2 Chapter 2: Determinants

2.2 Properties of Determinants

1. ELIMINATION METHOD: Finding determinant using row operations on a matrix \( A \):

   (a) \( B \) is obtained by interchanging two rows of \( A \) \( \Rightarrow \) \( \det(B) = -\det(A) \).

   (b) \( B \) is obtained by multiplying a row of \( A \) by \( \alpha \neq 0 \) \( \Rightarrow \) \( \det(B) = \alpha \cdot \det(A) \).

   (c) \( B \) is obtained by adding a multiple of a row of \( A \) to another \( \Rightarrow \) \( \det(B) = \det(A) \).

2. Table 1 compares the two methods (cofactor and elimination). Note that Elimination method is faster if \( n > 3 \).

3. \( A \) is singular \( \iff \) \( \det(A) = 0 \)

4. \( \det(AB) = \det(A) \cdot \det(B) \)

5. more relationships:

<table>
<thead>
<tr>
<th>matrix ( B ) obtained from matrix ( A ) through</th>
<th>relation for determinants</th>
</tr>
</thead>
<tbody>
<tr>
<td>swapping two rows</td>
<td>( \det B = -\det A )</td>
</tr>
<tr>
<td>multiplying a row by ( \alpha \neq 0 )</td>
<td>( \det B = \alpha \det A )</td>
</tr>
<tr>
<td>( B = A^T )</td>
<td>( \det B = \det A )</td>
</tr>
<tr>
<td>triangular</td>
<td>( \det B = \Pi_i a_{i,i} )</td>
</tr>
<tr>
<td>diagonal</td>
<td>( \det B = \Pi_i a_{i,i} )</td>
</tr>
<tr>
<td>has one row or column of 0</td>
<td>( \det B = 0 )</td>
</tr>
<tr>
<td>has two identical rows or columns of 0</td>
<td>( \det B = 0 )</td>
</tr>
<tr>
<td>singular</td>
<td>( \det B \neq 0 )</td>
</tr>
<tr>
<td>non-singular</td>
<td>( \det B = \det C \cdot \det D )</td>
</tr>
<tr>
<td>( B = C \cdot D )</td>
<td>( \det B = \frac{1}{\det A} )</td>
</tr>
<tr>
<td>( B = A^{-1} )</td>
<td>( \det B = \alpha^n \det A ) (where ( A ) and ( B ) are ( nn ))</td>
</tr>
<tr>
<td>( B = \alpha A )</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Caption