Abstracts for the graph theory and applications session at Mathfest, Portland, OR

August 6, 2009

Meeting room: SALON D

Co-chairs: Ralucca Gera (talks 1-9) and Gary MacGillivray (talks 10-17)

1. (1:00pm) Set Colorings in graphs, by Ralucca M Gera
   Let $G = (V(G), E(G))$ be a connected graph, and let $c : V(G) \rightarrow \{1, 2, \ldots, n\}$ be a vertex coloring (not necessarily a proper one). The neighborhood color set $NC(u)$ is the set of colors of the neighbors of $u$. We then define a set coloring to be a coloring in which $N(u) \neq N(v)$, for every pair $u, v \in V(G)$ for which $uv \in E(G)$. This concept was introduced by Chartrand, Okamato, Rasmussen, and Zhang. We present results on the set coloring.

2. (1:20pm) THE TALK “Maximal Irregular Colorings of Regular Graphs” HAS BEEN MOVED TO 5PM. NEW TALK:
   On The Number of Vertices of a Tashkinov Tree, by Oguz Kurt
   In optimization problems, the fractional solution is usually found in polynomial time while the integer solution many times ends up NP-complete. In 1967, Gupta and later Goldberg asked the following question for a multi-graph $G$: "What is the smallest function $f(\Delta)$ of $\Delta = \Delta(G)$, the maximum degree of a graph, such that if $\chi > f(\Delta)$ then $|\chi(G) - \chi^*(G)| < 1|$, where $\chi$ and $\chi'$ denote the chromatic index and fractional chromatic index of $G$?". They also conjectured that $f(\Delta) = \Delta + 1$. In 2000, Tashkinov found a very nice tree algorithm that finds a subgraph of $G$ which almost satisfies the conjecture their conjecture. Since then the so-called Goldberg-Gupta Conjecture is studied more than its earliest days. In 2006, Scheide showed that "If the number of vertices, $t(G)$, of a largest Tashkinov tree in $G$ is less than 11 then $G$ is elementary" meaning that Goldberg-Gupta Conjecture holds. We show that $G$ is elementary whenever $t(G) < 13$ improving the result of Scheide. Moreover, together with another result of ours, this gives the best known function $g(\Delta) = \frac{19}{18}\Delta(G) + \frac{16}{18}$ such that if $\chi' > g(\Delta)$ then $|\chi(G) - \chi^*(G)| < 1|$. 


3. (1:40pm) Evolutionary games on graphs, by Stephen Devlin
Evolving biological systems are often modeled in the context of the social structure of the population, which is represented by a (usually big) graph. We will give examples of such models that are based on game theory, and show how various graph parameters can be used to effectively predict the dynamics of the system.

4. (2:00pm) A Counter Example for a Chromatic Uniqueness Theorem, by Abdul Jalil M Khalaf and Yee-Hock Peng
Let \( P(G, \lambda) \) denote the chromatic polynomial of a graph \( G \). Two graphs \( G \) and \( H \) are chromatically equivalent, written \( G \sim H \), if \( P(G, \lambda) = P(H, \lambda) \). A graph \( G \) is chromatically unique written \( \chi - \)unique, if for any graph \( H \), \( G \sim H \) implies that \( G \) is isomorphic with \( H \). In this paper we introduce a counter example against one chromatic uniqueness theorem of 5-bridge graphs.

5. (2:20pm) A Closure for Claw-free Graphs, by Bill Linderman
A graph is said to be claw-free if it does not contain a copy of \( K_{1,3} \) as an induced subgraph. Ryjáček has described a closure for claw-free graphs which preserves the length of the longest cycle. Thus, a claw-free graph with a complete closure of this type is hamiltonian. The closure is also a line graph for some graph. We examine this closure for claw-free graphs and some results related to hamiltonicity and line graphs, including a result on the minimum number of edges a graph can have if it has a complete closure of this type.

6. (2:40pm) Edge Geodetic Covers in the Cartesian Product of Graphs, by Rochelleo Esios Mariano
For any two vertices \( u \) and \( v \) of a graph \( G = (V(G), E(G)) \), the set \( I_G^e[u, v] \) consists of all edges of \( G \) lying in any \( u-v \) geodesic in \( G \). If \( S \subseteq V(G) \), then the set \( I_G^e[S] \) denotes the union of all \( I_G^e[u, v] \), where \( u, v \in S \). A subset \( S \) of \( V(G) \) is an edge geodetic set or an edge geodetic cover of \( G \) if every edge of \( G \) is contained in \( I_G^e[S] \), that is, \( I_G^e[S] = E(G) \). The edge geodetic number of \( G \), denoted by \( g_e(G) \), is the minimum cardinality of an edge geodetic cover of \( G \). Any edge geodetic cover of \( G \) of cardinality \( g_e(G) \) is called an edge geodetic basis of \( G \). If \( G \) is a graph and \( S = \{x_1, \ldots, x_k\} \) an edge geodetic set of \( G \), then \( S \) is a linear edge geodetic set of \( G \) if for any edge \( e \in E(G) \), there exists an \( i, 1 \leq i < k \), such that \( e \in I_G^e[x_i, x_{i+1}] \). Moreover, a set of vertices \( S \) in a graph \( G \) is a double edge geodetic set of \( G \) if for any pair of edges, \( ab, xy \in E(G) \), there exist \( p, q \in S \) such that both \( ab, xy \in I_G^e[p, q] \).

In this paper, we established the lower and upper bounds of the edge geodetic covers of the cartesian product of two connected graphs \( G \) and \( H \) where these bounds are sharp. In particular, if \( G \) and \( H \) are graphs with \( g_e(G) = p \geq g_e(H) = q \geq 2 \), then \( g_e(G \times H) \leq pq - q \) while if \( G \) and \( H \) are connected graphs, then \( \max\{g_e(G), g_e(H)\} \leq g_e(G \times H) \). If we let \( G \) and \( H \) be connected graphs with \( g_e(G) = p \) and \( g_e(H) = q \) and if both \( G \) and \( H \) contain linear minimum edge geodetic covers, then \( g_e(G \times H) \leq \left\lfloor \frac{pq}{2} \right\rfloor \). For any trees \( T_1 \)}
and \( T_2, g_e(T_1 \times T_2) = \max\{g_e(T_1), g_e(T_2)\} \). Lastly, if a graph \( G \) has a double minimum edge geodetic cover, then \( g_e(G \times G) = g_e(G) \).

7. (3:00pm) **Turning Lights Out, by Crista Arangala**
The Tiger Electronics hand-held Lights Out toy is traditionally played on a board of 25 buttons set up in a 5 x 5 grid. Each button can start in an on or off state and the object of the game is to get all of the lights off by pressing the different buttons. However each time a button is pressed, it not only changes it’s state, but it also changes the state of all adjacent buttons. This problem consumes hours of the average players time but with this module, one can determine solutions of any binary Lights Out game. In this talk the history behind the mathematics of Lights Out and a unique graph theory approach, to show that every Lights Out toy has a solution when all buttons start in the on state will be presented.

8. (3:20pm) **Graph Theory Applications in Code Analyses and Developing Software Test Strategies by Vladimir Riabov**
The graph theory is effectively used in analyses of logical complexity of various computer-programming codes. It is shown that three branches of the graph theory (algebraic, predicative-logical, and topological approaches) provide simple algorithms for calculating the cyclomatic complexity of flowgraphs that represent the code structures. The main graph-based metrics (cyclomatic complexity, essential purely-structured-logic complexity, module design complexity, system design complexity, and system integration complexity parameters) are reviewed and applied for studying the C-programming code complexity and estimating the number of system integration and unit tests for two networking systems: Carrier Networks Support system with switches and routers, and Aggregation System for networking services. Comparing different code releases, it is found that the reduction of the code complexity leads to significant reduction of errors and maintainability efforts. The test and code coverage issues for these systems are also discussed.

9. (3:40pm) **Characterizations of cop-win graphs, by Nancy E. Clarke, and Gary MacGillivray**
The perfect information graph game “Cops and Robber” was introduced around 1980 by Quilliot, and Nowakowski and Winkler, independently. There are two sides: a collection of \( k > 0 \) cops and a single robber. The cops begin the game by each choosing a vertex to occupy. The robber then chooses a vertex and the two sides move alternately, with the cops moving first. A move for the cops consists of each cop traversing an edge to a neighbouring vertex, or remaining at his current vertex. A move for the robber is defined analogously. The cops win if any cop occupies the same vertex as the robber after a finite number of moves (some cop catches the robber), and otherwise the robber wins.

Both Quilliot, and Nowakowski and Winkler gave a characterization, in terms of a certain vertex ordering, of the finite graphs in which one cop has a winning
strategy. Nowakowski and Winkler also gave a relational characterization of these graphs that extends to the infinite case. We describe an extension of these characterizations to the case of $k$ cops.

10. (4:00pm) **Considering Symmetries of the Middle Levels (An interesting approach to a special case),** by Dov Zazkis

Let $k$ be a positive integer. We define $M_k$ to be the graph with a vertex set consisting of all binary strings of length $2k+1$ which have either $k$ or $k+1$ ones and edge set consisting of all pairs of these binary strings which differ in exactly one bit. Showing that the graph $M_k$ is Hamiltonian for all $k$ is known as the Middle Levels problem. This problem was first posed in the early 1980’s and to this day remains unsolved. In this paper we explore the symmetries of $M_k$ and graphs related to it. We then use these symmetries to propose a method for finding Hamiltonian cycles in $M_k$ when $2^k + 1$ and $k$ are prime. We believe that our method is more efficient than methods proposed by previous authors.

11. (4:20pm) **Extensions of Newman’s Conjecture and Applications to Prime Trees**, by Ben Small and Leanne Robertson

In 1980, Carl Pomerance and J. L. Selfridge proved D. J. Newman’s coprime mapping conjecture: If $n$ is a positive integer and $I$ is a set of $n$ consecutive integers, then there is a bijection $f : \{1, 2, \ldots, n\} \rightarrow I$ such that $\gcd(i, f(i)) = 1$ for $1 \leq i \leq n$. Around the same time, Roger Entringer conjectured that all trees are prime, that is, that if $T$ is a tree with vertex set $V$, then there is a bijection $L : V \rightarrow \{1, 2, \ldots, |V|\}$ such that $\gcd(L(x), L(y)) = 1$ for all adjacent vertices $x$ and $y$ in $V$. There has been little progress so far towards a proof of this conjecture. In this talk, I will discuss extensions of Newman’s conjecture and how they can be used to prove that various families of trees are prime, including palm trees, banana trees, binomial trees, and certain families of spider colonies.

12. (4:40pm) **Prism Complement Hamiltonian Graphs**, by Jonathan Adler, Nicholas LeCompte, and Peter Christopher

Let $G$ be a graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and let $\bar{G}$ be its complement with vertex set $\bar{V} = \{\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_n\}$. The complementary prism is defined to be the graph $GG$ with vertices $V \cup \bar{V}$ and with edges consisting of the edges in $G$ and $\bar{G}$ together with edges $\{v_i, \bar{v}_i\}$ for $i = 1, 2, \ldots, n$. $G$ is said to be **prism complement Hamiltonian** if $GG$ is Hamiltonian. We examine the classification of such graphs and consider how prism complement Hamiltonian relates to other traversibility problems.

13. (5:00pm) **Maximal Irregular Colorings of Regular Graphs**, by Mark Anderson, Richard Vitray, and Jay Yellen

An irregular coloring of a graph is a proper vertex coloring that distinguishes vertices in the graph either by their own colors or by the colors of their neighbors. In algebraic graph theory, groups with a certain amount of symmetry are
specified in terms of a group and a smaller graph (e.g. voltage graphs). Radcliffe and Zhang found a bound for the irregular chromatic number of a graph on \( n \) vertices. In this paper we create voltage graphs achieving that bound.

14. (5:20pm) **The Radio Number of Ladder Graphs, by Josefina Flores**
   The radio labeling of graphs originated from the real world problem of radio transmitter frequency assignment, which depends on distance between transmitters. For a connected graph \( G \), let \( d(u, v) \) denote the distance between any two vertices \( u \) and \( v \). The diameter, \( diam(G) \), is the longest distance in \( G \). A radio labeling \( c \) of \( G \) is an assignment of positive integer values to the vertices of \( G \) that satisfies \( d(u, v) + |c(u) - c(v)| \geq diam(G) + 1 \), for all vertex pairs \( u \) and \( v \). The maximum integer produced by the labeling is the span of the labeling. The radio number of \( G \), \( rn(G) \), is the minimum achievable span. Let \( L_n \) be a ladder graph with \( n \) rungs and \( 2n \) vertices. As stated by Liu and Zhu, “...determining the radio number seems a difficult problem even for some basic families of graphs.” We determine the radio number of ladder graphs.

15. (5:40pm) **Threshold Graphs, Linear Forests, and Hamiltonian Paths, by Jennifer Gorman**
   There are well known necessary and sufficient conditions for a Threshold graph to have a Hamiltonian Path. In this talk we will investigate Hamiltonian paths that contain a given linear forest. We will give necessary and sufficient conditions for determining if a threshold graph \( G \) contains a specified linear forest \( P \). We will also discuss an efficient algorithm to construct such a Hamiltonian path if one exists.

16. (6:00pm) **A methodology for converting a two-way road network to a one-way road network, by Sin-Chye Lew**
   It is not uncommon in the field of traffic engineering for two-way roads to be converted to one-way roads and vice versa. In these situations, the traffic engineer is faced with the problem of selecting directions for all the one-way roads so that a strongly-connected network can be achieved. This problem is named the one-way road network problem. Graph theory and minimum traverse-time algorithms provide a model of the network. This coupled with minimum cut graph model form a model referred to as the one-way road network model for solving the above-mentioned problem. A real-life case study is used for testing and validating the model.

   Two-way road networks are represented by undirected graphs and one-way road networks by digraphs. The problem then becomes one of converting an undirected graph to a strongly-connected digraph. A brute-force algorithm was initially used for obtaining feasible digraphs. In order to obtain a true-to-live, efficient one-way road network, the topological aspects of network efficiency incorporating realistic delay times at junctions is explored.

   Dijkstra’s algorithm is modified to calculate the network shortest total travel time in the search for a suitable initial road network layout. With the selected
road network layouts, flipping of the various one-way roads at identified critical junctions on the network is tested using a minimum-cut graph model. This allows efficient consideration of not only each direction but combinations of directions from different roads as well.

A theoretically optimal one-way road network for the given topology and traffic loading is obtained for the selected case study. The one-way road network is tested against the original two-way road network by means of simulation. Results of the simulation runs showed that the one-way road network performed better than the original two-way road network under given traffic load of the case study. This thus validates the one-way road network model.

17. (6:20pm) On Groups Admitting a Cayley Prism Mapping, by Kathryn Weld
A group $G$ is said to admit a Cayley prism mapping $\sigma$ provided that $\sigma$ is a bijection on $G$ and moreover, for each Cayley subset $S \subset G$, $\sigma$ induces an automorphism of the associated Cayley graph $\text{Cay}(G, S)$. The identity map on $G$ is a trivial example. In the case that $G$ is isomorphic to direct product of the quaternion group and an elementary abelian two group, the inversion map is an example of a prism map on $G$. In this paper we classify the groups that admitting a Cayley prism mapping which is neither the identity mapping nor the inversion mapping. In this paper we classify all groups that admit a Cayley prism mapping. (Preliminary report.)