In *proceedings of GLOBECOM Ad Hoc and Sensor Networking Symposium,* Washington DC, November 2007

Performance Limits of Fair-Access in Sensor Networks with Linear and Selected Grid Topologies

John Gibson* **Geoffrey G. Xie*** **Yang Xiao+**
*Computer Science Department, Naval Postgraduate School
+Department of Computer Science, The University of Alabama
{jhgibson, xie}@nps.edu yangxiao@ieee.org

Abstract—**This paper investigates fundamental performance limits of medium access control (MAC) protocols for multi-hop sensor networks. A unique aspect of this study is the modeling of a fair-access criterion requiring that sensors have an equal rate of frame delivery to the base station. Tight upper bounds on network utilization and tight lower bounds on minimum time between samples are derived for fixed linear and grid topologies. The significance of these bounds is two-fold: First, they are universal, i.e., they hold for any MAC protocol. Second, they are provably tight, i.e., they can be achieved by a version of time division multiple access (TDMA) protocol that is self-clocking, and therefore does not require system-wide clock synchronization. The paper also examines the implication of the end-to-end performance bounds regarding the traffic rate and sensing time interval of individual sensors.**

I. INTRODUCTION

It is important to study fundamental performance limitations of wireless sensor networks, as establishing performance bounds of a network protocol is necessary for determining whether the protocol is appropriate for a particular network design choice. An inappropriate protocol can result in a network which cannot sustain expected traffic loads. The wireless sensor network considered in this paper is multi-hop: each sensor node performs sensing, transmission, and relay. All data frames are destined to a dedicated data-collection node, called the base station.

In particular, we consider regular topologies like the linear network designed by researchers from UC Santa Barbara for moored oceanographic applications [1], in which an array of equally spaced underwater marine sensors are suspended from a mooring buoy. All data in the network flows to a base station above water which is responsible for storing and relaying all collected data to a command center over an aerial radio link. During an event of interest, e.g., a storm, it is desirable that the command center acquire near real-time readings from all the sensors in order to calibrate them as the event progresses [1]. For such real-world applicable networks, we observe that it is critical for the MAC protocol to ensure each sensor has an equitable opportunity to forward its local observations to the command system. In this paper, we introduce a notion of fairness for sensor data delivery based on this observation and formally define a *fair-access* criterion for MAC protocols.

Employing a fair-access MAC protocol, however, may have a *negative* impact on the network performance in terms of reduced throughput of data delivery to the base station and increased average frame latency. This paper analyzes such an impact by deriving tight bounds on the network utilization and frame latency performance of fair-access MAC protocols for linear and specific grid topologies. The bounds are significant since they hold for any MAC protocol conforming to the fair-access criterion, such as contention-based protocols (e.g., Aloha or CSMA based) or contention-free protocols (TDMA, etc.). We show that these bounds are tight by proving that they can be achieved by a particular TDMA scheduling algorithm. The existence of a computationally tractable optimal fair-access protocol is interesting since it has been shown that the general problem of optimal scheduling for a multi-hop network is NP-complete [2]. It may be because we consider only particular topologies where the routing structure is simple. As future work, we will investigate if optimal schedules exist for irregular topologies and various routing schemes under the fair-access constraint. While a star topology may be of particular interest, a linear one is directly applicable to buoyed networks.

The data forwarding paths of a linear or grid network can be simply modeled as a tree. While tree-based scheduling may be too restrictive for arbitrary ad hoc networks [3], such an approach seems appropriate for networks where all traffic must flow to a base station. The flow of traffic along the branches of the tree must be de-conflicted with the flow of traffic along other branches that are within the collision or interference range of a given node. The scheduling of transmission opportunities may be adaptive [4] or static as described herein.

We also examined the implication of the end-to-end performance bounds on the traffic generation rate and sensing interval of individual sensors. This paper presents an analysis that shows the maximum feasible offered load by each sensor node is inversely proportional to the size of the network.

In short, the specific contributions of this paper include a formulation of the fair-access concept, a formal analysis of utilization and delay performance of specific linear and grid sensor networks that require fair-access, a scheduling algorithm to achieve the optimal utilization, and theoretical limits on the sustainable traffic load per sensor node for these particular sensor networks.

II. PROBLEM FORMULATION

Sensor Network Definition: Consider a wireless sensor network including a base station (BS) and n sensor nodes, denoted as O_i ; $i=1,2,\ldots,n$. Sensor nodes generate sensor data frames and send them to the BS. Some sensor nodes perform an additional role of forwarding/routing frames to the BS, i.e., a frame may need to be relayed by several nodes to reach the BS.

Note that the above definition is not limited by a particular topology. Let $U(n)$ denote the utilization of the above network, i.e., the fraction of time that the BS is busy with receiving data frames. Let G_i denote the contribution of (i.e., data generated by) sensor O_i to the total utilization. The following holds: $U(n)=\sum_{i=1}^{n} G_i$. Suppose the network *is required* to use a MAC protocol that conforms to a fair-access criterion as defined next.

Fair-access Criterion Definition: *A MAC protocol used by the sensor network satisfies the fair-access criterion if all sensor nodes contribute equally to the network utilization, i.e., the following condition holds:*

$$G_1 = G_2 = \ldots = G_n \,. \tag{1}$$

Optimization Objective and Assumptions: Consider a sensor network such as described above. The central optimization problem is to maximize $U(n)$ under the fair-access criterion. In the rest of the paper, we investigate this problem under the following assumptions:

a. All data frames are of the same size.

b. All sensor nodes have the same transmission capacity.

c. Acknowledgments are either implicit via piggyback or, if explicit, are out-of-band.

d. In-network processing is not used.

e. If two sensor nodes are within one-hop, one sensor node's transmission will interfere with the other's reception.

f. Propagation and processing delays are negligible.

III. Derivation of Utlization and Delay Bounds

In this section, we derive upper bounds on $U(n)$ and lower bounds on the minimum transmission delay, or time between samples, for two specific topologies, linear and 2-row grid, under the fair-access criteria. We first describe the details of the topologies. Then we present three theorems establishing the performance bounds. Finally, the proofs of the theorems are given for completeness.

Linear Topology: The topology is illustrated in Fig.1. There are n sensor nodes and a base station (BS) placed in a linear fashion. Assume the transmission range of each node is just one hop and the interference range is less than two hops. In other words, only neighboring nodes have overlapping transmission ranges. As shown in Fig.1, O_i generates sensor data frames and sends the frames to O_{i+1}. O_i also relays data frames received from O_{i-1} to O_{i+1}. Finally, O_n forwards data to the BS, which collects all the data frames.

2-row Grid Topology: The 2-row grid topology is illustrated in Fig.2. The transmission ranges are such that horizontal or vertical neighbors can hear each other but two diagonal neighbors cannot. Two different routing patterns are considered: (i) the two rows forward data frames independently, as illustrated in Fig 2a, or (ii) the bottom sensors forward data to the top row first, as illustrated in Fig. 2b. The results for this grid can be extended to grids of larger depth, in terms of rows, but such results are not included here due to limited space.

Theorem 1: *For the linear topology, under fair-access, $U(n)$ is upper bounded by the optimal utilization, $U_{opt}(n)$:*

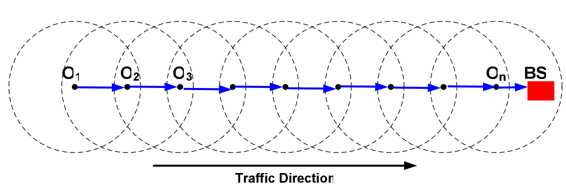

Fig. 1 A linear topology

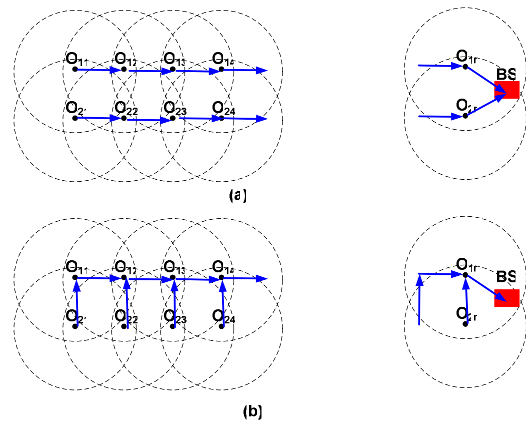

Fig. 2 Grid topology with two rows of sensors

$$U(n) \le U_{opt}(n) = \begin{cases} n/[3(n-1)], & n > 1 \\ 1, & n = 1 \end{cases} \tag{2}$$

An asymptotic lower limit for the optimal utilization exists and is $\frac{1}{3}$.

Moreover, the inter-sample time for each node, denoted by $D(n)$, is lower bounded by the minimum effective transmission delay for a node, or minimum cycle time, $D_{opt}(n)$:

$$D(n) \ge D_{opt}(n) = \begin{cases} 3(n-1)T, & n > 1 \\ T, & n = 1 \end{cases} \tag{3}$$

where T is the transmission time of one data frame.

.

Theorem 2: *For the 2-row grid topology with the routing pattern as illustrated in Fig.2a, under fair-access, $U(2n)$ is upper bounded by the optimal utilization, $U_{opt}(2n)$:*

$$U(2n) \le U_{opt}(2n) = n/(2n-1) \tag{4}$$

The asymptotic lower limit for the optimal utilization is $\frac{1}{2}$.

Moreover, $D(2n)$ is lower bounded by the minimum inter-sampling time, $D_{opt}(2n)$:

$$D(2n) \ge D_{opt}(2n) = \begin{cases} 2(2n-1)T, & n > 1 \\ T, & n = 1 \end{cases} \tag{5}$$

where T is the transmission time of one data frame.

Theorem 3: *For the 2-row grid topology with the routing pattern as depicted in Fig.2b, under the fair-access criterion, $U(2n)$ is upper bounded by the optimal utilization, $U_{opt}(2n)$:*

$$U(2n) \le U_{opt}(2n) = \begin{cases} n/(3n-2), & n > 2 \\ 1/2, & n = 2 \\ 2/3, & n = 1 \end{cases} \tag{6}$$

The asymptotic lower limit for the optimal utilization is $\frac{1}{3}$.

Moreover, $D(2n)$ is lower bounded by the minimum transmission delay, or time between samples, $D_{opt}(2n)$:

$$D(2n) \geq D_{opt}(2n) = 2(3n-2)T \quad (7)$$

where T is the transmission time of one data frame.

The significance of Theorems 1-3 is that they provide optimal bounds on utilization, independent of the MAC protocol employed. In other words, no matter which MAC protocol is used, whether contention-free (TDMA, token passing, etc.) or contention-based (CSMA, aloha, etc.), as long as the protocol conforms to the fair-access criterion, the bounds hold. To prove optimality, we must prove (i) the bounds hold for any fair-access conforming MAC protocol, and (ii) the bounds are indeed achievable by at least one protocol. We prove the former below and the latter in Section IV. Note that there are *n* nodes in Fig. 1, but *2n* nodes in Fig.2, as reflected in the notation for the network utilization and minimum inter-sample time, or transmission delay.

Before showing the actual proofs, let us provide some intuition behind them. The fair-access criterion requires that $G_1 = G_2 = ... = G_n$ for the linear network and $G_{12} = G_{21} = G_{13} = ... = G_{2n}$ for both of the grid networks. Let *x* denote the time period during which the BS successfully receives *at least* one original data frame from each sensor node in the network. It is clear that *x* is a random variable, but we can derive the minimum value of *x*, and when the minimum value of *x* is achieved, the maximum utilization is also achieved. During the time period *x*, the BS has busy time (denoted as *b*) receiving frames and idle time (denoted as *y*) while it is either blocked or waiting for its upstream neighbor to send. Thus, $x = b + y$. Note that *x* is the cycle time for the network under the fair-access criteria and determines the effective transmission delay for a node for a static ordering of relayed frames. For discussion purposes, we use a frame and the time period of transmitting/receiving a frame interchangeably in the following proofs. Since we assume no particular MAC protocol, frames may be lost, corrupted, or delayed due to collisions or queuing.

Proof of Theorem 1: 1) **<u>For n>2</u>**: During the time period *x*, the BS needs to receive at least *n* frames from O_n since frames may be lost or delayed as noted. Thus, O_n transmits at least *n* frames (including *n-1* relayed frames and one of its generated frames). We have $b \geq nT$. Likewise, in order for O_n to receive (*n-1*) frames from O_{n-1}, O_n needs to listen to at least (*n-1*) frames, during which the BS must be idle. Furthermore, when O_{n-2} transmits, O_n cannot transmit since they are within two-hops, i.e., O_n's transmissions will interfere with the frame reception by O_{n-1} from O_{n-2}. O_{n-2} needs to transmit at least (*n-2*) frames to O_{n-1}, during which O_n cannot transmit. So, the total time when O_n cannot transmit is $y \geq (n-1)T + (n-2)T$. Therefore, we have $x = b + y \geq nT + (n-1)T + (n-2)T$. Since $D(n) = x$, we have derived equation (3) for the case of *n>2*. During the time period *x*, the BS may receive more than *n* frames, but only *n* frames can be counted into the utilization under the fair-access criterion. Since we must minimize *x* to achieve the optimal utilization, we have

$$U(2n) = \frac{nT}{x} \leq \frac{nT}{nT + (n-1)T + (n-2)T} = \frac{n}{3(n-1)},$$

which proves equation (2) for the case of *n>2*.

Since $\lim_{n\to\infty} n/[3(n-1)] = 1/3$, $1/3$ is the asymptotic lower limit for the optimal utilization.

<u>For n=2</u>: Since we want $G_1 = G_2$, during the time period *x*, O_2 transmits at least two frames (one relayed frame and its own). We have $b \geq 2T$. O_2 needs to listen to at least one frame from O_1. We have $y \geq T$ and thus $x = b + y \geq 3T$. Since $D_{opt} = x$, we have derived equation (3). Since we must minimize *x* to achieve the optimal utilization, $U(n) = \frac{2T}{x} \leq \frac{2T}{3T} = \frac{2}{3}$, which proves equation (2) for this case.

<u>For n=1</u>: Obviously, $U(1) \leq 1$, and $D(1) \geq T$. #

Proof of Theorem 2: 1) **<u>For n>2</u>**: under the fair-access criterion, during the time *x*, the BS needs to receive at least *n* frames from O_{1n} since frames can collide, be corrupted, or be delayed, i.e., O_{1n} transmits at least *n* frames (including $n-1$ relayed frames and one of its generated frames) to the BS. Likewise, O_{2n} transmits at least *n* frames to the BS. We have $b \geq 2nT$. In order for O_{1n} to receive $n-1$ frames from $O_{1(n-1)}$, and for O_{2n} to receive $n-1$ frames from $O_{2(n-1)}$, O_{1n} and O_{2n} need to listen for at least $2(n-1)$ frames. Note that when $O_{1(n-2)}$ transmits, O_{1n} cannot transmit but O_{2n} can transmit. Similarly, when $O_{2(n-2)}$ transmits, O_{2n} cannot transmit but O_{1n} can. So, the total time when neither O_{1n} nor O_{2n} can transmit is $y \geq 2(n-1)T$. Thus, we have $x = b + y \geq 2nT + 2(n-1)T$. Since $D_{opt} = x$, we have derived equation (5) for this case. During the time *x*, the BS may receive more than $2n$ frames, but only $2n$ frames can be counted in the utilization under the fair-access criterion. To achieve the optimal utilization, we minimize *x*, yielding

$$U(2n) = \frac{2nT}{x} \leq \frac{2nT}{2nT + 2(n-1)T} = \frac{n}{(2n-1)}.$$

The rest of the proof is omitted for brevity. #

Proof of Theorem 3: 1) **<u>For n>2</u>**: under the fair-access criterion, during the time *x*, the BS needs to receive at least *2n* frames from O_{1n}, as above. We have $b \geq 2nT$. In order for O_{1n} to receive $2(n-1)$ frames from $O_{1(n-1)}$ and one frame from O_{2n}, O_{1n} must listen for at least $2(n-1)+1$ frames. Furthermore, when either $O_{1(n-2)}$ or $O_{2(n-1)}$ transmits, O_{1n} cannot transmit. O_{1n-2} must transmit at least $2(n-2)$ frames and O_{2n-1} must transmit at least one frame (if frames collide, are corrupted, or delayed more frames are needed). Thus, we have $y \geq 2(n-1) + 1 + 2(n-2) + 1 = 2(2n-2)$. During this time the BS may receive more than *2n* frames, but only $2n$ frames can be counted into the utilization under the fair-access criterion. Minimizing x to achieve the optimal utilization, yields

$$U(2n) \le \frac{2n}{2n+2(2n-2)} = \frac{n}{3n-2}.$$

The rest of the proof is omitted for brevity. #

IV. Bound Achievability via Optimal Fair Scheduling

In this section we prove that the performance bounds introduced in Theorems 1-3 are indeed achievable. Particularly, we present a TDMA scheduling algorithm that conforms to the fair-access criterion and show that it achieves the performance bounds. Note that herein the optimal utilization is under the constraint of the fair-access criterion. Otherwise, by simply allowing *only* O_n to transmit, the optimal utilization is 1. Recall that we assume a fixed data frame size and negligible propagation and processing delays. Thus, for the following discussion we divide the time into equal-duration timeslots with the duration being the time of transmitting one frame. The TDMA algorithm, which we term *optimal fair scheduling,* is described below.

Optimal Fair Scheduling for Linear Topology: Three tables containing the optimal schedules for the cases of n = 1, 2, or 3, respectively, are shown in Fig 3. Each row of the tables depicts node actions at a specific time slot. For example as shown in the table of Fig.3(b): at slot 1, O_1 transmits while O_2 receives and the BS is idle; at slot 2, O_2 relays the frame received in the previous slot to the BS; etc. It is straightforward to show these schedules achieve the bounds for the cases of n = 1, 2, or 3, respectively.

For the general case of $n>3$, let $d = D_{opt} = 3(n-1)$. A schedule with cycle d can be created as follows. O_1 transmits in timeslots $(d \cdot j)+1; j = 0,1,...$; O_i ($i=2,..,n$) transmits relayed frames to O_{i+1} in timeslots from $(d \cdot j)+f(i)$ through $(d \cdot j)+f(i)+i-2$, and transmits one of its own frames to O_{i+1} in timeslot $(d \cdot j)+f(i)+i-1; j = 0,1,...$, where $f(i)$ is recursively defined as follows:

$$f(i) = \begin{cases} 1, & i = 1 \\ f(i-1)+(i-1), & i > 1 \end{cases} \tag{8}$$

An example of this schedule for the case of $n=7$ is illustrated in Table 1. The cycle period is $3\cdot(7-1)=18$ timeslots. The utilization of the BS is $7/18$ by tallying the number of "G" slots in each cycle, which is consistent with Theorem 1. To understand the schedule better, let us consider node O_7. It must be silent while its 1-hop and 2-hop neighbors, O_5 and O_6, each transmit. In one cycle, O_5 transmits in 5 slots and O_6 transmits in 6 slots, resulting in the 11 slots during which O_7 must be silent.

The proof of the schedule's optimality for arbitrary n is omitted for brevity.

Optimal Fair Scheduling for Fig 2a Grid Topology: Before considering a general case, we first consider some simple cases where n is small. A schedule is illustrated in

Table 1: optimal Schedule for 7-NODE Linear Topology
(Legend: R: relay traffic; T: transmit own traffic;
L: Listening or Receiving: G: received at BS)

	O_1	O_2	O_3	O_4	O_5	O_6	O_7	BS
t+1	T	L	L	L	L	R	L	
t+2	L	R	L	L	L	R	L	
t+3	L	T	L	L	L	T	L	
t+4	L	L	R	L	L	L	R	G
t+5	L	L	R	L	L	L	R	G
t+6	L	L	T	L	L	L	R	G
t+7	L	L	L	R	L	L	R	G
t+8	L	L	L	R	L	L	R	G
t+9	L	L	L	R	L	L	R	G
t+10	L	L	L	T	L	L	T	G
t+11	L	L	L	L	R	L	L	
t+12	L	L	L	L	R	L	L	
t+13	L	L	L	L	R	L	L	
t+14	L	L	L	L	R	L	L	
t+15	L	L	L	L	T	L	L	
t+16	L	L	L	L	L	R	L	
t+17	L	L	L	L	L	R	L	
t+18	L	L	L	L	L	R	L	
t+19	T	L	L	L	L	R	L	

	O_1	BS
1	T	G
2	T	G

(a) n=1

	O_1	O_2	BS
1	T	L	
2	L	R	G
3	L	T	G
4	T	L	
5	L	R	G
6	L	T	G

(b) n=2

	O_1	O_2	O_3	BS
1	T	L	L	
2	L	R	L	
3	L	T	L	
4	L	L	R	G
5	L	L	R	G
6	L	L	T	G
7	T	L	L	
8	L	R	L	
9	L	T	L	
10	L	L	R	G
11	L	L	R	G
12	L	L	T	G

(c) n=3

Fig. 3 Optimal schedules for small linear topologies (Legend: R: relay traffic; T: transmit own traffic; L: listening or receiving: G: frame received at BS)

	O_{11}	O_{21}	BS
1	T	L	G
2	L	T	G
3	T	L	G
4	L	T	G

(a) n=1

	O_{11}	O_{12}	O_{21}	O_{22}	BS
1	T	L	L	L	
2	L	R	L	L	G
3	L	T	L	L	G
4	L	L	T	L	
5	L	L	L	R	G
6	L	L	L	T	G

(b) n=2

Fig.4 Optimal schedules for small Fig 2a grid networks

Fig. 4a for *n=1.* The utilization is 1. With *n=2*, when O_{11} transmits, O_{12}, O_{21}, and O_{22} cannot transmit; a schedule is illustrated in Fig. 4b. The utilization is $2/3$. These are consistent with Theorem 1 and, thus, are optimal.

Intuitively, if we let the first row perform a linear schedule first, followed by the second row, the utilization can be at least as good as that of the linear topology in equation (2) for the same number of nodes. This is not optimal, however. Note that while O_{11} and O_{13} cannot transmit at the same time, O_{21} and O_{13} can. Therefore, the utilization can be improved. Table 2 provides an improved scheduling for *n=7*. It reduces the transmission cycle from 36 (i.e., $2[3(n-1)]$), as in Table 1, to only 26. The utilization is $7/13$, which is consistent with Theorem 2, and is thus optimal. As $n \to \infty$, the utilization goes to $1/2$, which is better than that for a simple linear topology (Theorem 1), due to parallelism in the transmission.

Optimal Fair Scheduling for Fig.2b: We first consider some simple cases, where n is small. For Fig.3b, with $n=1$, one scheme is shown in Fig. 5a. The utilization is $2/3$. For $n=2$, when O_{21} transmits O_{12}, O_{11}, and O_{22} cannot. One possible scheme is shown in Fig. 5b. The utilization is $1/2$. With $n=3$, the only nodes that can transmit at the same time are O_{21} and O_{23}. One scheme is shown in Fig. 5c and the utilization is $3/7$. Each of these is consistent with Theorem 2.

Now consider the general case. To fully utilize parallel transmissions, in the first slot, we let $O_{2(2j+1)}$ (j=0,..,n-1) transmit, and in the second slot, we let $O_{2(2j+2)}$ (j=0,..,n-1) transmit. For the remainder of the cycle the second row waits while the first row forwards the traffic to the BS. This portion is simply a linear topology with double loads.

Therefore, the achievable utilization is $\dfrac{2n}{2n+2(n-1)+2(n-2)+2}=\dfrac{n}{3n-2}$, which is consistent with Theorem 3. Since the bound is achievable it is optimal. We can verify Fig.5 when n=1, 2, or 3. Interestingly, when $n \to \infty$, the asymptotic limit for the upper bound of the optimal utilization is $1/3$, which is less than $1/2$, the bound for traffic forwarded across the rows first, as in Fig.2a.

The optimal scheduling algorithm introduced above, while TDMA in nature, can be implemented without global clock synchronization. This is because a node's reception of a frame originated by its immediate upstream neighbor triggers that node's own transmission for the same cycle, thereby achieving self-clocking.

TABLE 2: OPTIMAL SCHEDULE FOR FIG 2A TOPOLOGY (N=7)

	O_{11}	O_{12}	O_{13}	O_{14}	O_{15}	O_{16}	O_{17}	O_{21}	O_{22}	O_{23}	O_{24}	O_{25}	O_{26}	O_{27}	BS
t+1	T	L	L	L	L	L	R	L	L	L	L	R	L	L	G
t+2	L	R	L	L	L	L	T	L	L	L	L	T	L	L	G
t+3	L	T	L	L	L	L	L	L	L	L	L	L	R	L	
t+4	L	L	R	L	L	L	L	L	L	L	L	L	R	L	
t+5	L	L	R	L	L	L	L	L	L	L	L	L	R	L	
t+6	L	L	T	L	L	L	L	L	L	L	L	L	R	L	
t+7	L	L	L	R	L	L	L	L	L	L	L	L	R	L	
t+8	L	L	L	R	L	L	L	L	L	L	L	L	T	L	
t+9	L	L	L	R	L	L	L	L	L	L	L	L	L	R	G
t+10	L	L	L	T	L	L	L	L	L	L	L	L	L	R	G
t+11	L	L	L	L	R	L	L	L	L	L	L	L	L	R	G
t+12	L	L	L	L	R	L	L	L	L	L	L	L	L	R	G
t+13	L	L	L	L	R	L	L	L	L	L	L	L	L	R	G
t+14	L	L	L	L	R	L	L	T	L	L	L	L	L	R	G
t+15	L	L	L	L	T	L	L	L	R	L	L	L	L	T	G
t+16	L	L	L	L	L	R	L	L	T	L	L	L	L	L	
t+17	L	L	L	L	L	R	L	L	L	R	L	L	L	L	
t+18	L	L	L	L	L	R	L	L	L	R	L	L	L	L	
t+19	L	L	L	L	L	R	L	L	L	T	L	L	L	L	
t+20	L	L	L	L	L	R	L	L	L	L	R	L	L	L	
t+21	L	L	L	L	L	T	L	L	L	L	R	L	L	L	
t+22	L	L	L	L	L	L	R	L	L	L	R	L	L	L	G
t+23	L	L	L	L	L	L	R	L	L	L	T	L	L	L	G
t+24	L	L	L	L	L	L	R	L	L	L	L	R	L	L	G
t+25	L	L	L	L	L	L	R	L	L	L	L	R	L	L	G
t+26	L	L	L	L	L	L	R	L	L	L	L	R	L	L	G
t+27	T	L	L	L	L	L	R	L	L	L	L	R	L	L	G

	O_{11}	O_{21}	BS
1	L	T	
2	R	L	G
3	T	L	G

(a) n=1

	O_{11}	O_{12}	O_{21}	O_{22}	BS
1	L	L	T	L	
2	R	L	L	L	
3	T	L	L	L	
4	L	L	L	T	
5	L	R	L	L	G
6	L	R	L	L	G
7	L	R	L	L	G
8	L	T	L	L	G

(b) n=2

	O_{11}	O_{12}	O_{13}	O_{21}	O_{22}	O_{23}	BS
1	L	L	L	T	L	T	
2	R	L	L	L	L	L	
3	T	L	L	L	L	L	
4	L	L	L	L	T	L	
5	L	R	L	L	L	L	
6	L	R	L	L	L	L	
7	L	R	L	L	L	L	
8	L	T	L	L	L	L	
9	L	L	R	L	L	L	G
10	L	L	R	L	L	L	G
11	L	L	R	L	L	L	G
12	L	L	R	L	L	L	G
13	L	L	R	L	L	L	G
14	L	L	T	L	L	L	G

(c) n=3

Fig.5 Optimal schedules for small Fig. 2b grid networks

V. TRAFFIC LOAD AND SENSOR DATA SAMPLING LIMITS

This section addresses the impact of end-to-end performance bounds on the traffic load limitation of each sensor. Let ρ denote the traffic load generated by each sensor node. For Fig. 1, Fig.2a, and Fig. 2b networks, since each node can transmit at most one original frame, which requires a period of T in every $3(n-1)T$ time period, $2(2n-1)T$ time period, and $2(3n-2)T$ time period, respectively, then, we must have that $\rho \le T/x = 1/[3(n-1)]$, $\rho \le T/x = 1/\left[2\left(2n-1\right)\right]$, and $\rho \le T/x = 1/\left[2\left(3n-2\right)\right]$, respectively, if $n>2$. Furthermore, a data frame contains protocol overhead (because of header and/or trailer). Thus, ρ must be adjusted to account for this overhead. Denote α to be the fraction of actual data bits in a frame. We have the following three theorems.

***Theorem 4:**, For the linear topology illustrated in Fig,1, under the fair-access criterion, the maximum feasible per node traffic load is*

$$\frac{\alpha}{3(n-1)}, \text{ if } n>2 \tag{9}$$

***Theorem 5:** For the 2-row grid topology depicted in Fig. 2a, under the fair-access criterion, the maximum feasible per node traffic load is*

$$\frac{\alpha}{2(2n-1)}, \text{ if } n>2 \tag{10}$$

***Theorem 6:** For the 2-row grid topology depicted in Fig. 2b, under the fair-access criterion, the maximum feasible per node traffic load is:*

$$\frac{\alpha}{2(3n-2)}, \text{ if } n>2\,. \tag{11}$$

These three theorems not only tell us the traffic limitation of the sensor network, but also provides lower bounds on the average sensor sampling rate/intervals, i.e., the minimum supportable time T/ρ between samples. The proofs are omitted.

VI. PERFORMANCE EVALAUTION

In this section, we provide some projected performance for various sized linear and 2-row grid topologies. Note that for this section, the optimal utilizations have been multiplied by α, which is the fraction of actual data bits in a frame, to account for protocol overhead.

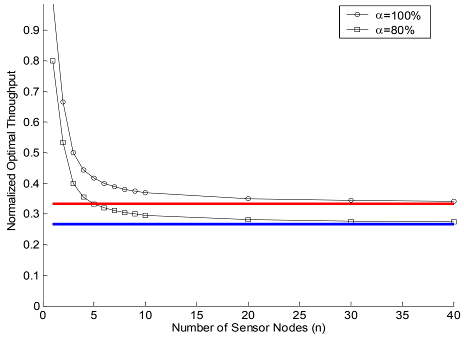

Fig 6 Optimal Utilization (linear topology)

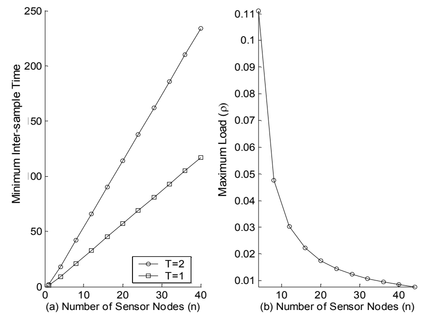

Fig. 7 Minimum Cycle Time and Maximum per Node Load (linear topology)

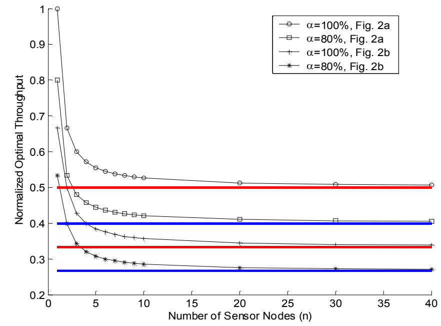

Fig. 8 Optimal Utilization (2 row grid)

A. Linear Topology

Fig.6 shows the optimal utilization vs. number of nodes for different α values for the basic linear topology based on the bounds of Theorem 1. The optimal utilization decreases quickly as n increases and approaches the asymptotic lower limit of optimal utilization, as suggested by the theorem. When $n = 5$, the optimal utilization is already near the asymptotic bound, which is indicated by the horizontal, colored lines.

Figure 7 shows the more significant impact of increasing the network size.. The effective transmission delay for each node increases linearly with n, as shown in Fig. 7a. The traffic limit, per sensor node, decreases quickly as n increases, as shown in Fig. 7b, approaching the asymptotic limit of zero.

B. Grid Topology

Fig. 8 shows the optimal utilization vs. n with different α values for the two-row topologies of Fig. 2, as derived from Theorems 2 and 3. Fig.8 shows that the topology of Fig. 2a may achieve much higher utilization than the topology of Fig. 2b. The delay and load characteristics of the two-row grid topology are illustrated by Fig. 9.

C. Linear Topology vs. 2-row Grid

Fig. 10 compares the optimal utilization of the linear topology of Fig. 1 with that of the horizontal-first-forwarding, 2-row grid of Fig. 2a. It is noteworthy that the optimal utilization of the Fig. 2a topology is better than that of Fig. 1, due to parallel transmissions of diagonal neighbors. This suggests that a 2-row grid may be preferable to a linear topology for some applications where a linear topology might have been the first consideration. This issue is left for further study. Note, however, that the vertical-first grid (Fig. 2b) actually performs worse, albeit insignificantly, than the linear topology, in terms of network utilization.

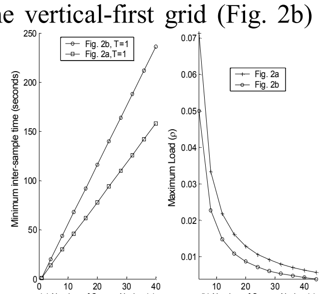

Fig. 9 Min Cycle Time and Max per Node Load (2-row grid)

VII. Conclusion

In this paper, we explored fundamental limits for sustainable loads, utilization, and delays in specific multi-hop sensor network topologies. We derived upper bounds on network utilization and lower bounds for minimum sample time in fixed linear and two-row grid topologies, under the fair-access criterion. This fair-access criterion ensures the data of all sensors is equally capable of reaching the base station. We proved that under some conditions/assumptions, these bounds are achievable, and therefore optimal. From the limitation on the sustainable traffic loads derived, one can determine a lower bound for the sampling interval for such networks. The significance of these limits is that these bounds are independent of the selection of MAC protocols. Thus, the performance bounds for specific implementations of such network topologies can be explicitly determined to ensure the proposed networks are capable of satisfying the networks' specified utilization and delay requirements. Further, a self-clocking implementation was described that achieves the performance bounds.

References

[1] B. Benson, G. Chang, D. Manov, B. Graham, and R. Kastner, "Design of a Low-cost Acoustic Modem for Moored Oceanographic Applications," Proc. of WUWNet'06, September 25, 2006, Los Angeles, California,USA.

[2] Ramaswami, R. and K.K. Parhi, Distributed Scheduling of Broadcasts "in a radio network," *Proceedings of IEEE INFOCOM* (1989).

[3] Ramanathan S., "A unified framework and algorithm for channel assignment in wireless network" *Wireless Networks*, Volume 5, Number 2, 1999, pp. 81-94(14)

[4] V. Rajendran, K. Obraczka, and J.J. Garcia-Luna-Aceves: "Energy-efficient, collision-free medium access control for wireless sensor networks," in *Proc.ACM Conference on Embedded Networked Sensor Systems (Sensys 2003)*, PP. 181–192 (Nov. 2003)

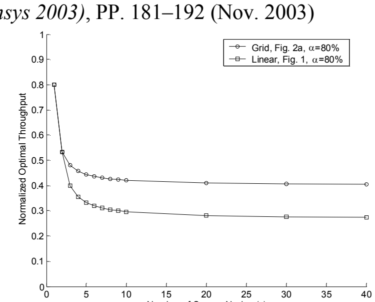

Fig. 10 Optimal Utilization (linear vs. 2-row grid of Fig. 2a)