame from O_1. We have $y \ge T$ and thus $x=b+y \ge 3T$. So we must minimize x to achieve the optimal utilization, $U(n)=\frac{2T}{x} \le \frac{2T}{3T}=\frac{2}{3}$, which proves equation (6) for this case. Note that the propagation delay can be ignored since it is possible to send the frame from O_1 such that it arrives at O_2 just as O_2 finishes its transmission of the previous frames.

3) For $n=1$: Obviously, $U(1) \le 1$.

We will prove that the performance bounds $U_{opt}(n)$ are indeed achievable in a special case next.

Note that herein the optimal utilization is under the constraint of the fair-access criterion when $\tau \le T/2$. We first give the algorithm for the optimal fair scheduling. Then we show the optimal fair scheduling for the cases of $n=3,5$ in Fig. 4 and Fig. 5, respectively. Before showing the algorithm, we need give some notations. Let A_i denote the frame generated by O_i, $1 \le i \le n$.

Algorithm for optimal Fair Scheduling for Linear Topology: First, we define a cycle. Let t_0 denote the time from which O_n begins transmission of its own frame A_n. Thus, the BS receives frame A_n at time $t_0+\tau$. As we mentioned above, x is the cycle time for the network under the fair-access criteria. Thus we define a cycle as $(t_0+\tau, t_0+\tau+x)$. Thus, the next cycle is $(t_0+\tau+x, t_0+\tau+2x)$.

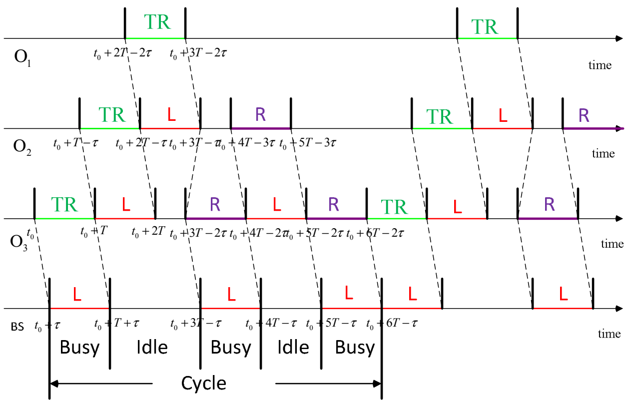

Figure 4. Bottom-up approach for Linear topology (n=3) [Legend: TR: transmit own traffic; R: relay traffic (note: actually relay latest received frame from upstream nodes); L: receiving

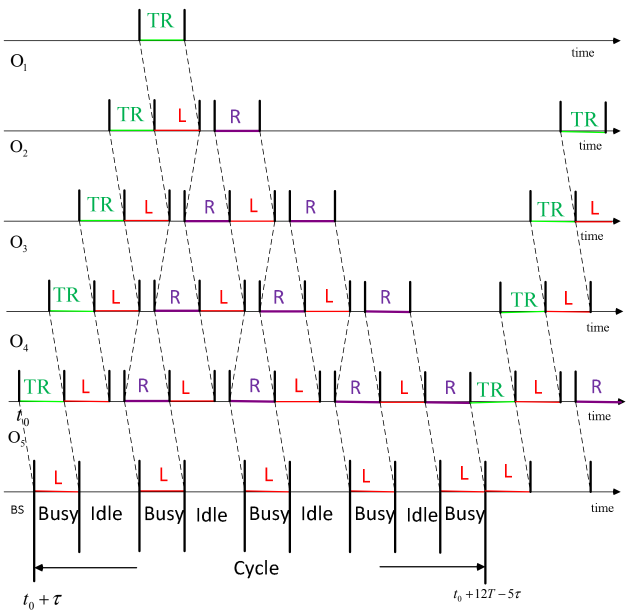

Figure 5. Bottom-up approach for Linear topology (n=5)

Second, any node O_i , $1 \le i \le n$ in the cycle $(t_0+\tau, t_0+\tau+x)$, has a start time (the time at which O_i starts to transmit its own frame, A_i) and an end time (the time at which O_i just completes A_i's transmission). We denote the start time and the end time by s_i and d_i , respectively. s_i and d_i are defined as follows:

$$S_i = \begin{cases} t_0+(n-i)T-(n-i)\tau & 1\le i<n \\ t_0 & i=n \end{cases}$$

$$d_i = \begin{cases} S_i+T+(i-1)(3T-2\tau) & 1\le i<n \\ t_0+x & i=n \end{cases}$$

where $x = 3(n-1)T - 2(n-2)\tau$.

Third, we defined (s_i, d_i) as an active period for node O_i , $1 \le i \le n$ in the cycle $(t_0+\tau, t_0+\tau+x)$. In period (s_i, d_i) , O_i includes a TR (transmit own traffic) period and $i-1$ subcycles. Their definitions are given as follows. $[S_i, S_i+T]$ denotes the TR period during which O_i transmits its own frame A_i . $[S_i+T, d_i]$ is divided into $i-1$ subcycles. We denote a subcycle by $[u_{i,j}, u_{i,j+1}]$, $j=1,\dots,i-1$ during which O_i receives and relays a frame from upstream nodes. Thus, we have

$$\begin{cases} u_{i,1} = S_i + T \\ u_{i,j} = (j-1)(3T-2\tau) + u_{i,1} & j = 2,\dots,i-1 \\ u_{i,i} = d_i \end{cases}$$

Finally, for any subcycle $[u_{i,j}, u_{i,j+1}]$, there are three phases. We give them as follows: In phase $[u_{i,j}, u_{i,j}+T]$, O_i receives a frame from O_{i-1} , $2 \le i \le n$. In phase $[u_{i,j}+T, M]$, O_i is idle (neither receiving a frame nor transmitting a frame), where

$$M = \begin{cases} u_{i,j}+T & i=n \text{ and } j=n-1 \\ u_{i,j}+T+T-2\tau & \text{others} \end{cases}$$

In phase $[M, u_{i,j+1}]$, where $u_{i,j+1} = M+T$, O_i relays a frame to O_{i+1} , $2 \le i \le n$. Note, when $i=n$, O_{n+1} represents the base station.

Two examples of this schedule are illustrated in Fig. 4 and Fig. 5. We show the case $n=3$ in Fig. 4, the cycle period is $6T-2\tau$ and the utilization of the BS is $3T/6T-2\tau$, which is consistent with Theorem 3. The theorem also holds for the case, $n=5$, as shown in Fig. 5, where the cycle period is $12T-6\tau$ and the utilization of the BS is $5T/(12T-6\tau)$. It is straight-forward to verify the case of n nodes, and is thus omitted. The performance bounds are indeed achievable in a special case under the algorithm above.

Theorem 4: *For the linear topology, under fair-access,* $U(n)$ *is upper bounded by* $\frac{nT}{nT+(n-1)T}$ *for all* τ $(\tau > T/2)$.

Proof of Theorem 4: 1) For $n>2$: During the time period x , the BS needs to receive at least n frames from O_n (including $n-1$ relayed frames and one of its generated frames). Thus, O_n transmits at least n frames. We have. $b \ge nT$. In order for O_n to receive $(n-1)$ frames from O_{n-1} , O_n needs to listen for at least $(n-1)$ frames, during which the BS must be idle. There is also a delay of τ from the beginning of each frame's transmission until the start of its reception. Thus, $y \ge (n-1)T$. Furthermore, when O_{n-2} transmits, O_n must not transmit during a period such that its traffic arrives at O_{n-1} while O_{n-1} is receiving the frame from O_{n-2} . As with Theorem 3, we note that the time in which O_n is blocked due to potential interference with reception at O_{n-1} or for its own reception of traffic from O_{n-1} may overlap. The first intuition of maximizing the overlapping period, and thus minimizing the blocked (idle) time is the same as the case when $\tau \le T/2$. Namely, we strive to maximize the throughput of O_{n-1}. Let O_{n-1} just complete transmission of frame A, then begin to receive frame B. This process is illustrated in Fig. 6.

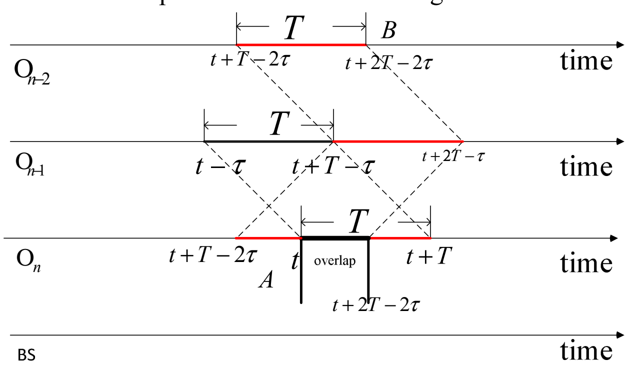

Figure 6. $\frac{T}{2} < \tau < T$

But from Fig. 6, it is clear to see that we can maximize the overlapping period further. Fig. 7 shows the potential optimal situation for maximizing overlapping periods. Since O_n 's transmission during $(t, t+T)$ will interfere with O_{n-1} 's reception of O_{n-2} 's transmission during $(t, t+T)$, the best case may be to let O_n receive a frame during the same period. But this potential optimal situation may (or may not) be achieved by all $(n-2)$ frames when they are sent from O_{n-2} to O_{n-1} under the constraint of the fair-access criterion. If this optimal situation is achieved by all $(n-2)$ frames from O_{n-2}, the minimum of the idle time generated by each frame independently is 0. Thus, we have $x \ge nT + (n-1)T$. So we have following inequality

$$U(n) \le \frac{nT}{nT + (n-1)T} = \frac{n}{2n-1}$$

2) For $n = 2$: Since we want $G_1 = G_2$, during the time period x, O_2 transmits at least two frames (one relayed frame and its own). We have $b \ge 2T$. O_2 needs to listen to at least one frame from O_1. We have $y \ge T$ and thus, $x = b + y \ge 3T$. So, minimizing x yields the optimal utilization, $U(n) = \frac{2T}{x} \le \frac{2T}{3T} = \frac{2}{3}$, which proves the inequality for this case.

3) For $n = 1$. Obviously, $U(1) \le 1$.#

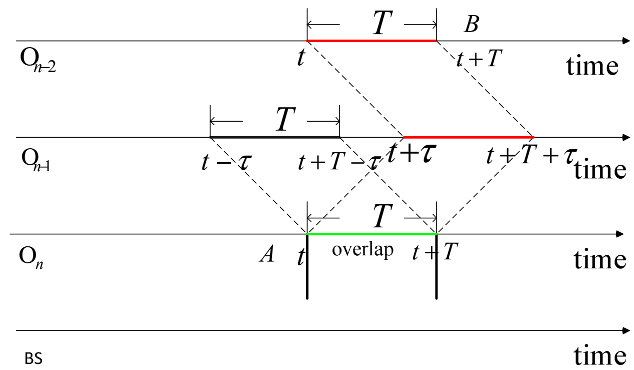

Figure 7. $\frac{T}{2} < \tau < T$

Next, we address the impact of end-to-end performance bounds on the traffic load limitation of each sensor. Let ρ denote the traffic load generated by each sensor node. We express the propagation delay τ, in normalized time units, as $\alpha = \tau/T$. For a linear network under the constraint of the criterion, since each node can transmit at most one original frame, which requires a period of T in every $3(n-1)T - 2(n-2)\tau$ time period, then, we must have that $\rho \le T/x = 1/[3(n-1) - 2(n-2)\alpha]$ where $0 \le \alpha \le 1/2$, if $n \ge 2$. Denote m to be the fraction of actual data bits in a frame. We have following theorem.

Theorem 5*: For the linear topology under the fair-access criterion, for all* τ $(\tau \le T/2)$, *the maximum feasible per node traffic load is*

$$\frac{m}{3(n-1) - 2(n-2)\alpha} \quad \text{if } n \ge 2$$

IV. PERFORMANCE EVALAUTION

In this section, we present some selected numeric results, derived from these theorems, for underwater sensor networks. The optimal utilizations have been multiplied by m, which is the fraction of actual data bits in a frame, to account for protocol overhead. Define $\alpha = \tau/T$ as propagation delay factor, the classic ratio of propagation delay to transmission delay.

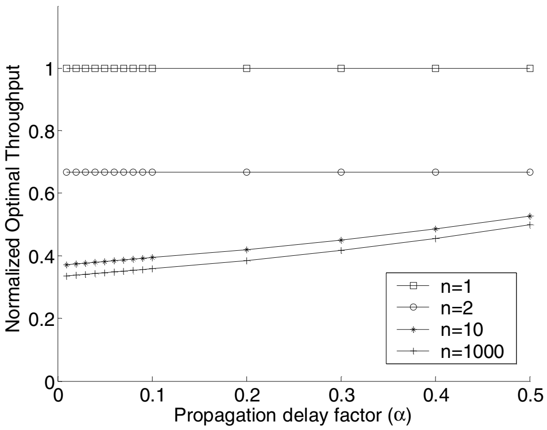

Figure 8. Optimal Utilization

Fig. 8 shows the optimal utilization vs. propagation delay factor α, for different n values (number of nodes) based on the bounds of Theorem 3, when $m=1$. We can see that $\alpha=0.5$, the throughput achieves the maximum in this range of α, for different n values. When n goes infinite, there is a limit $1/(3-2\alpha)$.

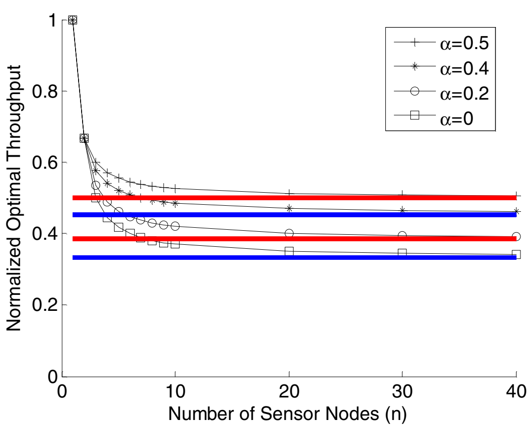

Figure 9. Optimal Utilization

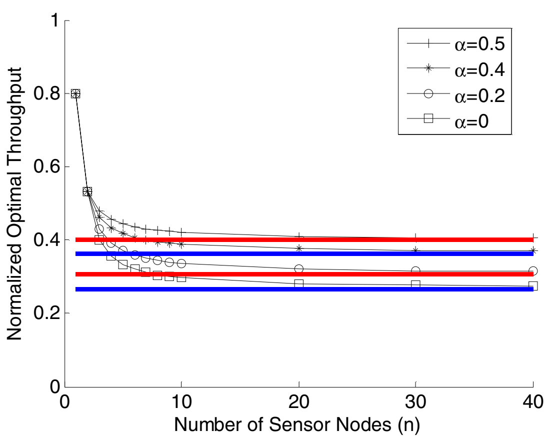

Figure 10. Optimal Utilization

Fig. 9 and Fig.10 show the optimal utilization vs. number of nodes when $m=1$ and $m=0.8$, respectively, for different α values based on the bounds of Theorem 3. The optimal utilization decreases quickly as n increases and approaches the asymptotic lower limit of optimal utilization, as suggested by the theorem. We also can see that when $\alpha=0.5$, the throughput achieves the maximum in this range of α.

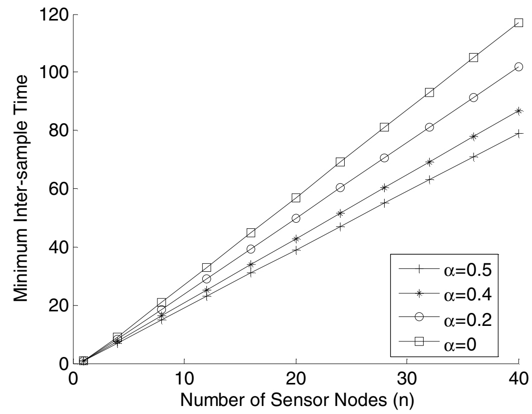

Figure 11. Minimum Cycle Time

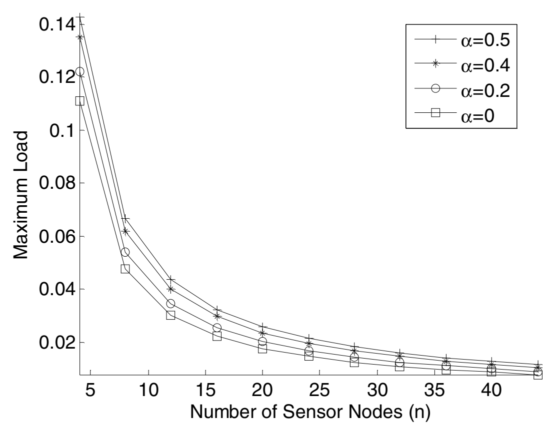

Figure 12. Maximum per Node Load

Fig. 11 shows that the effective transmission delay increases linearly with n for different value of α values. Fig. 12 shows that the traffic limit, per sensor node, decreases quickly as n increases for different values of α, approaching the asymptotic limit of zero.

V. CONCLUSION

In this paper, we explored fundamental limits for sustainable loads, utilization, and delays in specific multi-hop sensor network topologies for underwater sensor networks. We derived upper bounds on network utilization and lower bounds for minimum sample time for fixed linear topologies under the fair-access criterion. This fair-access criterion ensures the data of all sensors is equally capable of reaching the base station. We proved that under some conditions/assumptions, these bounds are achievable, and therefore optimal. From the limitation on the sustainable traffic loads derived, one can determine a lower bound for the sampling interval for such networks. The significance of these limits is that these bounds are independent of the selection of MAC protocols. Thus, the performance bounds for specific implementations of such network topologies can be explicitly determined to ensure the proposed networks are capable of satisfying the networks' specified utilization and delay requirements.

ACKNOWLEDGMENT

This work is supported in part by the US National Science Foundation (NSF) under the grant numbers CCF-0829827, CNS-0716211, and CNS-0737325.

REFERENCES

[1] B. Benson, G. Chang, D. Manov, B. Graham, and R. Kastner, "Design of a Low-cost Acoustic Modem for Moored Oceanographic Applications," Proc. of WUWNet'06, Sep. 25, 2006, Los Angeles, California,USA.

[2] R. Ramaswami and K. K. Parhi, "Distributed Scheduling of Broadcasts in a radio network," *Proc. of IEEE INFOCOM* 1989.

[3] S. Ramanathan "A unified framework and algorithm for channel assignment in wireless network" *Wireless Networks*, Volume 5, Number 2, 1999, pp. 81-94(14)

[4] V. Rajendran, K. Obraczka, and J.J. Garcia-Luna-Aceves: "Energy-efficient, collision-free medium access control for wireless sensor networks," in *Proc. of ACM Sensys 2003*, PP. 181–192, Nov. 2003.

[5] J.H. Gibson, G.G. Xie, and Y. Xiao, "Performance Limits of Fair-Access in Sensor Networks with Linear and Selected Grid Topologies," in Proc. GLOBECOM 2007, Nov. 2007.